Watershed Hydrology

Frequency Analysis of Point Rainfall
What is Frequency Analysis?

Deals with the chance of occurrence of an event equal to or greater than a specified magnitude

- The objective of frequency analysis of hydrologic data is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions

Utility of Frequency Analysis

- Floodplain management & design of flood control structures
- Design of industrial & domestic water-supply systems
Suppose, $P$ is the probability of occurrence of an event (rainfall) whose magnitude is equal to or in excess of a specified magnitude $X$.

The recurrence interval (return period) is related to $P$ as follows:

$$T = \frac{1}{P}$$

This represents the average interval between the occurrence of a rainfall of magnitude equal to or greater than $X$. 
If return period of rainfall of 20 cm in 24 hour is 10 years at a certain station A

- It implies that on an average rainfall magnitudes equal to or greater than 20 cm in 24 hour occur once in 10 year period, i.e., in long period say 100 years, 10 such events can be Expected.

However, this does not mean that such rainfall events will be separated by 10-year intervals

- It is much more likely that two or more events may occur within one year or month
If the probability of an event is $P$, the probability of the event not occurring in a given year is $q = (1 - P)$

$$P_{r,n} = \binom{n}{r} P^r q^{n-r} = \frac{n!}{(n-r)!r!} P^r q^{n-r}$$

Where $P_{r,n} =$ probability of a random hydrologic event (rainfall) of a given magnitude and exceedence probability $P$ occurring $r$ times in $n$ successive years.

**Example**

- The probability of an event not occurring at all in $n$ successive year is:
  $$P_{0,n} = q^n = (1 - P)^n$$

- The probability of an event occurring at least once in $n$ successive year is:
  $$P_1 = 1 - q^n = 1 - (1 - P)^n$$
Problem

Analysis of the data on maximum one day rainfall depth at Madras indicated that a depth of 280mm had a return period of 50 years. Determine the probability of a one-day rainfall depth equal to or greater than 280mm at Madras occurring

a) Once in 20 successive years,
b) Two times in 15 successive years, and
c) At least once in 20 successive years.
Purpose of Frequency Analysis
It helps to obtain relation between the magnitude of event and its probability of occurrence.

Methods for Frequency Analysis

Empirical Method
The exceedence probability of the event is obtained by the use of empirical formula, known as plotting position formula
Plotting position formulae

<table>
<thead>
<tr>
<th>Method</th>
<th>P (probability)</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>( \frac{m}{N} )</td>
</tr>
<tr>
<td>Hazen</td>
<td>( \frac{m - 0.5}{N} )</td>
</tr>
<tr>
<td>Weibull</td>
<td>( \frac{m}{N + 1} )</td>
</tr>
<tr>
<td>Chegodayev</td>
<td>( \frac{m - 0.3}{N + 0.4} )</td>
</tr>
<tr>
<td>Blom</td>
<td>( \frac{m - 0.44}{N + 0.12} )</td>
</tr>
</tbody>
</table>

Where \( m \) is rank assigned to the data after arranging them in descending order of magnitude

- Thus, the maximum value is assigned \( m = 1 \), the second largest value \( (m = 2) \), and the lowest value \( m = N \), \( N \) being the number of records.
Having calculated $P$ and $T$ for all the events in the series, the variation of rainfall magnitude is plotted against the corresponding $T$ on semi-log or log-log paper.

The rainfall magnitude for any recurrence interval can be determined by extrapolating the plot between magnitude and recurrence interval.

Empirical procedures can give good results for small extrapolations but the errors increase with the amount of extrapolation.