



# HAMMING CODE

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# INTRODUCTION

- A code is said to be linear if any two code words in the code can be added in modulo-2 arithmetic to produce a third codeword in the code.
- A codeword is divided into two parts, one of which occupied by the message bits and other by the parity bits.
- For an  $(n, k)$  linear block code,  $k$  bits of the  $n$  bits are message bits and  $n-k$  bits are parity bits.



Structure of codeword

# HAMMING DISTANCE AND HAMMING WEIGHT

- Consider a pair of code vectors  $c_1$  and  $c_2$  that have the same number of elements.
- The **Hamming distance**  $d(c_1, c_2)$  between such a pair of code vectors is defined as the number of locations in which their respective elements differ.
- The **Hamming weight**  $w(c)$  of a code vector  $c$  is defined as the number of nonzero elements in the code vector. We may state that the Hamming weight of a code vector is the distance between the code vector and the all-zero code vector.
- The minimum distance  $d_{\min}$  of a linear block code is defined as the smallest Hamming distance between any pair of code vectors in the code.
- We may state that the minimum distance of a linear block code is the smallest Hamming weight of the nonzero code vectors in the code.

# HAMMING CODES

- Hamming code is a set of error-correction codes that can be used to **detect and correct the errors** that can occur when the data is moved or stored from the transmitter to the receiver.
- This technique was developed by **R.W. Hamming** for error correction.
- **Parity bits** are extra binary bits that are generated and added to the information-carrying (message) bits of data transfer to ensure that no bits were lost during the data transfer.
- The number of parity bits can be calculated using the following formula:

Where  $p$  is the no. of parity bits and  $m$  is the no. of message bits  $2^p \geq p + m + 1$

- Here,  $P \geq 3$ , block length= $n=2^p-1$ , no. of message bits= $m=k=2^p-p-1$ , no. of parity bits= $p=n-k$  for block code of  $(n, k)$ .

# HAMMING CODES

- Hamming codes are single-error correcting binary perfect codes.
- No. of parity bits calculation formula

$$2^p \geq p + m + 1$$

Where p is the no. of parity bits and m is the no. of message bits

- All bit positions that are power of 2 are marked as parity bits (1, 2, 4, 8,.....,  $2^n$ ), other bits are message bits.
- So block code C(7,4) means total no. of bits in the codeword is 7. The no. of message bits=4 and no. of parity bits=7-4=3. message= $m_1m_2m_3m_4$  and parity bits are  $p_1, p_2, p_3$ .
- Hamming code format is  $p_1 p_2 m_1 p_3 m_2 m_3 m_4$ .
- Bit positions are 1 , 2, 3, 4, 5, 6, 7.

# GENERATION OF HAMMING CODE

- Generate the hamming code for the message 1110.
- Here, message bits= $m=4$ .
- To calculate no. of parity bits  $p$ , we have to use the formula  $2^p \geq p + m + 1$
- To satisfy above condition, the minimum value of  $p$  is 3. So, the no. of parity bits= $p=3$
- Parity bits are  $p_1, p_2, p_3$ .
- Hamming code is  $p_1 p_2 m_1 p_3 m_2 m_3 m_4$ . i.e.  $p_1 p_2 1 p_3 1 1 0$ .
- Now we have to find the value of parity bits i.e.  $p_1$  value,  $p_2$  value and  $p_3$  value.
- $p_1$  value,  $p_2$  value and  $p_3$  value must satisfy even parity or odd parity.

# HAMMING CODE GENERATION

- $P_1 \rightarrow 1, 3, 5, 7 \rightarrow P_1 = 1 \oplus 1 \oplus 0$ . To make even parity  $P_1 = 0$ . Because for even parity, the total of 1's present must be even.
- $P_2 \rightarrow 2, 3, 6, 7 \rightarrow P_2 = 1 \oplus 1 \oplus 0$ . To make even parity  $P_2 = 0$ . Because for even parity, the total of 1's present must be even.
- $P_3 \rightarrow 4, 5, 6, 7 \rightarrow P_3 = 1 \oplus 1 \oplus 0$ . To make even parity  $P_3 = 0$ . Because for even parity, the total of 1's present must be even.
- So, all parity bits are 0.
- Hence, Hamming code is 0 0 1 0 1 1 0 for even parity transmission.
- For odd parity transmission, Hamming code is 1111110.
- Since 3 parity bits are there, so total combination values, we have 000, 001, 010, 011, 100, 101, 110, 111.
- For  $P_1$ , we have taken which have 1 in 1<sup>st</sup> place in above combinations i.e. 1, 3, 5, 7.
- For  $P_2$ , we have taken which have 1 in 2<sup>nd</sup> place in above combinations i.e. 2, 3, 6, 7.
- For  $P_3$ , we have taken which have 1 in 3<sup>rd</sup> place in above combinations i.e. 4, 5, 6, 7.

# ERROR CORRECTION IN HAMMING CODING

C(7,4) hamming code is received as 1010111. Determine the correct code when even parity is there.

- Here, no. message bits=4, no. of parity bits=7-4=3.

1	0	1	0	1	1	1
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- Error place depends on the no. of parity bits. So , here error place= $E_3 E_2 E_1$ .
- $E_1 \rightarrow 1, 3, 5, 7 \rightarrow 1 1 1 1$ , To make even parity, here  $E_1=0$  , as four 1's are present.
- $E_2 \rightarrow 2, 3, 6, 7 \rightarrow 0 1 1 1$ , To make even parity, here  $E_2=1$  , as three 1's are present.
- $E_3 \rightarrow 4, 5, 6, 7 \rightarrow 0 1 1 1$ , To make even parity, here  $E_2=1$  , as three 1's are present.

## CORRECT HAMMING CODE

- Here, error place can be defined as  $E_3 E_2 E_1$  i.e. 1 1 0 whose decimal equivalent is 6.
- So, in 6<sup>th</sup> bit there is an error. Since '1' is received in 6<sup>th</sup> bit, the corrected bit value in 6<sup>th</sup> bit is 0.
- Hence, the corrected Hamming code is 1 0 1 0 1 0 1.



THANK YOU