Axial Compressors

Session delivered by:

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Session Objectives

This session is intended to introduce the following:

• Basic Theory of Axial Flow Compressors
• Velocity Triangles and Degree of Reaction
• Three Dimensional Flow and Vortex Theory
• Compressor Efficiency
• Performance Characteristics
• Stall and Surge Phenomena in Compressors
Axial Compressor

Compressor

Combustor

Turbine

Intake

Exhaust
Axial Compressor Blading

ROTOR BLADES

VARIABLE STATOR VANES

ROTOR BLADES
Introduction

- In axial flow compressors, flow enters the first blade row and leaves the last blade row in axial direction.
- Flow through the machine is parallel to the axis of the shaft.
- Axial compressors are characterised by lower pressure ratio per stage at higher mass flow rates compared to centrifugal compressors; and hence these machines are preferred for civil and military aero engines as well as for industrial gas turbines.
- Higher mass flow rate produces higher thrust.
- Axial compressors are classified as *subsonic*, *transonic* and *supersonic* depending on whether the relative flow Mach number at rotor inlet is *fully subsonic*, *partly subsonic and partly supersonic*, or *fully supersonic* along the blade height.
Mechanical Features

- Basic components are: **rotor** and **stator**.
- **Rotor** carries moving blades.
- **Stator** has stationary rows of blades, which convert kinetic energy of air into pressure energy and also redirect the flow at an angle suitable for entry to the next row of moving blades.
- Both, rotor and stator blade passages, are basically diffusers.
- A stage comprises one rotating row followed by a stator row.
- Sometimes, a row of so-called **Inlet Guide Vanes (IGV)** is provided upstream of the first rotor, forming an additional row of stator blades. These IGV serve to direct the axially approaching flow correctly into the first row of rotor blades to meet the design and off-design requirements.
Flow through an Axial Compressor

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Axial Compressor Stage

A *compressor stage* is defined as a rotor blade row followed by a stator blade row. The rotor blades (black) are fixed to the rotor drum and the stator blades are fixed to the outer casing. The blades upstream of the first rotor row are inlet guide vanes. These are not considered to be a part of the first stage and are treated separately. Their function is quite different from the other blade rows since, by directing the flow away from the axial direction, they act to *accelerate* the flow rather than diffuse it. Functionally, inlet guide vanes are the same as turbine nozzles; they increase the kinetic energy of the flow at the expense of the pressure energy.
Compression Process
Compression Efficiency

\[ \eta_{isen} = \frac{\text{Isentropic enthalpy rise}}{\text{Actual enthalpy rise}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} \]

\[ \frac{c_p(T_{02s} - T_{01})}{c_p(T_{02} - T_{01})} \]

\[ (T_{02} - T_{01}) = \frac{1}{\eta_c}(T_{02s} - T_{01}) = \frac{T_{01}}{\eta_c} \left( \frac{T_{02s}}{T_{01}} - 1 \right) \]

\[ \therefore \quad \frac{T_{02s}}{T_{01}} = \left( \frac{p_{02}}{p_{01}} \right)^{(\gamma-1)/\gamma} \]

\[ T_{02} - T_{01} = \frac{1}{\eta_c} \left[ \left( \frac{p_{02}}{p_{01}} \right)^{(\gamma-1)/\gamma} - 1 \right] \]
Elementary Theory

- Total and static pressure and temperature rise across the rotor blades due to addition of external work.
- Absolute flow undergoes acceleration across rotor blade rows.
- There is flow diffusion across stator blade rows, converting kinetic energy into pressure.
- The design pressure rise is achieved in a number of stages.
Velocity Triangles

Two-Dimensional Approach:

$U$: Tangential blade speed

$C$: Absolute flow velocity

$C_a$: Axial component of $C$

$C_w$: Whirl or tangential component of $C$

$V$: Relative flow velocity

$\alpha$: Absolute flow angle

$\beta$: Relative flow angle
Euler Turbine Equation

Work done per unit mass flow rate or Specific Work

\[ \dot{W} = (U_2 C_{w_2} - U_1 C_{w_1}) \]

For \( U_2 = U_1 \), and \( C_{a1} = C_{a2} = C_a \), we can write

\[ \dot{W} = U \left( C_{w_2} - C_{w_1} \right) \]
\[ = U C_a \left( \tan \alpha_2 - \tan \alpha_1 \right) \]
\[ = U C_a \left( \tan \beta_1 - \tan \beta_2 \right) \]

Also

\[ \frac{U}{C_a} = \tan \alpha_1 + \tan \beta_1 \] \hspace{1cm} (1)

\[ \frac{U}{C_a} = \tan \alpha_2 + \tan \beta_2 \] \hspace{1cm} (2)
Euler Turbine Equation

The input energy is absorbed usefully in raising the pressure, temperature and velocity of the air and wastefully in overcoming various frictional losses.

\[ \dot{W} = c_p \Delta T_{os} = UC_a (\tan \beta_1 - \tan \beta_2) \]

\[ \Delta T_{os} = \frac{UC_a}{c_p} (\tan \beta_1 - \tan \beta_2) \]

And, if \( C_3 = C_1 \)

\[ \Delta T_{os} = \Delta T_s = \frac{UC_a}{c_p} (\tan \beta_1 - \tan \beta_2) \]

Total pressure ratio,
\[ \frac{p_{03}}{p_{01}} = \left[ 1 + \eta_s \frac{\Delta T_{os}}{T_{01}} \right]^{\frac{\gamma}{\gamma-1}} \]

\( \eta_s = \) stage isentropic efficiency
\( T_{01} = \) inlet stagnation temperature
Work Done Factor

• Axial velocity is not uniform along the blade height.

• The end wall boundary layers are responsible for the nature of spanwise axial velocity distribution, as shown.

• The axial velocity profile becomes more and more peaky as the flow proceeds downstream and settles down in the fourth stage.

• There is no appreciable change in the axial velocity profile beyond fourth stage.
Work Done Factor

Work Done Factor is the ratio of the actual work absorbing capacity of the stage to its ideal value as calculated from the Euler turbine equation.

\[ W = UC_a (\tan \beta_1 - \tan \beta_2) \]

\[ = U[(U - C_a \tan \alpha_1) - C_a \tan \beta_2] \]

\[ = U[U - C_a (\tan \alpha_1 + \tan \beta_2)] \]

Hence, less work is done at the region where \( C_a \) is high and the actual temperature rise is given by

\[ \Delta T_{os} = \Delta T_s = \frac{\lambda UC_a}{c_p} (\tan \beta_1 - \tan \beta_2) \]

\( \lambda \) is the work done factor, which is less than unity. Its value may range from 0.96 at the first stage to about 0.85 at the fourth and subsequent stages.
Work Done Factor

\[ W = UC_a \left[ \tan \beta_1 - \tan \beta_2 \right] \]
\[ = U \left[ (U - C_a \tan \alpha_1) - C_a \tan \beta_2 \right] \]
\[ = U \left[ U - C_a (\tan \alpha_1 + \tan \beta_2) \right] \]

Since \( \alpha_1 \) and \( \beta_2 \) are approximately constant for a given design, an increase in \( C_a \) will result in a decrease in ‘\( W \)’ and vice versa, hence less work at the region where \( C_a \) is high.

Actual temperature rise

\[ \Delta T_{os} = \frac{\lambda}{C_p} \left[ \tan \beta_1 - \tan \beta_2 \right] \]

and pressure ratio

\[ R_s = \left[ 1 + \eta_s \frac{\Delta T_{os}}{T_{o1}} \right]^{\frac{\gamma}{\gamma - 1}} \]

\( \eta_s \): Stage isentropic efficiency

\( T_{o1} \): Inlet stagnation temperature
Degree of Reaction

- Degree of reaction is the **ratio** of static enthalpy rise in the rotor to static enthalpy rise in the whole stage

\[ \Delta T_A : \text{Static temperature rise in the rotor} \]
\[ \Delta T_B : \text{Static temperature rise in the stator} \]

\[ W = c_p (\Delta T_A + \Delta T_B) = c_p \Delta T_s \]
\[ = UC_a (\tan \beta_1 - \tan \beta_2) \]
\[ = UC_a (\tan \alpha_2 - \tan \alpha_1) \]

- Since all the work input to the stage is transferred to the air by means of the rotor, the SFEE yields

\[ W = c_p \Delta T_A + \frac{1}{2} \left( C_2^2 - C_1^2 \right) \]
\[ c_p \Delta T_A = UC_a (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} \left( C_2^2 - C_1^2 \right) \]

\[ C_1 = C_a \sec \alpha_1 \]
\[ C_2 = C_a \sec \alpha_2 \]
Degree of Reaction

\[ c_p \Delta T_A = UC_a (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} C_a^2 (\sec^2 \alpha_2 - \sec^2 \alpha_1) \]

\[ = UC_a (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} C_a^2 (\tan^2 \alpha_2 - \tan^2 \alpha_1) \]

\[ R = \text{Degree of Reaction} \]

\[ = \frac{\Delta T_A}{\Delta T_A + \Delta T_B} \]

\[ = \frac{UC_a (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} C_a^2 (\tan^2 \alpha_2 - \tan^2 \alpha_1)}{UC_a (\tan \alpha_2 - \tan \alpha_1)} \]

\[ = 1 - \frac{C_a}{2U} (\tan \alpha_2 + \tan \alpha_1) \]
Symmetrical Blading

By adding equations (1) and (2)

\[
\frac{2U}{C_a} = \tan \alpha_1 + \tan \beta_1 + \tan \alpha_2 + \tan \beta_2
\]

\[
R = \frac{C_a}{2U} \left[ \frac{2U}{C_a} - \frac{2U}{C_a} + \tan \beta_1 + \tan \beta_2 \right]
\]

\[
= \frac{C_a}{2U} (\tan \beta_1 + \tan \beta_2)
\]

If \( R = \frac{1}{2} \) then \( \tan \beta_1 + \tan \beta_2 = \frac{U}{C_a} \)

From equation (1) & (2)

- \( \alpha_1 = \beta_2 \)
- \( \beta_1 = \alpha_2 \)

This results in symmetrical velocity triangles across the rotor

- It is assumed that \( \lambda = 1 \)
- Since \( \lambda \) cannot be 1, the degree of reaction achieved will be slightly different from 0.5
Un-symmetrical Blading

If $R > 0.5$, then $\beta_2 > \alpha_1$ and the velocity diagram is skewed to the right. The static enthalpy rise in the rotor exceeds that in the stator (this is also true for the static pressure rise).

If $R < 0.5$, then $\beta_2 < \alpha_1$ and the velocity diagram is skewed to the left. The stator enthalpy (and pressure) rise exceeds that in the rotor.
Three Dimensional Flow

- Two dimensional flow analysis is reasonable when blade height is small i.e. hub/tip radius ratio $\geq 0.8$.

- Three dimensional analysis is to be considered when hub/tip radius ratio $\approx 0.4$.

- In three dimensional flow, the radial component of velocity also needs to be considered alongwith axial and tangential components.

- However, the radial velocity can be ignored if the flow is assumed to be in radial equilibrium from hub to tip.
Vortex Theory

• The Vortex theory is based on the radial equilibrium between centrifugal forces and pressure forces experienced by the flowing medium in a blade passage.

• It is used to obtain the axial velocity distribution across the blade rows from hub to tip by specifying a whirl distribution. With the knowledge of whirl and axial velocity distribution across the radius, one can complete the velocity triangles.
Radial Equilibrium of Fluid Element

Radial equilibrium flow through a rotor blade row

A fluid element in radial equilibrium ($C_r = 0$)
Radial Equilibrium Equation

The basic assumption of the radial equilibrium is that the radial velocity $C_r$ is zero at entry and exit from a blade row.

Starting from the equation of motion in cylindrical coordinates, the variation in $C_r$ is written as

$$C_r \frac{\partial C_r}{\partial r} + \frac{C_\theta}{r} \frac{\partial C_r}{\partial \theta} + C_x \frac{\partial C_r}{\partial x} - \frac{C_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}$$

If there are large number of blades, then variations in $\theta$ direction may be neglected.

$$C_r \frac{\partial C_r}{\partial r} + C_x \frac{\partial C_r}{\partial x} - \frac{C_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}$$

Further, if there is no component of velocity in the radial direction, i.e. if there is radial equilibrium, then $C_r = 0$, and the above equation reduces to

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{C_\theta^2}{r}$$

Radial equilibrium equation indicating that the pressure forces on the fluid particles are balanced by the centrifugal forces.
Alternate Approach

Consider a small element of fluid of mass $dm$ of unit depth and subtending an angle $d\theta$ at the axis, rotating about the axis with tangential velocity $C_\theta$ at radius $r$. The element is in radial equilibrium so that the pressure forces balance the centrifugal forces.

\[
(p + dp)(r + dr)d\theta - prd\theta - (p + \frac{1}{2}dp)drd\theta = dmc_\theta^2/r
\]

Writing \( dm = \rho r d\theta dr \)

and ignoring terms of the second order of smallness, the above equation reduces to:

\[
\frac{1}{\rho} \frac{dp}{dr} = \frac{c_\theta^2}{r}
\]
Axial Velocity Distribution

For incompressible flow:

\[ p_0 = p + \frac{1}{2} \rho \left( C_x^2 + C_\theta^2 \right) \]

and

\[ \frac{1}{\rho} \frac{dp_0}{dr} = \frac{1}{\rho} \frac{dp}{dr} + C_x \frac{dC_x}{dr} + C_\theta \frac{dC_\theta}{dr} \]

\[ = \frac{C_\theta^2}{r} + C_x \frac{dC_x}{dr} + C_\theta \frac{dC_\theta}{dr} \]

\[ = C_x \frac{dC_x}{dr} + \frac{C_\theta}{r} \frac{d}{dr} \left( r.C_\theta \right) \]

If the total pressure is assumed constant along the radius, then

\[ C_x \frac{dC_x}{dr} + \frac{C_\theta}{r} \frac{d}{dr} \left( r.C_\theta \right) = 0 \]

or

\[ \frac{d}{dr} \left( C_x^2 \right) + \frac{1}{r^2} \frac{d}{dr} \left( r.C_\theta \right)^2 = 0 \]

Gives variation of axial velocity with radius
Axial Velocity Distribution

Similarly, for compressible flow:

\[ h_0 = h + \frac{1}{2} \left( C_x^2 + C_\theta^2 \right) \]

\[
\frac{dh_0}{dr} = \frac{dh}{dr} + C_x \frac{dC_x}{dr} + C_\theta \frac{dC_\theta}{dr}
\]

But

\[
T \frac{ds}{dr} = \frac{dh}{dr} + \frac{1}{\rho} \frac{dp}{dr}
\]

\[
\frac{dh_0}{dr} - T \frac{ds}{dr} = \frac{1}{\rho} \frac{dp}{dr} + C_x \frac{dC_x}{dr} + C_\theta \frac{dC_\theta}{dr}
\]

\[
= C_x \frac{dC_x}{dr} + C_\theta \frac{d}{dr} \left( rC_\theta \right)
\]

If \( \frac{dh_0}{dr} = 0 \) and \( T \frac{ds}{dr} = 0 \)

Then

\[
C_x \frac{dC_x}{dr} + C_\theta \frac{d}{dr} \left( rC_\theta \right) = 0
\]

or

\[
\frac{d}{dr} \left( C_x^2 \right) + \frac{1}{r^2} \frac{d}{dr} \left( rC_\theta \right)^2 = 0
\]

Gives variation of axial velocity with radius
Types of Whirl Distribution

The whirl (vortex) distributions normally used in compressor design practice are:

• Free vortex \( r \ C_\theta = constant \)

• Forced vortex \( C_\theta / r = constant \)

• Constant reaction \( R = constant \)

• Exponential \( C_\theta_1 = a - b/r \hspace{1cm} (after \ stator) \)
\[ C_\theta_2 = a + b/r \hspace{1cm} (after \ rotor) \]

• Free vortex whirl distribution results in highly twisted blades and is not advisable for blades of small height.

• The current design practice for transonic compressors is to use constant pressure ratio across the span.
General Whirl Distribution

\[ c_{\theta 1} = ar^n - b/r \text{ (before rotor)}, \]
\[ c_{\theta 2} = ar^n + b/r \text{ (after rotor)}. \]

If \( h_{01} \) and \( h_{02} \) are constant along the radius, then

\[ \dot{W} = h_{02} - h_{01} = U(c_{\theta 2} - c_{\theta 1}) = 2b\Omega = \text{constant} \]

- \( n = 0 \) ➞ Exponential design
- \( n = 1 \) ➞ Constant reaction design
- \( a = 0 \) ➞ Free vortex design
- \( b = 0 \) and \( n = 1 \) ➞ Forced vortex design
Free Vortex Design

\[ r C_\theta = \text{constant} \]

Putting this in the equation for axial velocity distribution, we get

\[ \frac{dC_x}{dr} = 0 \implies C_x = \text{constant}, \]

enabling the radial variation in flow angles, reaction and work to be found.

Let \( r C_{\theta 1} = K_1 \) before the rotor and \( r C_{\theta 2} = K_2 \) after the rotor.

Then

\[ \dot{W} = U(c_{\theta 2} - c_{\theta 1}) = \Omega r(K_2/r - K_1/r) = \text{constant} \]

\[ \tan \beta_1 = \frac{U}{c_x} - \tan a_1 = \frac{\Omega r - K_1/r}{c_x} \]

\[ \tan \beta_2 = \frac{U}{c_x} - \tan a_2 = \frac{\Omega r - K_2/r}{c_x} \]
Free Vortex Design

Degree of reaction

\[ R = \frac{c_x}{2U} (\tan \beta_1 + \tan \beta_2) \]

For \( C_{x1} = C_{x2} = C_x \) and \( \alpha_1 = \alpha_3 = \alpha \)

\[ R = 1 - \frac{k}{r^2} \]

where \( k = (K_1 + K_2)/(2\Omega) \)

- Since \( k \) is positive, the reaction increases from root to tip.
- Likewise, as \( \frac{c_{\theta}^2}{r} \) is always positive, so the static pressure increases from root to tip.
- For the free-vortex flow \( r \ C_\theta = \) constant. Hence, the static pressure variation is

\[ p/\rho = \text{constant} - K/(2r^2) \]
Free Vortex Design

Radial variation of air angles
Constant Reaction Design

\[ n = 1 \quad \Rightarrow \quad c_{\theta 1} = ar - b/r \quad \text{and} \quad c_{\theta 2} = ar + b/r \]

Degree of reaction

\[ R = \frac{c_x}{2U} (\tan \beta_1 + \tan \beta_2) \]
\[ = 1 - \frac{c_x}{2U} (\tan \alpha_1 + \tan \alpha_2) \]
\[ = 1 - a/\Omega = \text{constant} \]

Implicit is the assumption that the axial velocity across the rotor remains constant, which is tantamount to ignoring radial equilibrium in this case.

Assuming constant stagnation enthalpy at entry to the stage and integrating the equation for axial velocity, the distribution of \( C_x \) before and after the rotor is given by:

\[ c_{x1}^2 = \text{constant} - 4a\left(\frac{1}{2}ar^2 - b \ln r\right) \]
\[ c_{x2}^2 = \text{constant} - 4a\left(\frac{1}{2}ar^2 + b \ln r\right) \]
Constant Reaction Design

Radial variation of air angles, constant 50% reaction
Comparison of rotor air angles for free vortex, exponential and constant reaction designs
Comparison of Vortex Designs

- The air angles for free vortex, constant reaction and exponential designs are compared in the figure (previous slide) both at inlet and exit to the rotor.

- The free vortex design exhibits most marked twist over the blade span, with the constant reaction showing the least; the exponential design gives a compromise between the two.

- The aerodynamic loading at the root section of the free vortex is substantially higher than that for either of the other two designs.

- The constant reaction design looks quite attractive, but the radial equilibrium is ignored. This will result in flow velocities not in agreement with the predicted air angles, leading to some loss in efficiency.

- The exponential design results in a substantial variation in axial velocity, both across the annulus and through the stage.
Compressibility Effects

- The effect of excessive air velocities past the blades can be detrimental to the compressor performance.
- Variation of entry Mach Numbers are shown in the figure.
Blade Section-Free Vortex

Rotor

Root

Tip

Stator
Blade Section Constant Reaction
## Choice of Whirl Distribution

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<tr>
<th>Method of design</th>
<th>Work variation with radius</th>
<th>Whirl distribution</th>
<th>Axial velocity variation with radius</th>
<th>Variation of reaction with radius</th>
<th>Radial equilibrium</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free vortex</td>
<td>Constant</td>
<td>( r.C_\theta = \text{constant} )</td>
<td>Constant</td>
<td>Increases with radius</td>
<td>yes</td>
<td>Highly twisted rotor blades</td>
</tr>
<tr>
<td>Forced vortex</td>
<td>Increases with ( r^2 )</td>
<td>( C_\theta /r = \text{constant} )</td>
<td>From radial equilibrium</td>
<td>Varies with radius</td>
<td>yes</td>
<td>Rarely used</td>
</tr>
<tr>
<td>Constant reaction</td>
<td>Constant</td>
<td>( C_\theta = a r + b/r )</td>
<td>From radial equilibrium</td>
<td>Constant</td>
<td>yes</td>
<td>A logical design method. Highly twisted blades</td>
</tr>
<tr>
<td>Exponential</td>
<td>Constant</td>
<td>( C_\theta = a \pm b/r )</td>
<td>From radial equilibrium</td>
<td>Varies with radius</td>
<td>yes</td>
<td>A logical design method.</td>
</tr>
<tr>
<td>Constant ( \alpha_2 )</td>
<td>Supposed constant</td>
<td>Fixed by the condition that ( C_{\theta 2} = \text{constant}; C_{\theta 1} = a - b/r )</td>
<td>Supposed constant</td>
<td>Approx. constant</td>
<td>Ignored</td>
<td>Blades with lesser twist</td>
</tr>
</tbody>
</table>
Deviation Angle $\delta$
Deviation Angle $\delta$

• Referring to cascade notations, if $i = 0$, then $\alpha'_1 = \alpha_1$; but the blade outlet angle $\alpha'_2$ can not be obtained from the air outlet angle $\alpha_2$ until the deviation angle $\delta = \alpha_2 - \alpha'_2$ has been determined.

• Ideally, the mean direction of the air leaving the cascade would be that of the outlet angle of the blades, but in practice it is found that there is a deviation which is due to reluctance of the air to turn through the full angle required by the shape of the blades (shown in the figure).

• The analysis of the relation between the air and the blade outlet angles from cascade tests shows that their difference is dependent mainly on the blade camber and the pitch/chord ratio. It is also dependent on the shape of the camber line of the blade section and on the air outlet angle itself.
Deviation Angle $\delta$

Deviation angle

$$\delta = m \theta \sqrt{\frac{s}{c}}$$

where

$$m = 0.23 \left( \frac{2a}{c} \right)^2 + 0.1 \left( \frac{\alpha_2}{50} \right)$$

- $a$ is the distance of the point of maximum camber from the leading edge of the blade.
- The formula for $m$ is valid for all blade camber line shapes, including circular arc, parabolic arc, etc.
- For circular arc camber line, $2a/c = 1$
- For inlet guide vanes, which are essentially nozzle vanes giving accelerating flow, the deviation angle is given by

$$\delta = 0.19 \theta \left( \frac{s}{c} \right)$$
The isentropic efficiency $\eta_c$ or $\eta_i$ of axial compressor is expressed as the ratio of isentropic work of compression to actual work with friction.

$$\eta_i = \frac{h'_{02} - h'_{01}}{h_{02} - h_{01}} = \frac{T'_{02} - T'_{01}}{T_{02} - T_{01}} = \frac{T'_{2} - T_{1}}{T_{2} - T_{1}}$$

For small velocities the total temperature and static temperature isentropic efficiencies are same.
In Multistage compressors, the designer tries to obtain same efficiency for each stage.

The small stage or polytropic efficiency is defined as the isentropic efficiency of an elemental stage (infinitesimal) of the compressor, which remains constant throughout the whole process of compression.

On a $T$-$s$ diagram, the vertical distance increases with an increase in entropy. The isentropic temperature rise is more for an elemental stage at higher entropy than the temperature rise of another elemental stage at lower entropy.

The sum of the isentropic temperature rise for all the elemental stages is greater than the single overall isentropic rise.
Polypotropic efficiency is the efficiency of a compressor stage operating between infinitesimal pressure differential $\Delta p$. It is used in comparing the performance of two compressors having the same pressure ratio but operating at different temperature levels.

In multistage compressors, the polypotropic efficiency is used in defining the isentropic efficiency of individual stages.
Polytropic Efficiency

Relation between **Polytropic Efficiency** and **Isentropic Efficiency** of a compressor

\[ \eta_c = \frac{\left( \frac{p_{02}}{p_{01}} \right)^{\frac{\gamma-1}{\gamma}}}{\left( \frac{p_{02}}{p_{01}} \right)^{\frac{\gamma-1}{\gamma} \eta_p} - 1} \]

Variation of small stage (polytropic) efficiency of compressor with pressure ratio

Pressure ratio, \( p_{02}/p_{01} \)

Isentropic efficiency, \( \eta_c \)

Polytropic Efficiency

\( \eta_p = 0.9 \)

\( \eta_p = 0.8 \)

\( \eta_p = 0.7 \)
Polytropic Index

Polytropic index $n$ is defined such that

$$\frac{\gamma - 1}{\gamma} \frac{1}{\eta_p} = \frac{n - 1}{n}$$

or

$$\eta_p = \frac{\gamma - 1}{\gamma} \times \frac{n}{n - 1}$$

From consideration of small stage efficiency

$$\frac{T_{02}}{T_{01}} = \left( \frac{p_{02}}{p_{01}} \right)^{(\gamma - 1)/\eta_p \gamma}$$

For ideal compression process

$$\frac{T_{02}'}{T_{01}} = \left( \frac{p_{02}}{p_{01}} \right)^{(\gamma - 1)/\gamma}$$

Stage polytropic efficiency can now be written as

$$\eta_p = \frac{\gamma - 1}{\gamma} \times \frac{n}{n - 1} = \frac{\gamma - 1}{\gamma} \frac{\ln(p_{02}/p_{01})}{\ln(T_{02}/T_{01})}$$
Blade Loading Criteria

De Haller Number

\[
\frac{V_2}{V_1} \geq 0.72 \quad \text{for rotor}
\]

\[
\frac{C_3}{C_2} \geq 0.72 \quad \text{for stator}
\]

Lieblein’s Diffusion Factor

\[
D = 1 - \frac{V_2}{V_1} + \frac{\Delta V_w}{2V_1} \frac{s}{c}
\]

\[
D = \left(1 - \frac{\cos \beta_1}{\cos \beta_2}\right) + \frac{\cos \beta_1}{2} \left(\tan \beta_1 - \tan \beta_2\right) \frac{s}{c} \quad \text{for incompressible flow}
\]

\[
D \not\geq 0.4-0.45 \quad \text{(at rotor tip)} \quad D \not\geq 0.6 \\quad \text{(at rotor hub)}
\]
<table>
<thead>
<tr>
<th>Subsonic Compressors</th>
<th>Transonic Compressors</th>
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<tr>
<td>Inlet relative Mach number is subsonic from hub to tip</td>
<td>Inlet relative Mach number varies from subsonic at the hub to supersonic at the tip</td>
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<td>Pressure ratios up to ~1.2</td>
<td>Pressure ratios form 1.2 to 2.3</td>
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<td>Flatter pressure ratio-mass flow rate characteristics</td>
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<td>Good stall margin</td>
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<td>Thick blade sections, including leading and trailing ends</td>
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<td>Typical blade profiles used are: NACA 65, NACA 63, C4, Double Circular Arc (DCA), Controlled Diffusion Aerofoil (CDA)</td>
<td>Requires special blade profiles, like Multiple Circular Arc (MCA), Arbitrary Mean Camber Line (AMCL), Controlled Diffusion Aerofoil (CDA)</td>
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<td>Used in land based gas turbines, HP stages of aeroengines</td>
<td>Used in modern land based gas turbines, civil and military aeroengines (specially fan and LP stages)</td>
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# Subsonic and Transonic Compressors

<table>
<thead>
<tr>
<th>Category</th>
<th>Inlet</th>
<th>Passage</th>
<th>Exit</th>
<th>Application</th>
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<tr>
<td>I</td>
<td>Subsonic</td>
<td>Subsonic</td>
<td>Subsonic</td>
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<td>II*</td>
<td>Subsonic</td>
<td>Transonic</td>
<td>Subsonic</td>
<td>Rotor, Stator</td>
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<td>III</td>
<td>Supersonic</td>
<td>Transonic</td>
<td>Subsonic</td>
<td>Rotor</td>
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<tr>
<td>IV</td>
<td>Supersonic</td>
<td>Transonic</td>
<td>Supersonic</td>
<td>Rotor</td>
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<table>
<thead>
<tr>
<th>Rotor</th>
<th>Stator</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>High</td>
<td>Normally, but not necessary</td>
</tr>
</tbody>
</table>

* Usually referred to as supercritical flow conditions
Blade Profiles

• The manner of specifying the base profile is shown in the figure.
• The RAF profiles and C series profiles are widely used in British practice.
• NACA 65 series is used in USA.
• The method can be applied to a selected number of points along the blade height.
• Pitch at the mean diameter and the number of blades are fixed, and pitch values at the other points are determined.
• $s/c$ ratio is derived from the air angles; the chord length of the blade at any particular radius will be determined from the pitch.
• This usually results in a blade tapering from root to tip, which is desirable from the point of view of centrifugal stresses.
• By this means, a complete 3-D blade form can be built up.
Blade Profiles

- Controlled Diffusion Airfoil D
- NACA 65/CA(30)09
- Double Circular Arc Airfoil CA(30)09

Subsonic Blading

Transonic Blading

Multiple Circular Arc Airfoil (MCA)
Sources of Loss in Compressors

1. Profile Loss
2. End Wall Loss
3. Secondary Flow Loss
4. Tip Clearance Loss
5. Shock Loss
6. Shock Boundary Layer Interaction

All these losses result in reduction of pressure rise across the compressor stage and degradation of efficiency
Compressor Boundary Layers

- Viscous effects in turbomachines arise due to development of boundary layers on the blade surfaces and the end walls.

- In most compressor flows, the existence of turbulent shear stress is essential to surmount the adverse pressure gradients without separation.

- Generally, the performance of compressor improves as the turbulent stresses get stronger relative to the laminar viscous stresses, that is as the Reynolds number increases.

- Boundary layers are the regions in which the viscous effects are largest.
Blockage through the compressor passages is defined as

\[ B = 1 - \frac{\text{effective flow area}}{\text{geometric flow area}} \]

This can be rewritten in terms of the sum of the displacement thicknesses

\[ B = 1 - \frac{A - \Sigma \delta^*}{A} \]

where \( A \) is the total cross-sectional area and \( \delta^* \) is the displacement thickness, given by

\[ \delta^* = \int \left( 1 - \frac{\rho v}{\rho_\infty V} \right) dy \]

With a uniform flow region outside the viscous one, the evaluation of blockage is unambiguous. But, with the non-uniform flow across the whole passage, there is some arbitrariness in defining the conditions corresponding to the free stream. A general form useful of turbomachines is

\[ B = 1 - \left( \frac{\int \rho v dA}{\int \rho v dA} \right)_{\text{actual}} / \left( \int \rho v dA \right)_{\text{no viscous regions}} \]
Blockage

Total pressure contours in the blade to blade plane at $x/c_{ax} = 0.86$ showing flow blockage at the blade suction surface - hub corner

Bo Song, Ph.D. Diss., Virginia Polytechnic, USA, Nov. 2003

1. Vortex on the hub
2. In the vortex core, flow is transported out normal to surface
3. Vortex (c) is formed by sudden obstruction due to separation
4. Back flow inside the separated region moves upstream and coils up into another vortex (d)
Tip Clearance Flows

- Tip-Leakage Vortex
- Low Pressure
- High Pressure
- Rotor Blade
- Tip
- Hub
- Blade Movement
- Mixing
- Casing
- Separation bubble
- THICK BLADE
- PS
- SS
- VORTEX
Secondary Flows

Secondary flows at exit from a blade passage (viewed in upstream direction)

Secondary vorticity produced by a row of guide vanes
Movement of Shock with Back Pressure

Mach number contours near stall point

Mach number contours near peak efficiency point

Mach number contours for back pressure slightly below the choke value

Mach number contours for lowest back pressure operating point

G.S. Bloch and W.F. O'Brien, AGARD CP 571, May 1995
Complex Flow through an Axial Compressor Rotor
Axial Compressor Characteristics

![Diagram showing the characteristics of an axial compressor with the stagnation pressure ratio and mass flow rate plotted against each other. The diagram includes a surge line and relative values to the design value.]
Axial Compressor Characteristics
Overall Performance

For a multi-stage compressor consisting of a number of stages of the same blade form having a similar efficiency, it would at first appear that the overall pressure ratio will be equal to the stage pressure ratio raised to the power of the number of stages. That is, the overall pressure ratio $R$ for $N$ stages would be given by

$$R = (R_s)^N$$

But this is quite incorrect owing to the influence of the progressively rising temperature of the air as it passes through the compressor. For a fixed stage efficiency and temperature rise this increasing temperature will have the effect of decreasing the pressure ratios of successive stages because the pressure ratio of any one stage is given by

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{0s}}\right]^{\gamma/(\gamma - 1)}$$
Mechanism of Stall Cell Propagation
(Emmon’s Theory)

[Diagram showing stall zones and direction of flow]
Classification of Rotating Stall

Rotating Stall can be classified as:

- Part span and full span
- Progressive and abrupt
- Mild and deep
Classification of Rotating Stall

Hysteresis is an important aspect of compressor characteristics. If the width of the hysteresis loop is large, then it becomes difficult to bring the compressor out of stall regime.
Development of Surge

I. J. Day [7]

Greitzer, 1978
Stall and Surge in a Multistage Compressor
Definition of Stall Margin

Normally, **stall margin** \((SM)\) is defined as the difference of compressor mass flow rates at design point and stall point.

\[
SM = \dot{m}_{design} - \dot{m}_{stall}
\]

**NASA Definition**

\[
SM = 1 - \left\{ \left( \frac{PR_{design}}{PR_{stall}} \right) \ast \left( \frac{\dot{m}_{stall}}{\dot{m}_{design}} \right) \right\}
\]

- D: design point
- S: stall point
Off-Design Operation

Phenomena at off-design operating conditions
Choke Last Stage

• Axial compressors are normally designed for a constant axial velocity through all stages. This means that the annulus area should progressively decrease from inlet to exit because of the increasing density.

\[ \dot{m} = \rho \cdot A \cdot C_a = \text{constant} \]

• When the compressor is run at a speed lower than design, the temperature rise and pressure ratio will be reduced and the density at the rear stages will be lower than the design value. This will increase the axial velocity in the rear stages where choking will eventually occur and limit the mass flow. Thus at low speeds the mass flow will be determined by choking of the rear stages.
Choke First Stage

• As the speed of the compressor is increased, the density in the rear stages is increased to the design value and the rear stages of the compressor can pass all the flow provided by the early stages. Eventually, however, choking will occur at the inlet; the vertical constant speed line is due to choking at the inlet of the compressor.
Stall Last Stage

• When the compressor is operating at design point, all stages are operating at the correct value of $Ca/U$, and hence at the correct incidence. If the operating point is moved from design point A to surge point B at the design speed, the density at the compressor exit will increase due to the increase in pressure ratio. But there is slight reduction in the mass flow rate. Both these effects reduce the axial velocity in the last stage, thus increasing the incidence. A relatively small increase in incidence will cause the rotor blades to stall. Thus, surge at high speeds is due to stalling of the last stage.
Stall First Stage

• If the speed is reduced from A to C, the mass flow generally reduces more rapidly than the speed, thus decreasing the axial velocity at inlet and causing the incidence on the first-stage blade to increase. The axial velocity in the later stages, however, is increased because of the lower pressure and density, so causing the incidence to decrease. Thus at low speeds, surging is probably due to stalling of the first stage.

• It is possible for axial compressors to operate with several of the upstream stages stalled. And, this is thought to account for the ‘kink’ in the surge line which is often encountered in high performance compressors.
Stall due to Negative Incidence

• At conditions far removed from surge, the density will be much lower than required. The resulting high axial velocities will induce large decrease in incidence, which will eventually result in stalling at negative incidences. The efficiency will be very low under these operating conditions.
• As the design pressure ratio is increased, the difference in density between design and off design conditions will be increased and the probability of blades stalling due to incorrect axial velocities will be much higher. The effect of increased axial velocity towards the rear of the compressor can be alleviated by means of blow off, where air is discharged from the compressor at some intermediate stage to reduce the mass flow through the later stages.

• *Blow off* is wasteful, but sometimes it is necessary to prevent the engine running line intersecting the surge line.
Twin Spool Compressor

- Reduction of compressor speed from the design value will cause an increase of incidence in the first stage and a decrease of incidence in the last stage; clearly the effect will increase with pressure ratio. The incidence could be maintained at the design value by increasing the speed of the last stage and decreasing the speed of the first stage as indicated in the figure. These conflicting requirements can be met by splitting the compressor into LP and HP compressor driven by LP and HP turbines. The speed of the two spools are mechanically independent but a strong aerodynamic coupling exists, which has the desired effect on the relative speeds when the gas turbine is operating at an off-design point.
Rotor Construction

- Constant meanline radius
- Constant hub radius
- Constant tip radius
Comments

• The use of a constant outer diameter results in the mean blade speed increasing with stage number, and this in turn implies that for a given temperature rise, $\Delta C_w$ is reduced. The fluid deflection is correspondingly reduced with a beneficial increase in de Haller number.

• Alternatively, because of the higher blade speed, a higher temperature rise could be achieved in the later stages; this might permit the required pressure ratio to be obtained in less number of stages.

• Note that the simple equations derived on the basis of $U = \text{constant}$ are then not valid, and it would be necessary to use the appropriate values of $U_1$ and $U_2$; the stage temperature rise would then be given by $\lambda (U_2 C_{w2} - U_1 C_{w1})/c_p$.

• Compressors which use constant inner diameter, constant mean diameter or constant outer diameter will all be found in service.
Comments

• The use of a constant inner diameter is often found in industrial units, permitting the use of rotor discs of the same diameter, which lowers the cost.

• Constant outer diameter compressors are used where the minimum number of stages is required, and these are commonly found in aircraft engines.

• The compressor annulus of the Olympus 593 engine used in Concorde employs a combination of these approaches; the LP compressor annulus has a virtually constant inner diameter, while the HP compressor has a constant outer diameter.

• The accessories are packed around the HP compressor annulus and the engine when fully equipped is almost cylindrical in shape, with the compressor inlet and turbine exit diameters almost equal. In this application, frontal area is of critical importance because of the high supersonic speed.
Olympus 593 Mk 610 Engine

Compressor: Axial – 7 high pr. stages; 7 low pr. stages

Turbine: 1 low pr. stage; 1 high pr. stage

Weight: 3180 kg
Length: 7.11m
Diameter: 1.21m
Thrust: 170kN
Session Summary

- Axial compressors are used in almost all gas turbine systems.
- Velocity triangles represent the changes in flow parameters across the blade rows.
- Tip clearance, secondary flows, boundary layers and shocks are responsible for loss generation in blade rows.
- Stall and surge are important phenomena that limit the stable operating range of the compressors.
- It is a complex piece of equipment to design and manufacture.
Thank you