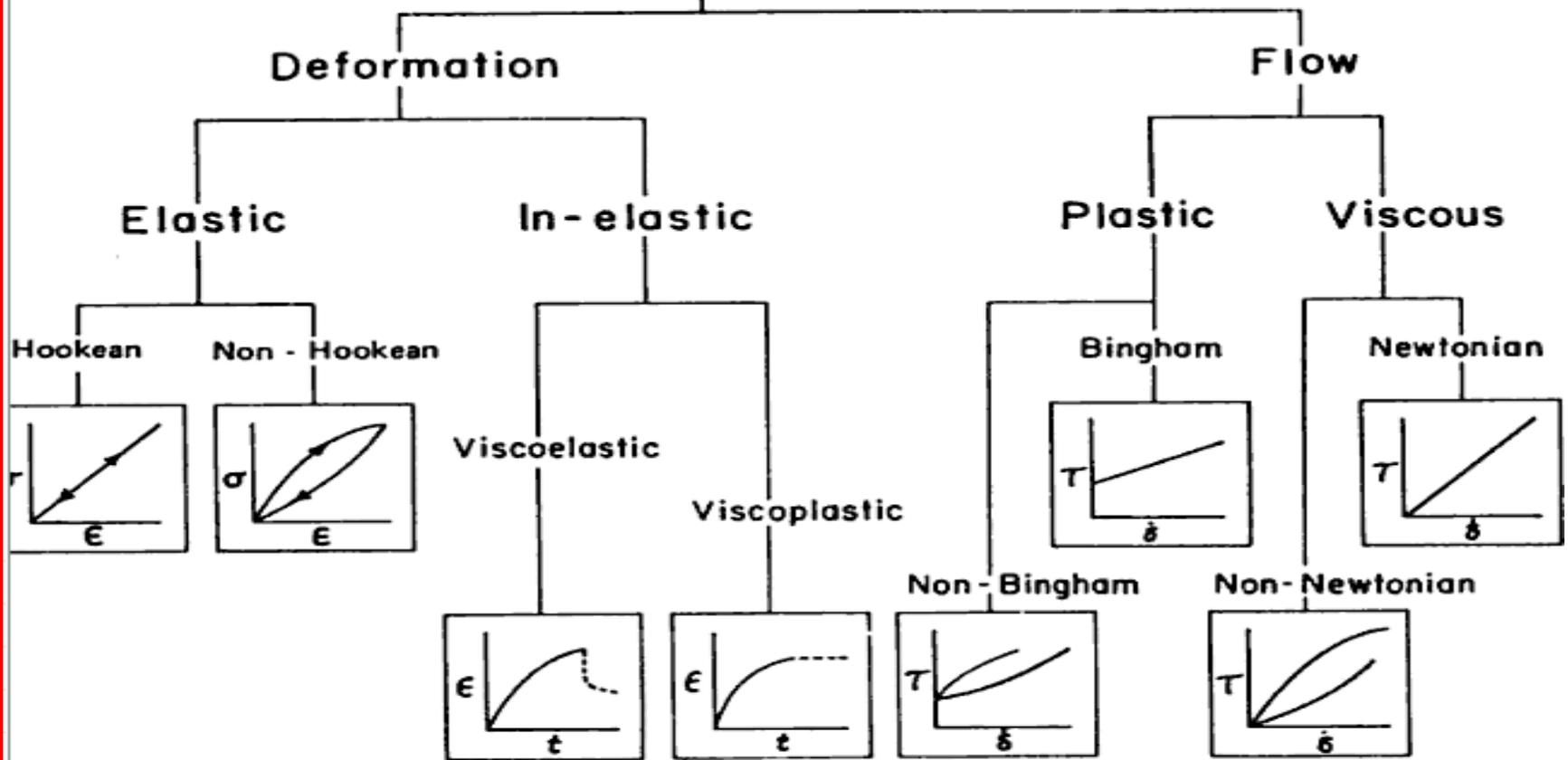


# **RHEOLOGICAL PROPERTIES (CLASSICAL MATERIAL)**

# Rheology



$\sigma$ : Stress    $\epsilon$ : Strain    $t$ : Time    $\dot{\epsilon}$ : Shear rate    $\tau$ : Shear stress

# CLASSICAL IDEAL MATERIALS

Elasticity, Plasticity and Viscosity are three fundamental properties by which the rheological behavior of a material can be described.

The **three classical ideal bodies** representing these properties are

1. IDEAL ELASTIC BEHAVIOR (HOOKEAN BODY),
2. IDEAL PLASTIC BEHAVIOR (ST. VENANT BODY),
3. IDEAL VISCOUS BEHAVIOR (NEWTONIAN LIQUID)

*Since no real material behaves perfectly elastic or perfectly plastic, the three ideal bodies have been chosen to serve as standards of comparison in the analysis of the behavior of any real material.*

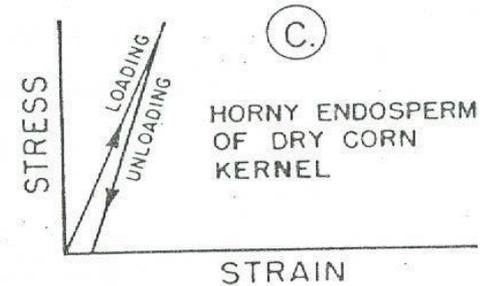
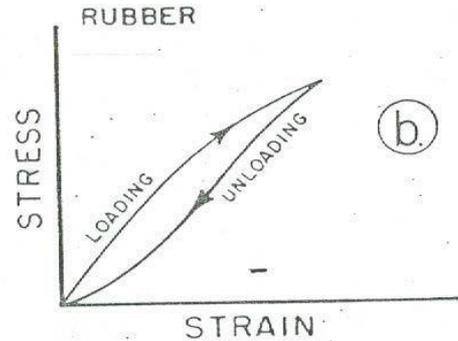
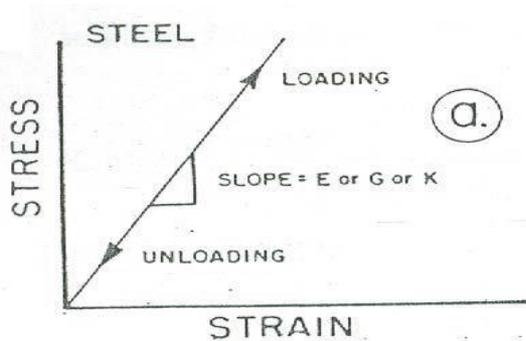
Elasticity is the tendency of solid material to temporarily deform when placed under stress and **regains** its original shape and size upon removal of stress. It spreads, it stretches; it alters to accommodate the strain.

Plasticity is the tendency of a material **deform permanently** under the application of external load .

Viscosity is a measure of a fluid's **resistance to flow**. A fluid with large viscosity resists motion. A fluid with low viscosity flows. For example, water flows easier than syrup because it has a lower viscosity. High viscosity materials might include honey, syrups, or gels – generally things that resist flow.

## ***Ideal elastic behavior (Hookean body)***

In a Hooke body, **stress is directly proportional to strain**. The relationship is known as the Hooke's law and the behavior is referred to as Hookean behavior (Elastic spring).



a) Linear elasticity in steel, (b) non-linear elasticity in rubber, (c) inelasticity in corn

## **Elastic Properties**

1. Young's Modulus
2. Shear Modulus
3. Bulk Modulus (Compressibility)
4. Poisson's Ratio

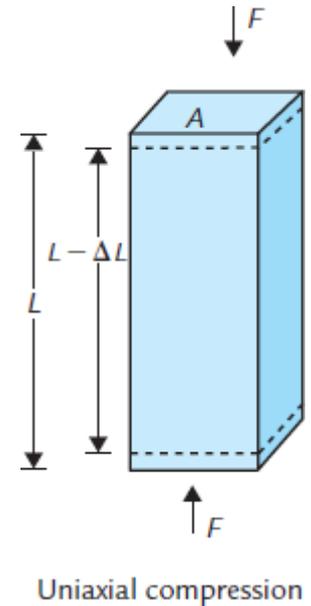


# Young's Modulus

(modulus of elasticity)

Young's modulus of elasticity is the **ratio of tensile or compressive stress to tensile or compressive strain**. It is a measure of stiffness and is described by equation:

$$E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L}$$

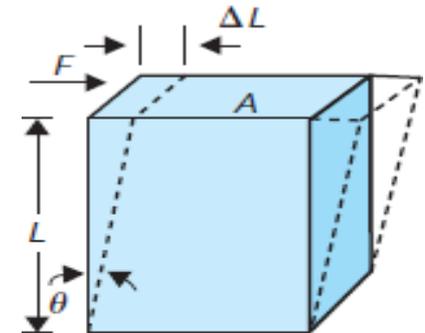


where  $E$  is Young's modulus,  $F$  is the applied force perpendicular to the area defined by the stress,  $A$  is the cross-sectional area of the test material,  $L$  is the length or height of the test specimen and  $\Delta L$  the change in length resulting from the application of force  $F$ .

## Shear Modulus (modulus of rigidity)

*Shear modulus* is the **ratio of shearing stress to shearing strain** and is described by the equation:

$$G = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A}{\gamma/L}$$



Shear

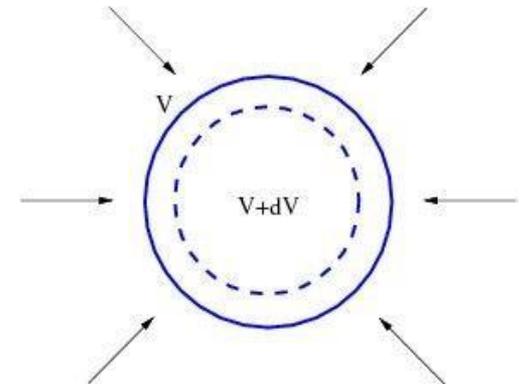
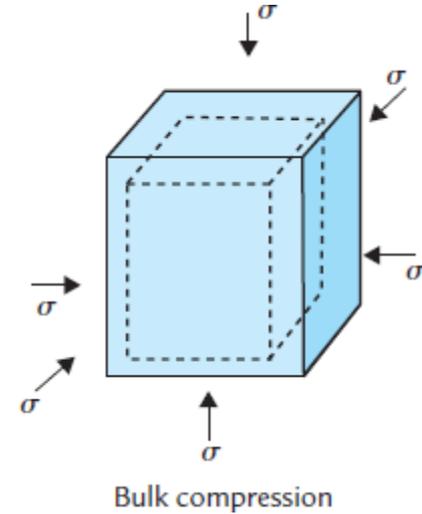
where G is the shear modulus, F is the applied force parallel to the area defined by the stress,  $\gamma$  (or  $\Delta L$ ) is the greatest distance of movement in the test material, and L is the length.

# Bulk Modulus (Compressibility)

*Bulk modulus* is the ratio of the stress to the change in volume w r t original volume:

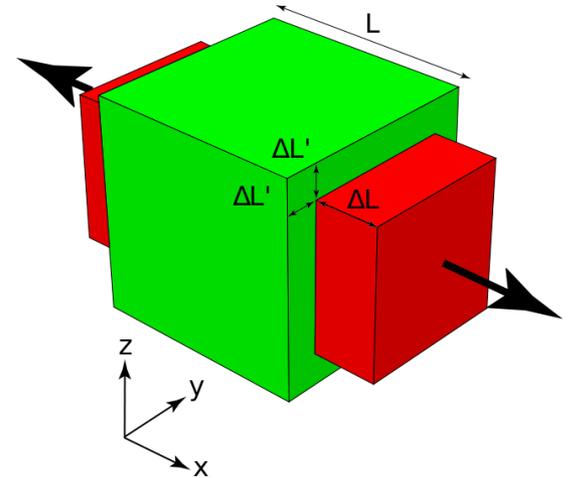
$$K = \frac{\text{hydrostatic pressure}}{\text{volumetric strain}} = \frac{P}{\Delta V/V}$$

where  $K$  is the bulk modulus,  $P$  the force per unit area (pressure) applied from all directions (isotropically),  $V$  is the original volume of the unstressed material and  $\Delta V$  the change in volume resulting from the application of pressure  $P$ .



# Poisson's Ratio

When a sample of material is stretched in one direction it tends to get thinner in the other two directions. Poisson's ratio is the **ratio of the relative contraction strain, or transverse strain normal to the applied load, to the axial strain in the direction of the applied load.**



$$\mu = \frac{\text{lateral strain}}{\text{axial strain}} = \frac{\text{change in width per unit width}}{\text{change in length per unit length}} = \frac{\Delta D/D}{\Delta L/L}$$

where  $\mu$  is Poisson's ratio,  $D$  is the width of the test specimen and  $L$  its length,  $\Delta D$  and  $\Delta L$  the changes caused by the application of a stress.

NB: If the volume is unchanged when the stress is applied, Poisson's ratio is 0.5. It is less than 0.5 if the volume changes. Since **water** is essentially incompressible, foods containing high water content such as fruits and vegetables have a Poisson's ratio close to 0.5 unless they also contain gas.

## Interrelations Between Moduli

$E$ ,  $G$ ,  $K$  and  $\mu$  are called 'material constants' because they are inherent to the material. Rheological theory shows that for elastic materials these four moduli are interrelated as follows:

$$\frac{1}{E} = \frac{1}{3G} + \frac{1}{9K}$$

$$E = 3K(1 - 2\mu)$$

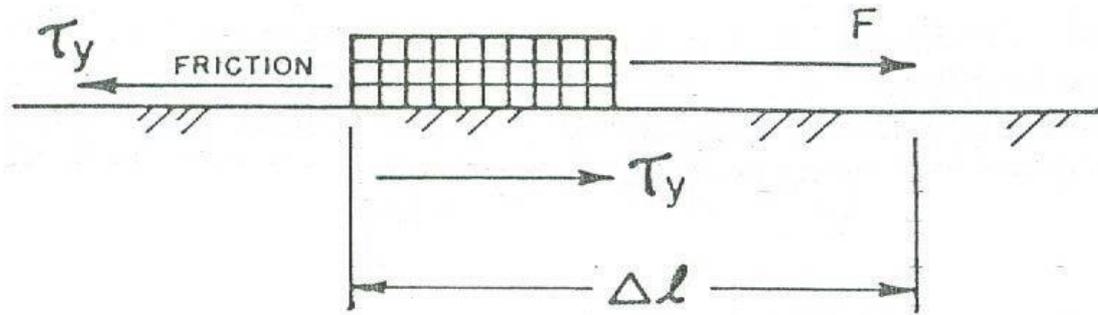
$$E = 2G(1 + \mu)$$

$$\mu = \frac{(3K - E)}{6K}$$

$$\mu = \frac{(E - 2G)}{2G}$$

## Ideal plastic behavior (St. Venant body)

A friction block can be used as the mechanical model for St. Venant Body representing ideal plasticity. The material does not flow until a limiting value of shearing stress  $\tau_y$ , called the "yield-stress," is reached. The material can sustain no stress greater than this value and flows indefinitely under this stress unless distortion is restricted by some other factor.



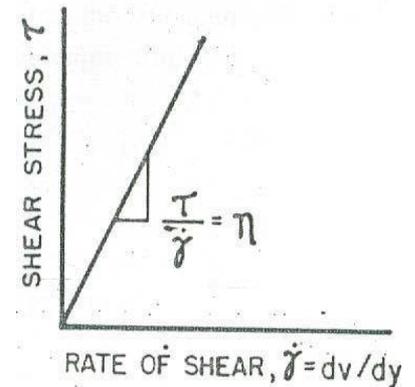
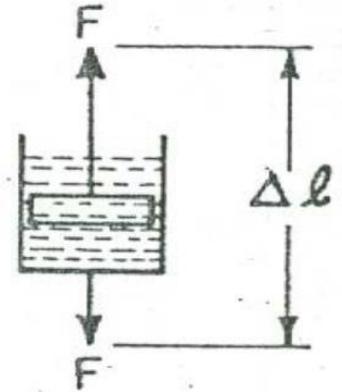
Friction block representing St. Venant Body for ideal plasticity



## Ideal viscous behavior (Newtonian liquid)

In a liquid, deformation and flow begin as soon as a shear stress is applied, and when the stress is removed, it will not return to its original state. In a liquid, strain is a function not only of stress but of time as well. The mechanical model **simple dashpot (damper)**, called the viscous element, is supposed to be a **weightless, loosely fitted piston moving in a medium filled with the liquid**.

✓ Newton's law of viscosity, The plot is a straight line passing through the origin. The slope of the line is by definition the viscosity.



# VISCOSITY

**Viscosity is a measure of a fluid's resistance to flow and is a measure of the adhesive/cohesive or frictional fluid property.**

The resistance is caused by intermolecular friction exerted when layers of fluids attempt to slide by one another.

The knowledge of viscosity is needed for proper design of required temperatures for storage, pumping or flow of fluids.

Thus, **water** and **honey** are "thin," having a lower viscosity, while **ghee** is "thick" having a higher viscosity.



In general, in any flow, layers move at different velocities and the fluid's viscosity arises from the shear stress between the layers that ultimately opposes any applied force.

Isaac Newton

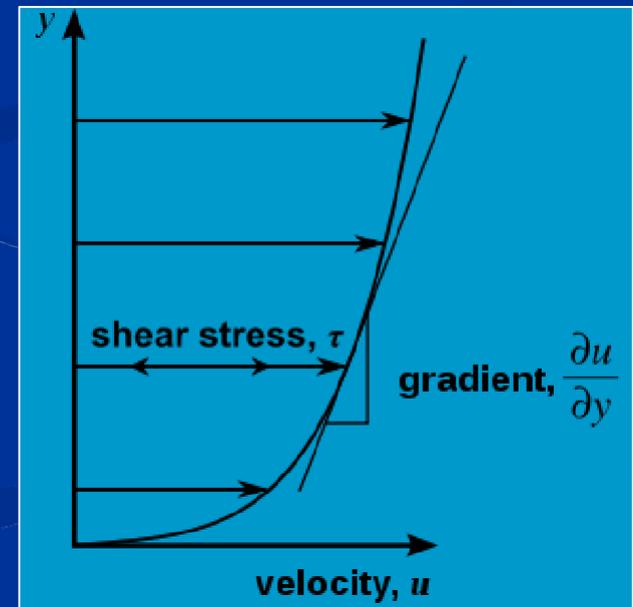
for straight, parallel and uniform flow, the shear stress,  $\tau$ , between layers is proportional to the velocity gradient,  $\partial u / \partial y$ , in the direction perpendicular to the layers

$$\tau \propto (du/dy);$$

$$\tau = \eta (du/dy);$$

$$\tau = (1/\phi) (du/dy)$$

$\phi$ : Fluidity, or  $1/\phi = \eta$ : Viscosity



## Dynamic (absolute) Viscosity

**SI system:**  $\text{N s/m}^2$ ,  $\text{Pa s}$  or  $\text{kg/m s}$  where

$$1 \text{ Pa s} = 1 \text{ N s/m}^2 = 1 \text{ kg/m. s} = 10 \text{ poise}$$

**CGS system:**  $\text{g/cm.s}$ ,  $\text{dyne.s/cm}^2$  or  $\text{poise}(p)$  where

$$1 \text{ poise} = \text{dynes/cm}^2 = \text{g/cm s} = 1/10 \text{ Pa s}$$

$$1 p = 100 \text{ cP} \quad (\text{centipoise})$$

## Kinematic Viscosity

is the ratio of dynamic viscosity to density

$$v = \mu / \rho \quad \text{Where, } v = \text{kinematic viscosity, } \mu = \text{absolute or dynamic viscosity, } \rho = \text{density}$$

**SI-system** the unit is  $\text{m}^2/\text{s}$  or commonly used **Stoke (St)** where

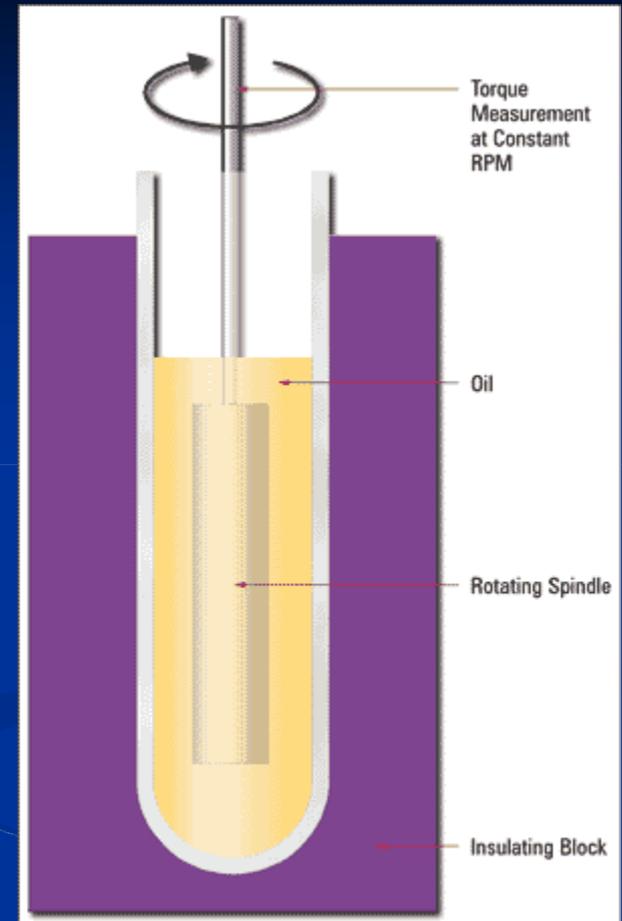
$$\bullet 1 \text{ St} = 10^{-4} \text{ m}^2/\text{s} ,$$

$$\bullet 1 \text{ St} = 100 \text{ cSt} (\text{Centistokes}), \text{ So } 1 \text{ cSt} = 10^{-6} \text{ m}^2/\text{s}$$

Viscosity of a fluid is highly temperature dependent and for either dynamic or kinematic viscosity to be meaningful, the **reference temperature** must be quoted. Water at  $(20.2^\circ\text{C})$  has an absolute viscosity  $1 \text{ centiPoise}$  and kinematic viscosity  $1.0 \text{ cSt}$ .

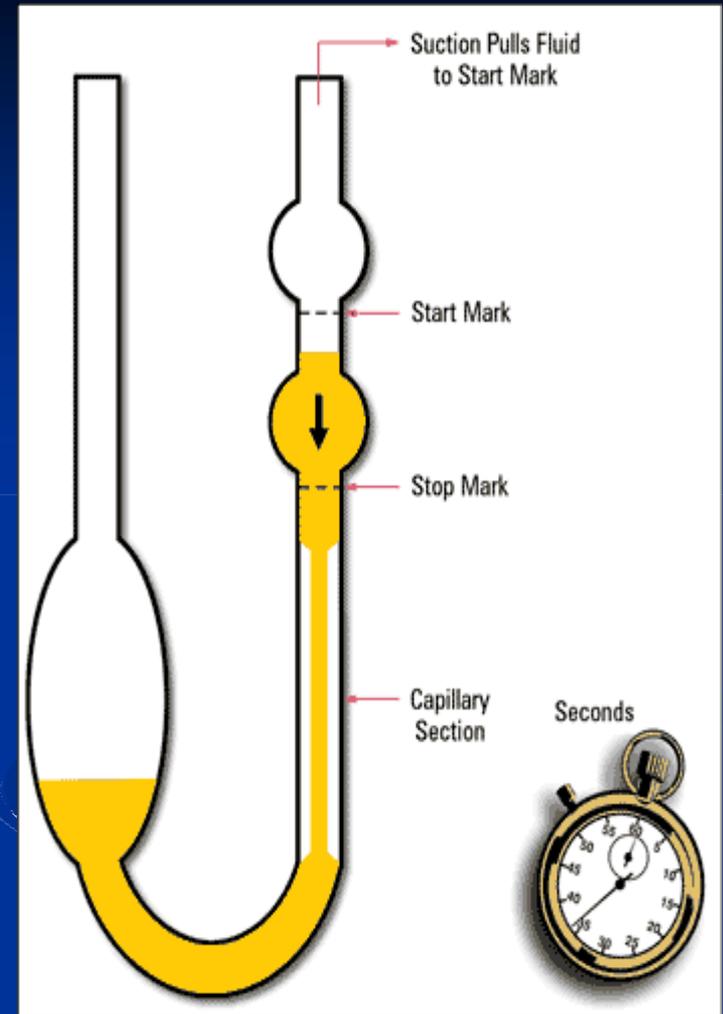
- ✓ For a liquid - the kinematic viscosity will **decrease** with higher temperature
- ✓ For a gas - the kinematic viscosity will **increase** with higher temperature

**Absolute viscosity** is measured as the resistance to flow when a body is forced through the fluid by an external and controlled force such as a spindle driven by a motor. In this case, the **rpm** (or shear) as a function of the **input force** is measured, which reflects the internal resistance of the sample to the applied force, or its dynamic viscosity.



**Rotary Viscometer**

**Kinematic viscosity** is traditionally measured by noting the time it takes oil to travel through the orifice of a capillary under the force of gravity. The orifice of the kinematic viscometer tube produces a fixed resistance to flow. Different sized capillaries are available to support fluids of varying viscosity. The **time** taken for the fluid to flow through the capillary tube can be converted directly **to a kinematic viscosity** using a simple **calibration** constant provided for each tube.



**Capillary U-Tube Viscometer**

**THANK YOU**