

# INTER SYMBOL INTERFERENCE (ISI)

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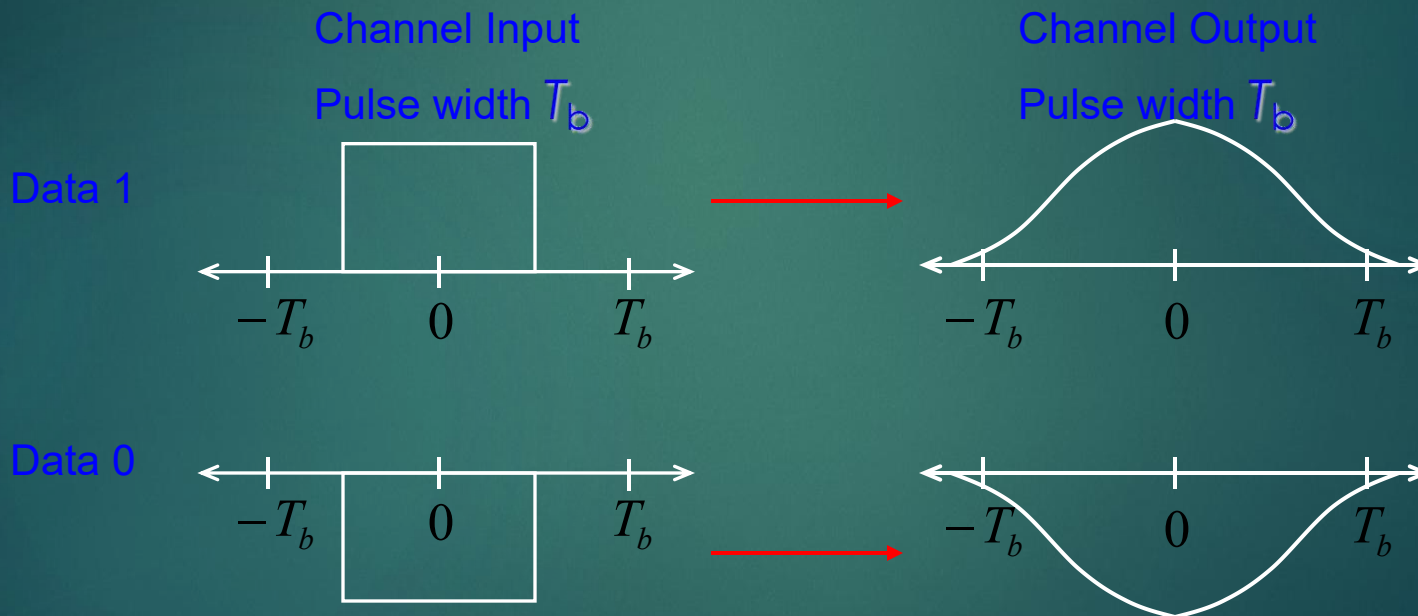
# INTERSYMBOL INTERFERENCE (ISI)<sup>2</sup>

- Intersymbol Interference
- ISI on Eye Patterns
- Combatting ISI
- Nyquist's First Method for zero ISI
- Raised Cosine-Rolloff Pulse Shape
- Nyquist Filter

# Intersymbol Interference

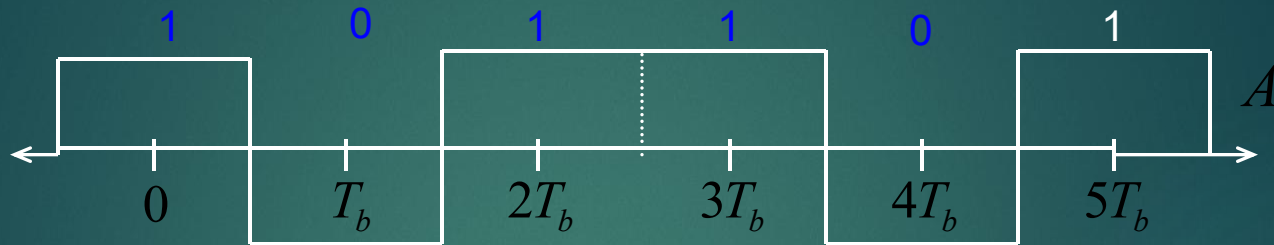
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- ▶ **Inter symbol interference (ISI)** occurs when a pulse spreads out in such a way that it interferes with adjacent pulses *at the sample instant*.
- ▶ Example: assume polar NRZ line code. The channel outputs are shown as spreaded (width  $T_b$  becomes  $2T_b$ ) pulses shown (Spreading due to band limited channel characteristics).

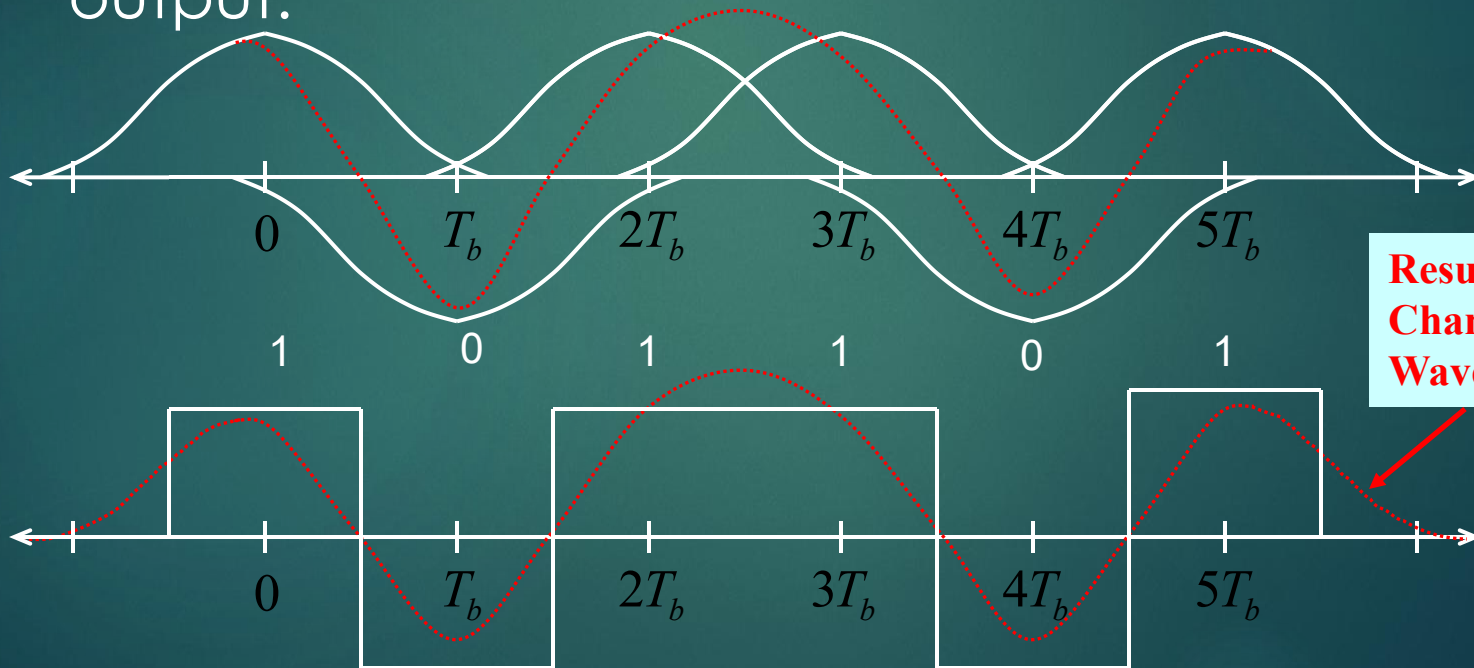


# Intersymbol Interference

- ▶ For the input data stream:



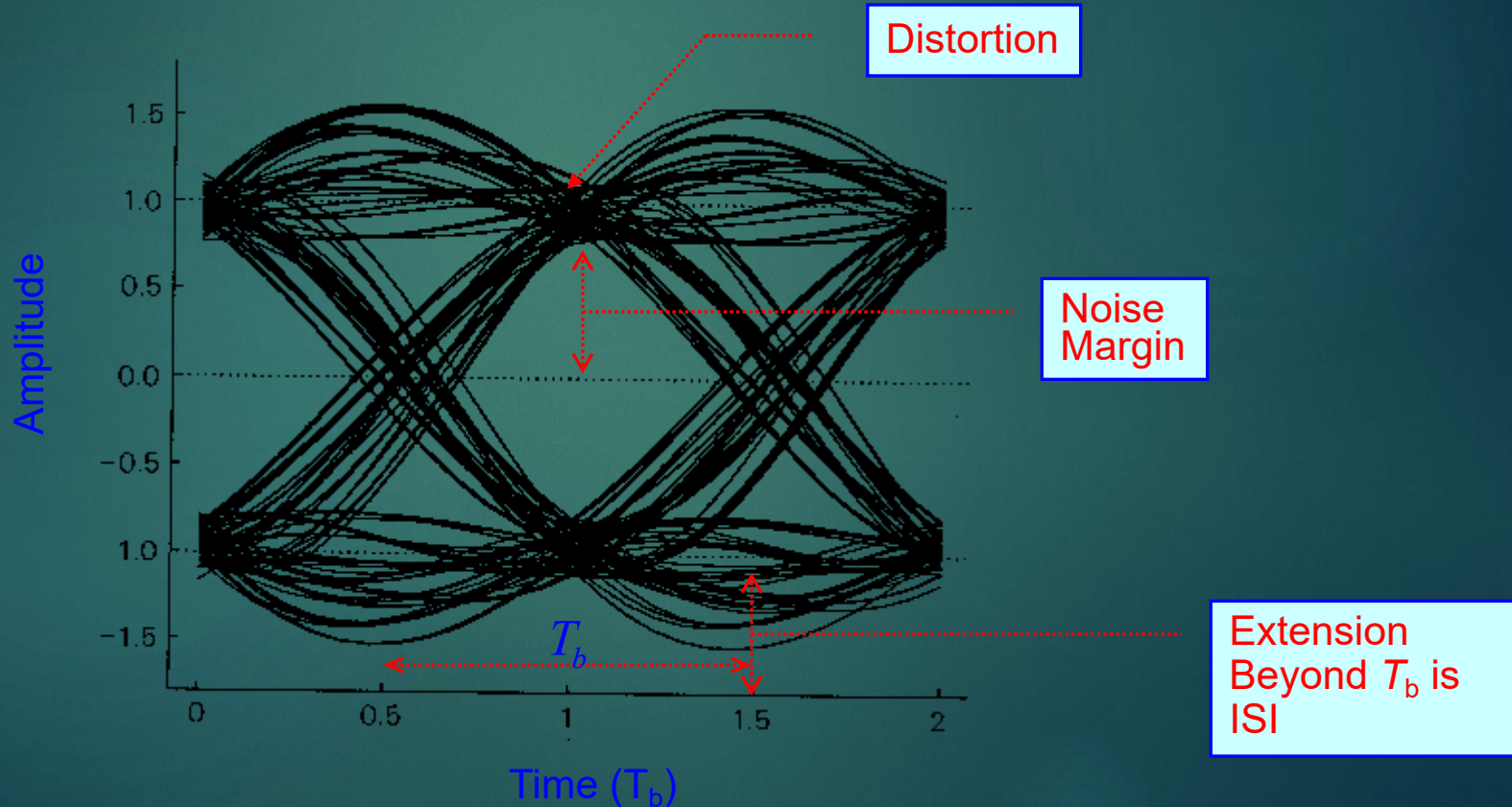
- ▶ The channel output is the superposition of each bit's output:



# ISI on Eye Patterns

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- ▶ The amount of ISI can be seen on an oscilloscope using an **Eye Diagram** or **Eye pattern**.



# Intersymbol Interference

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- If the rectangular multilevel pulses are filtered improperly as they pass through a communications system, they will spread in time, and the pulse for each symbol may be smeared into adjacent time slots and cause *Intersymbol Interference*.

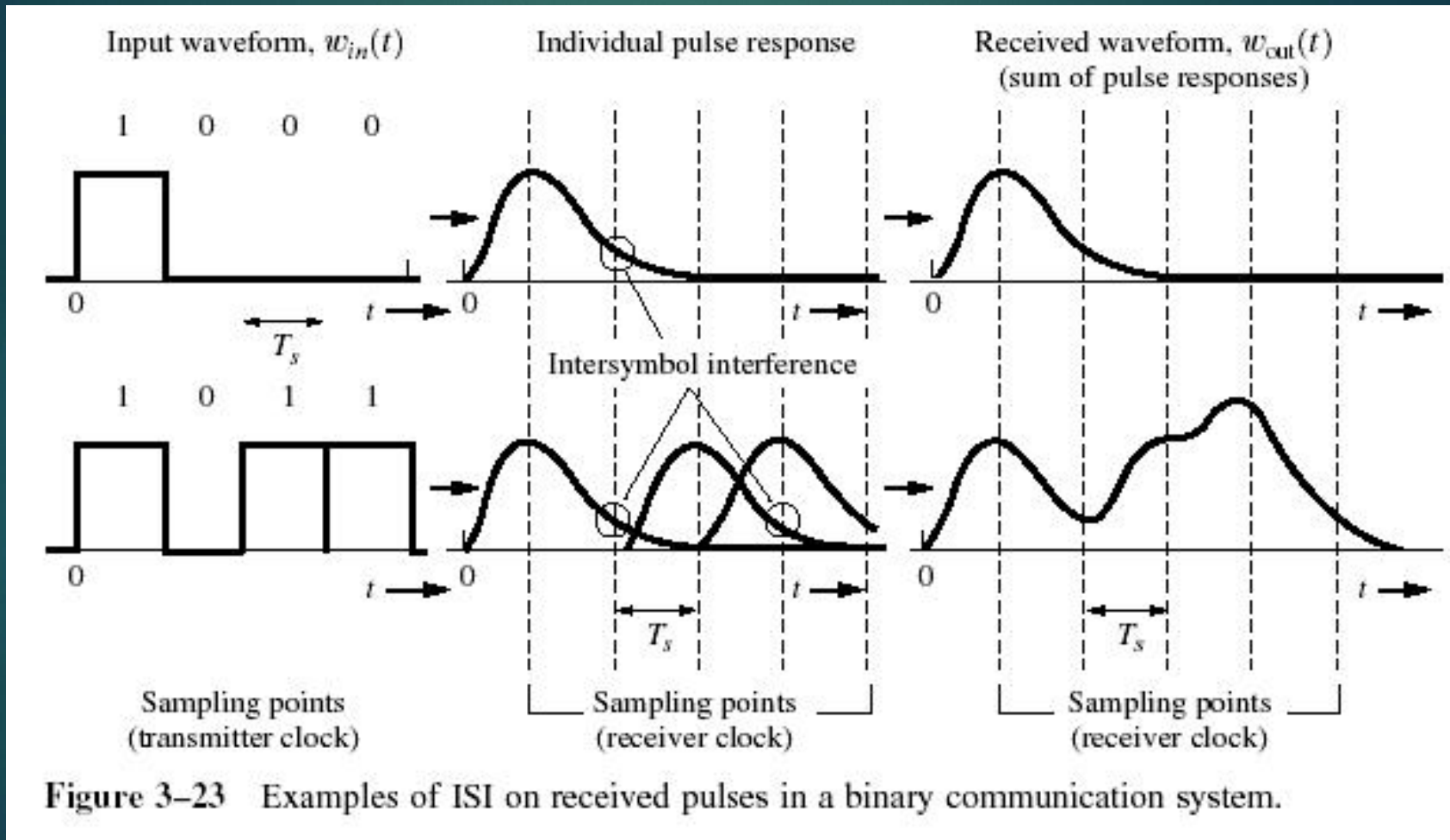


Figure 3-23 Examples of ISI on received pulses in a binary communication system.

- How can we restrict BW and at the same time not introduce ISI? 3 Techniques.

# Intersymbol Interference

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- Flat-topped multilevel input signal having pulse shape  $h(t)$  and values  $a_k$ :

$$w_{in}(t) = \sum a_n h(t - nT_s) = \sum a_n h(t) * \delta(t - nT_s) = \left[ \sum a_n \delta(t - nT_s) \right] * h(t)$$

Where  $h(t) = \text{rect}\left(\frac{t}{T_s}\right)$  Where  $D = \frac{1}{T_s}$  pulses/s

$$w_{out}(t) = \left[ \sum a_n \delta(t - nT_s) \right] * h_e(t) = \sum a_n h_e(t - nT_s)$$

Equivalent impulse response:  $h_e(t) = h(t) * h_T(t) * h_C(t) * h_R(t)$

- $h_e(t)$  is the pulse shape that will appear at the output of the receiver filter.

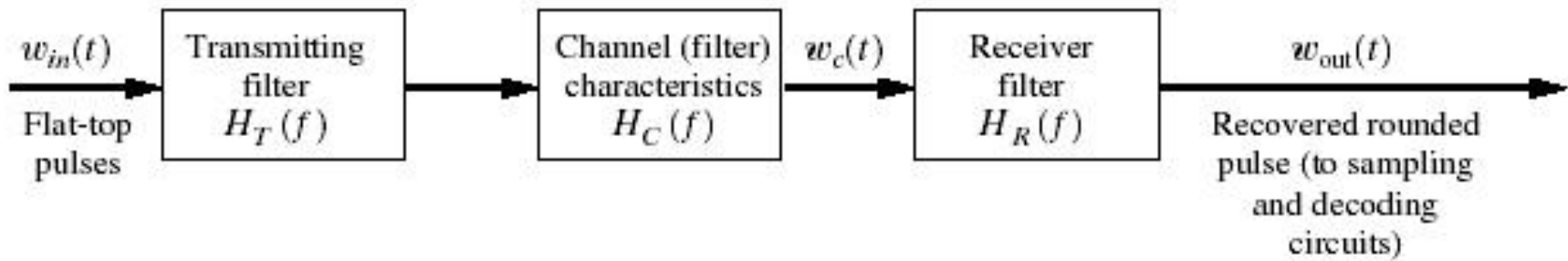


Figure 3-24 Baseband pulse-transmission system.

# Intersymbol Interference

- Equivalent Impulse Response  $h_e(t)$  :

$$h_e(t) = h(t) * h_T(t) * h_C(t) * h_R(t)$$

- Equivalent transfer function:

$$H_e(f) = H(f)H_T(f)H_C(f)H_R(f) \quad \text{Where} \quad H(f) = F \left[ \Pi \left( \frac{t}{T_s} \right) \right] = T_s \left( \frac{\sin \pi T_s f}{\pi T_s f} \right)$$

- Receiving filter can be designed to produce a needed  $H_e(f)$  in terms of  $H_T(f)$  and  $H_C(f)$ :

$$H_R(f) = \frac{H_e(f)}{H(f)H_T(f)H_C(f)}$$

- Output signal can be rewritten as:

$$w_{out}(t) = \sum_n a_n h_e(t - nT_s)$$

- $H_e(f)$ , chosen such to minimize ISI is called EQUALIZING FILTER)



# Combating ISI

- ▶ Three strategies for eliminating ISI:
  - ▶ Use a line code that is absolutely band limited.
    - ▶ Would require Sinc pulse shape.
    - ▶ Can't actually do this (but can approximate).
  - ▶ Use a line code that is zero during adjacent sample instants.
    - ▶ It's okay for pulses to overlap somewhat, as long as there is no overlap at the sample instants.
    - ▶ Can come up with pulse shapes that don't overlap during adjacent sample instants.
      - Raised-Cosine Roll-off pulse shaping
  - ▶ Use a filter at the receiver to “undo” the distortion introduced by the channel.
    - ▶ Equalizer.

# Nyquist's First Method for Zero ISI

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➤ ISI can be eliminated by using an equivalent transfer function,  $H_e(f)$ , such that the impulse response satisfies the condition:

$$h_e(kT_s + \tau) = \begin{cases} C, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

$k$  is an integer,  $T_s$  is the symbol (sample) period

$\tau$  is the offset in the receiver sampling clock times

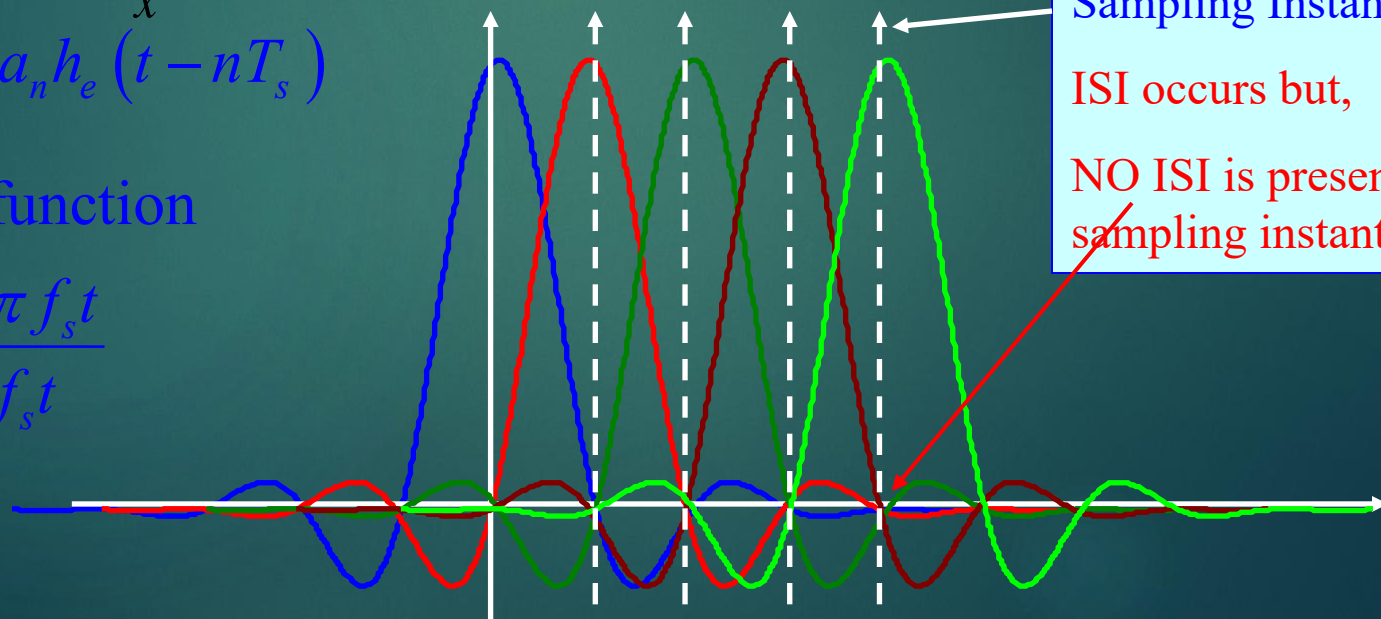
$C$  is a nonzero constant

Now choose the  $\frac{\sin x}{x}$  function for  $h_e(t)$

$$w_{out}(t) = \sum_n a_n h_e(t - nT_s)$$

$h_e$  is a Sa function

$$h_e(t) = \frac{\sin \pi f_s t}{\pi f_s t}$$



Sampling Instants

ISI occurs but,

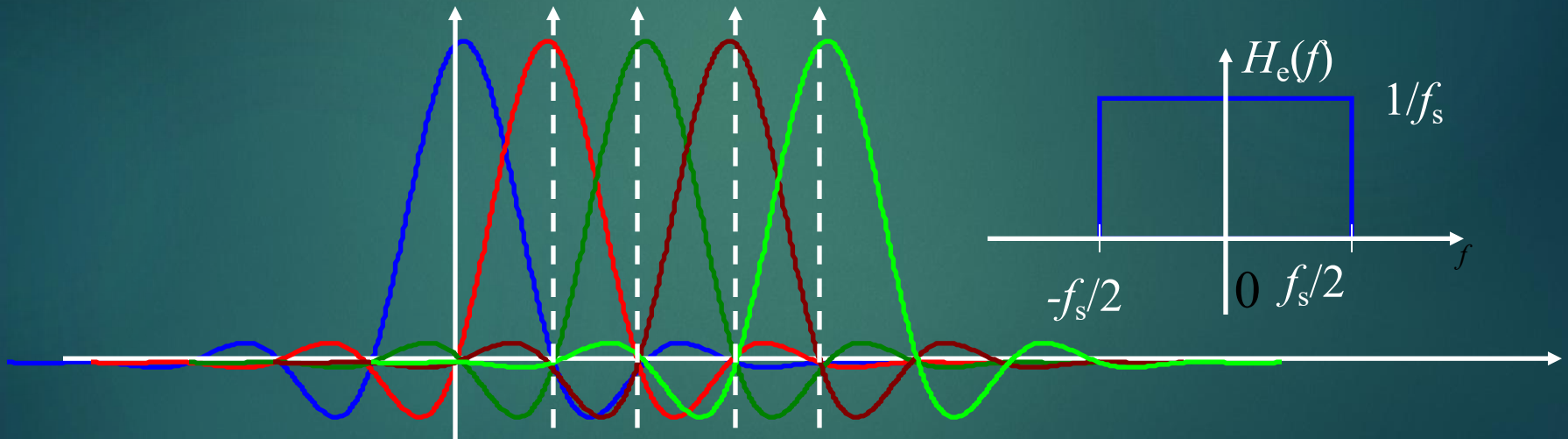
NO ISI is present at the sampling instants

# Nyquist's First Method for Zero ISI

- There will be **NO ISI** and the bandwidth requirement will be minimum (Optimum Filtering) if the transmit and receive filters are designed so that the overall transfer function  $H_e(f)$  is:

$$H_e(f) = \frac{1}{f_s} \Pi\left(\frac{f}{f_s}\right) \quad h_e(t) = \frac{\sin \pi f_s t}{\pi f_s t} \quad \text{Where } f_s = \frac{1}{T_s}$$

- This type of pulse will allow signalling at a baud rate of  $D=1/T_s=2B$  (for Binary  $R=1/T_s=2B$ ) where  $B$  is the absolute bandwidth of the system.



Absolute bandwidth is:  $B = \frac{f_s}{2}$  **MINIMUM BANDWIDTH**

Signalling Rate is:  $D=1/T_s = 2B$  Pulses/sec

# Nyquist's First Method for Zero ISI

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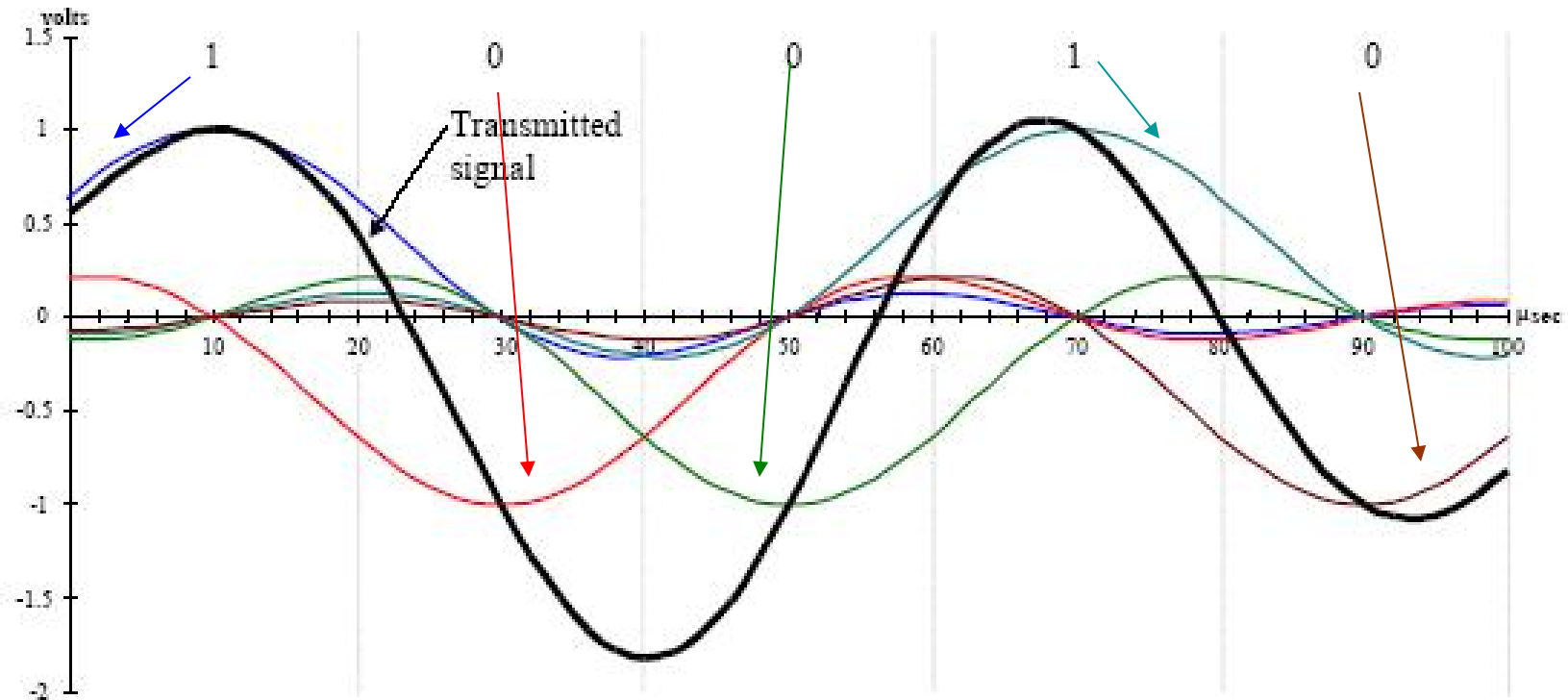
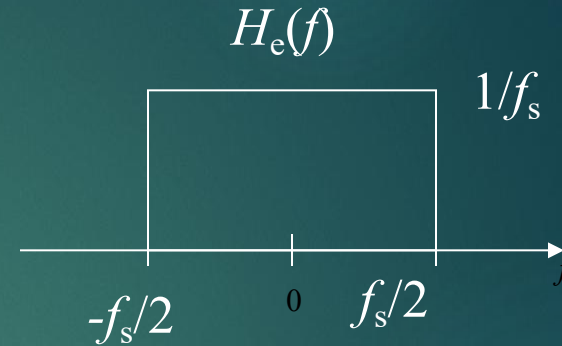
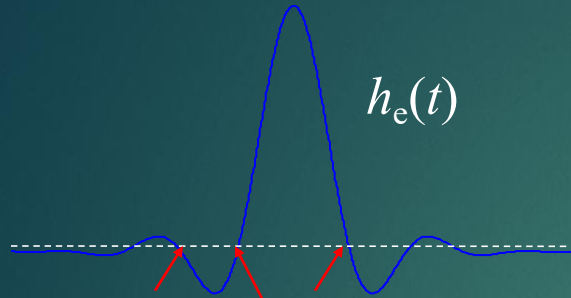


Figure 3.28 Transmitted binary PAM waveform for the data sequence “10010” using sinc-shaped pulses (raised cosine pulse shaping,  $\alpha = 0$ ) at a transmission speed of 50,000 bits/sec. Note that this is the same plot as Figure 3.20a.

# Nyquist's First Method for Zero ISI

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Zero crossings at non-zero integer multiples of the bit period

- ▶ Since pulses are not possible to create due to:
  - ▶ Infinite time duration.
  - ▶ Sharp transition band in the frequency domain.
- ▶ The Sinc pulse shape can cause significant ISI in the presence of timing errors.
  - ▶ If the received signal is not sampled at exactly the bit instant (Synchronization Errors), then ISI will occur.
- ▶ We seek a pulse shape that:
  - ▶ Has a more gradual transition in the frequency domain.
  - ▶ Is more robust to timing errors.
  - ▶ Yet still satisfies Nyquist's first method for zero ISI.

# Raised Cosine-Rolloff Nyquist Filtering

- Because of the difficulties caused by the Sa type pulse shape, consider other pulse shapes which require more bandwidth such as the Raised Cosine-rolloff Nyquist filter but they are less affected by synchronization errors.
- The Raised Cosine Nyquist filter is defined by its rolloff factor number  $r=f_{\Delta}/f_0$ .

$$H_e(f) = \begin{cases} 1, & |f| < f_1 \\ \frac{1}{2} \left\{ 1 + \cos \left[ \frac{\pi (|f| - f_1)}{2f_{\Delta}} \right] \right\}, & f_1 < |f| < B \\ 0, & |f| > B \end{cases} \quad \text{B is the Absolute Bandwidth}$$

$f_{\Delta} = B - f_0$     $f_1 \equiv f_0 - f_{\Delta}$    Where  $f_0$  is the 6-dB bandwidth of the filter

Rolloff factor:  $r = \frac{f_{\Delta}}{f_0}$    Bandwidth:  $B = \frac{R_b}{2} (1 + r)$

$$h_e(t) = F^{-1} [H_e(f)] = 2f_0 \left( \frac{\sin 2\pi f_0 t}{2\pi f_0 t} \right) \left[ \frac{\cos 2\pi f_{\Delta} t}{1 - (4f_{\Delta} t)^2} \right]$$

$$\text{Rolloff factor: } r = \frac{f_{\Delta}}{f_0} \quad \text{Bandwidth: } B = \frac{R_b}{2} (1 + r)$$

# Raised Cosine-Rolloff Nyquist Filtering

- Now filtering requirements are relaxed because absolute bandwidth is increased.
- Clock timing requirements are also relaxed.
- The  $r=0$  case corresponds to the previous Minimum bandwidth case.

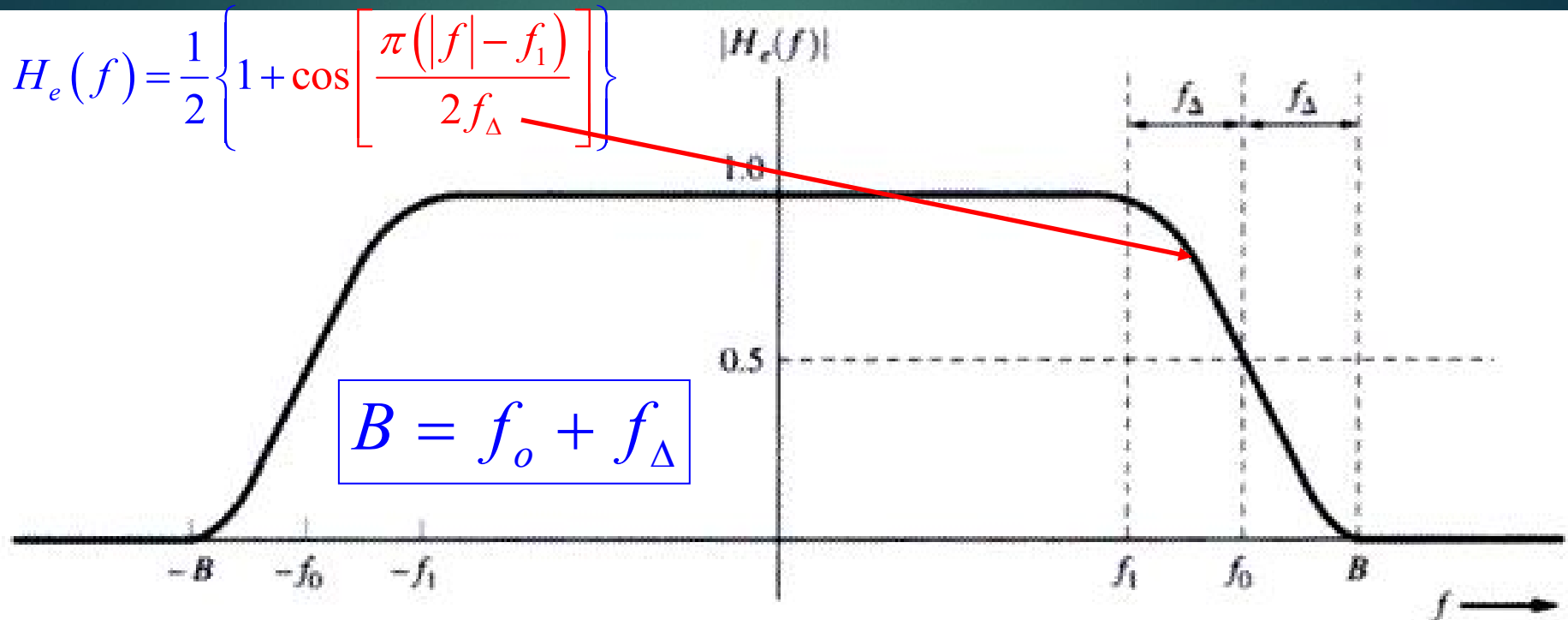


Figure 3-25 Raised cosine-rolloff Nyquist filter characteristics.

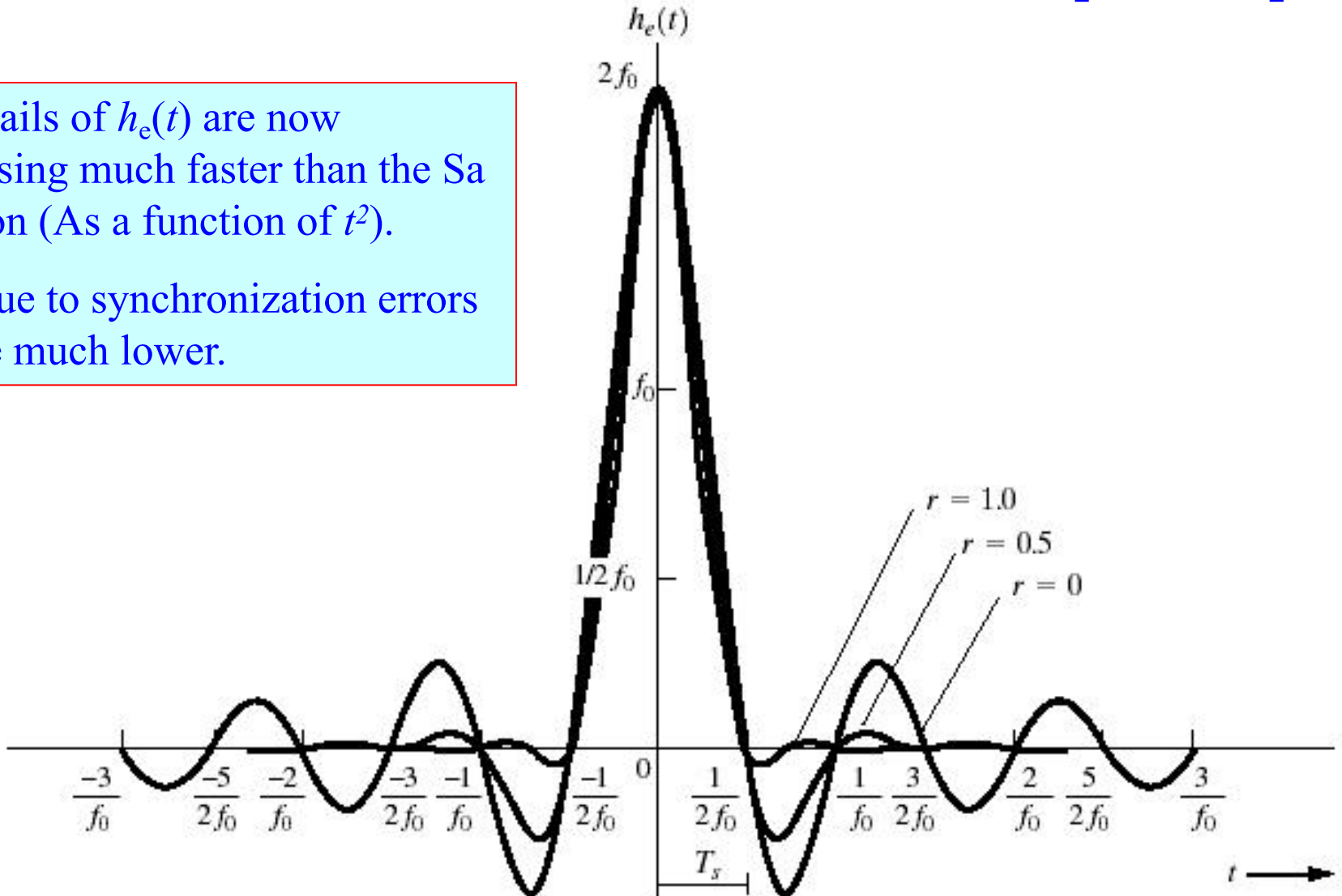
Rolloff factor:  $r = \frac{f_\Delta}{f_0}$  Bandwidth:  $B = \frac{R}{2}(1+r) = \frac{D}{2}(1+r)$

# Raised Cosine-Rolloff Nyquist Filtering

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➤ Impulse response is given by: 
$$h_e(t) = F^{-1}[H_e(f)] = 2f_0 \left( \frac{\sin 2\pi f_0 t}{2\pi f_0 t} \right) \left[ \frac{\cos 2\pi f_0 t}{1 - (4f_0 t)^2} \right]$$

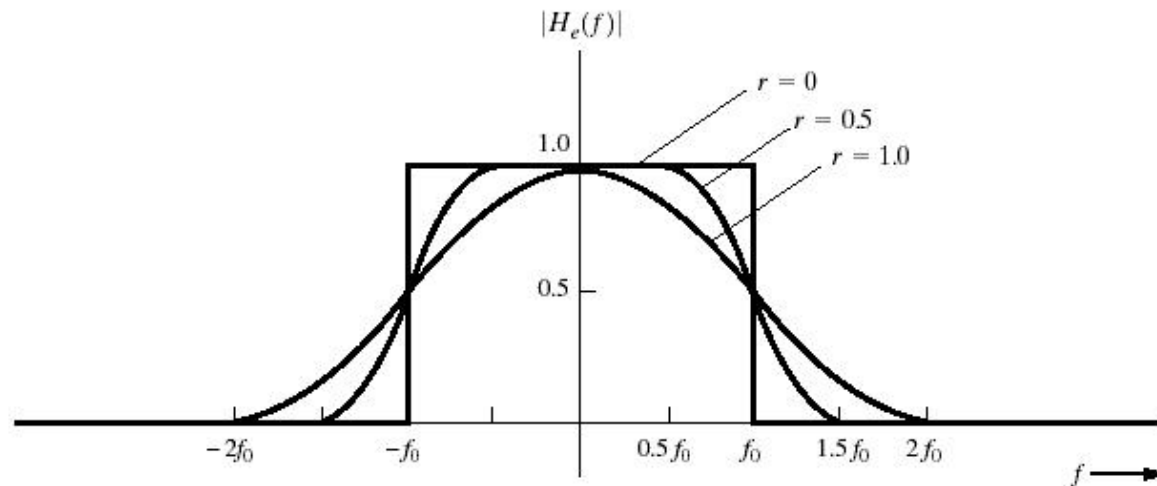
- The tails of  $h_e(t)$  are now decreasing much faster than the Sa function (As a function of  $t^2$ ).
- ISI due to synchronization errors will be much lower.



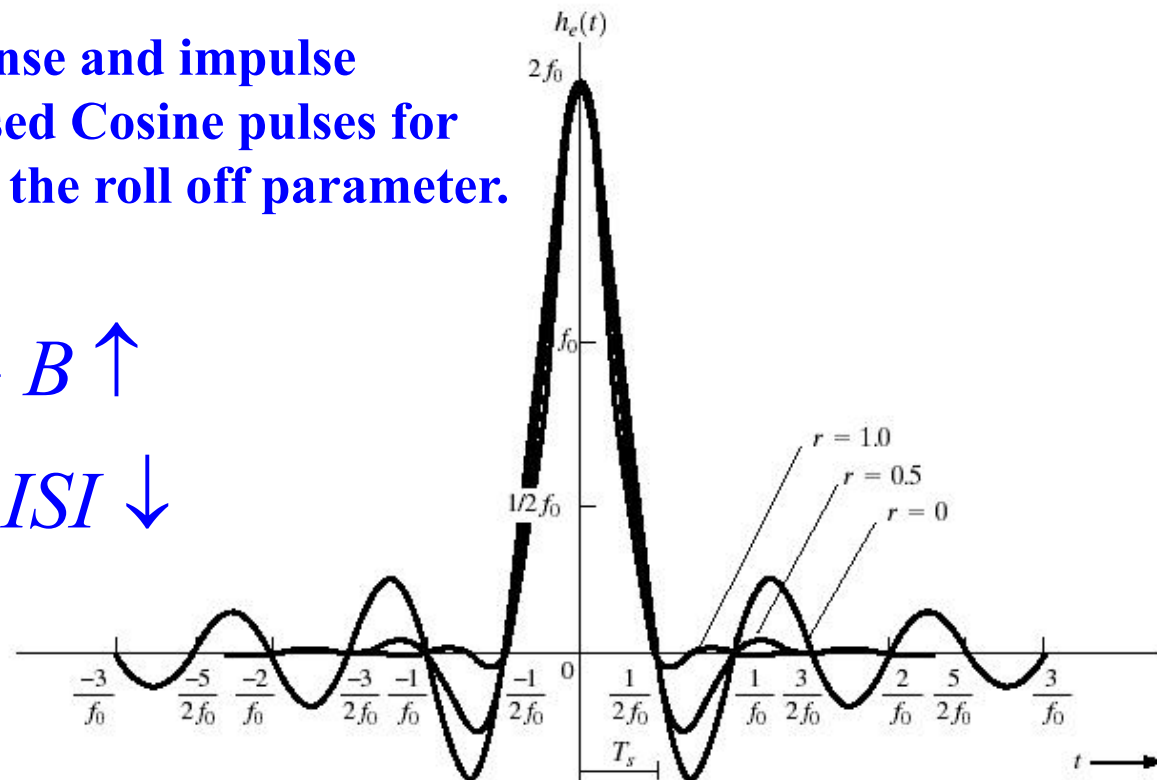


# Raised Cosine-Rolloff Nyquist Filtering

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(a) Magnitude Frequency Response



Frequency response and impulse responses of Raised Cosine pulses for various values of the roll off parameter.

$r \uparrow \rightarrow B \uparrow$

$r \uparrow \rightarrow ISI \downarrow$

# Raised Cosine-Rolloff Nyquist Filtering

➤ Illustrating the received bit stream of Raised Cosine pulse shaped transmission corresponding to the binary stream of 1 0 0 1 0 for 3 different values of  $r=0, 0.5, 1$ .

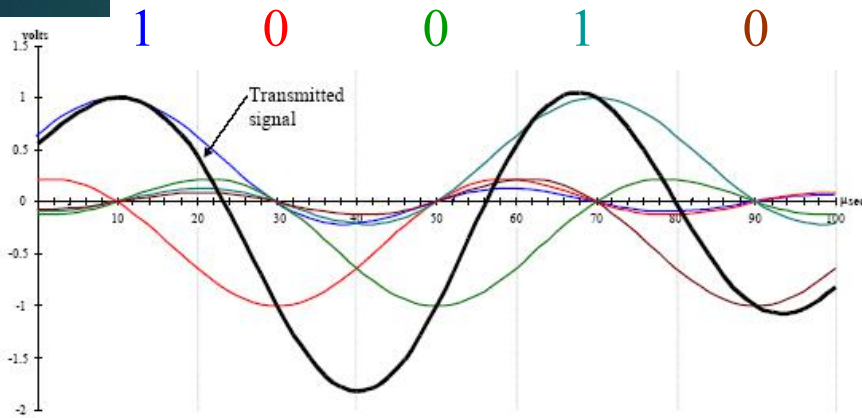


Figure 3.28 Transmitted binary PAM waveform for the data sequence "10010" using sinc-shaped pulses (raised cosine pulse shaping,  $\alpha = 0$ ) at a transmission speed of 50,000 bits/sec. Note that this is the same plot as Figure 3.20a.

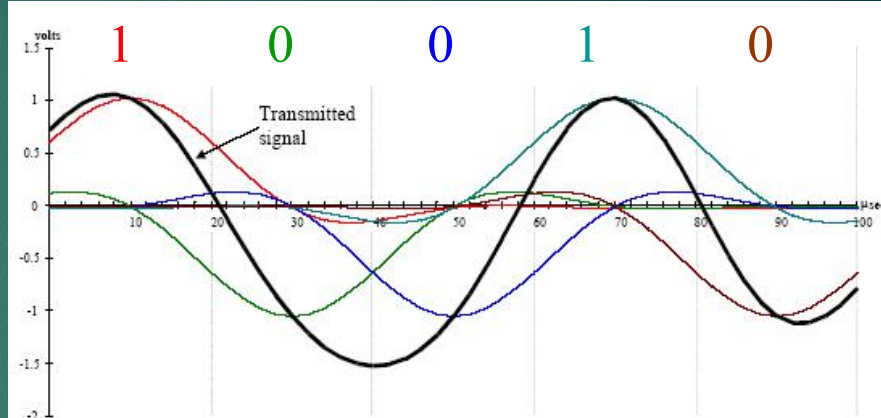


Figure 3.30 "10010" with raised cosine pulse shaping,  $\alpha = 0.5$

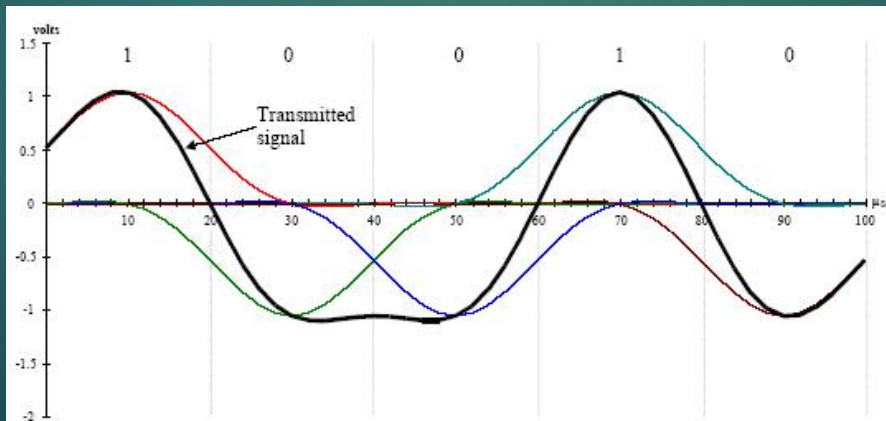


Figure 3.32 "10010" with raised cosine pulse shaping,  $\alpha = 1$

# Bandwidth for Raised Cosine Nyquist Filtering

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- ▶ The bandwidth of a Raised-cosine (RC) rolloff pulse shape is a function of the bit rate and the rolloff factor:

$$B = f_o + f_{\Delta} = f_o \left( 1 + \frac{f_{\Delta}}{f_o} \right) = f_o (1 + r)$$

$$B = \frac{R}{2} (1 + r)$$

$$B = \frac{D}{2} (1 + r) \quad \text{Multilevel Signalling}$$

- ▶ Or solving for bit rate yields the expression:

$$R = \frac{2B}{1 + r}$$

- ▶ This is the maximum transmitted bit rate when a RC-rolloff pulse shape with Rolloff factor  $r$  is transmitted over a baseband channel with bandwidth  $B$ .

# Nyquist Filter

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- **Raised Cosine Filter is also called a NYQUIST FILTER.**
- **NYQUIST FILTERS refer to a general class of filters that satisfy the NYQUIST's First Criterion.**
- **Theorem:** A filter is said to be a **Nyquist filter** if the effective transfer function is :

$$H_e(f) = \begin{cases} \Pi\left(\frac{f}{2f_0}\right) + Y(f), & |f| < f_0 \\ 0, & f \text{ Elsewhere} \end{cases}$$

$Y(f)$  is a real function and even symmetric about  $f = 0$ :

$$Y(-f) = Y(f), \quad |f| < 2f_0$$

$Y$  is odd symmetric about  $f = f_0$ :

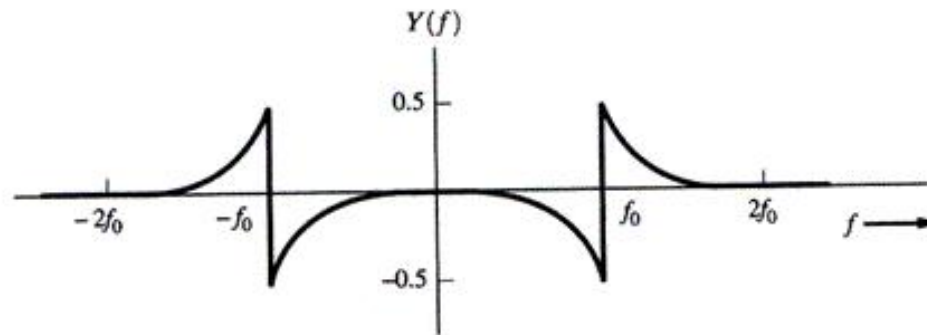
$$Y(-f + f_0) = -Y(f + f_0), \quad |f| < f_0$$

- There will be no intersymbol interference at the system output if the symbol rate is

$$D = f_s = 2f_0$$

# Nyquist Filter Characteristics

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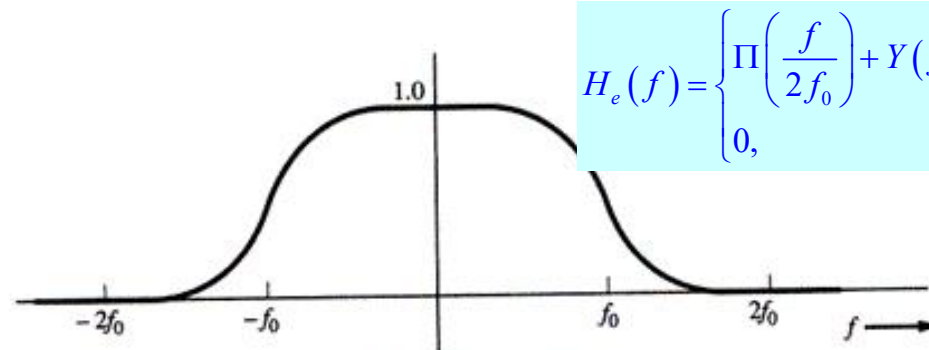
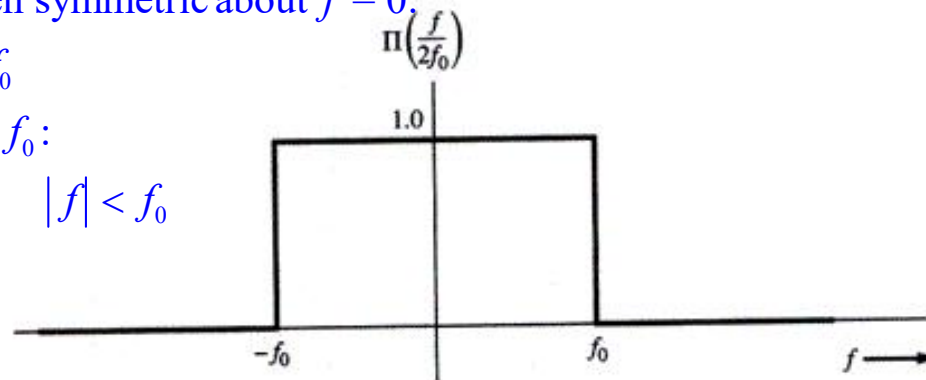


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Figure 3-27 Nyquist filter characteristic.

THANK YOU