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# System Modeling & Control

Presented by

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# Module III

# Time Response Analysis

# INTRODUCTION

The concept of **poles** and **zeros**, fundamental to the analysis of and design of control system, **simplifies the evaluation of system response**.

The **poles** of a transfer function are:

- i. Values of the Laplace Transform variables  $s$ , that cause the transfer function to become **infinite**.
- ii. Any **roots of the denominator** of the transfer function that are common to roots of the numerator.

The **zeros** of a transfer function are:

- i. The values of the Laplace Transform variable  $s$ , that cause the transfer function to become **zero**.
- ii. Any **roots of the numerator** of the transfer function that are common to roots of the denominator.

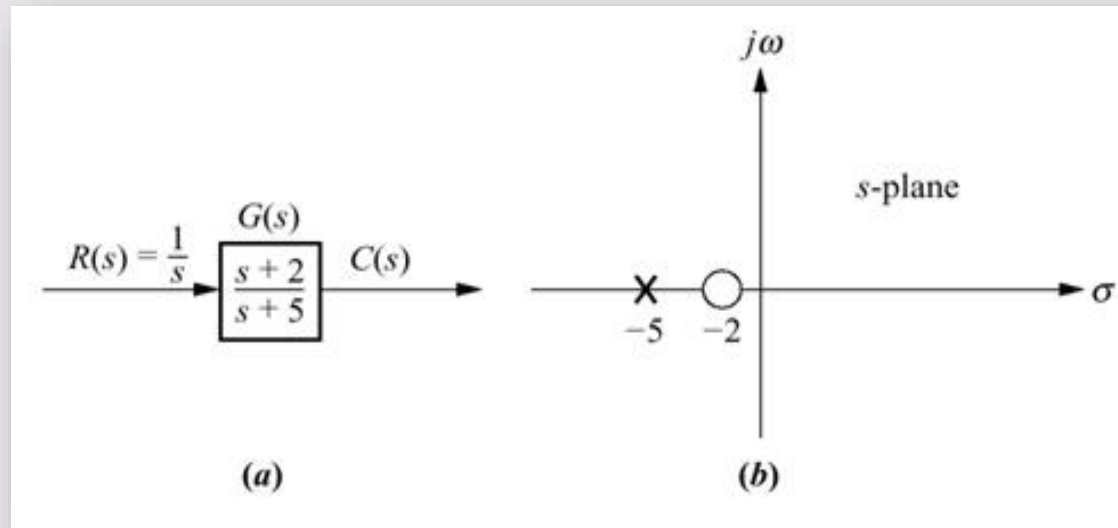


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# INFLUENCE OF POLES ON TIME RESPONSE

The **output response** of a system is a sum of

- i. **Forced** response
- ii. **Natural** response



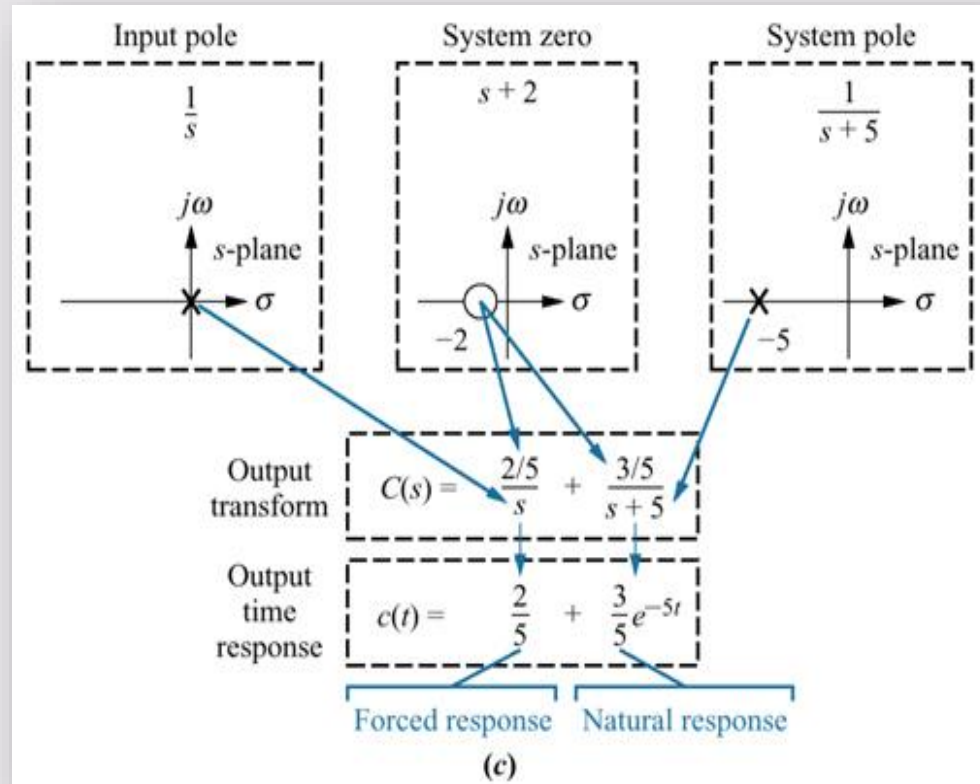
- a) System showing an input and an output
- b) Pole-zero plot of the system



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# INFLUENCE OF POLES ON TIME RESPONSE



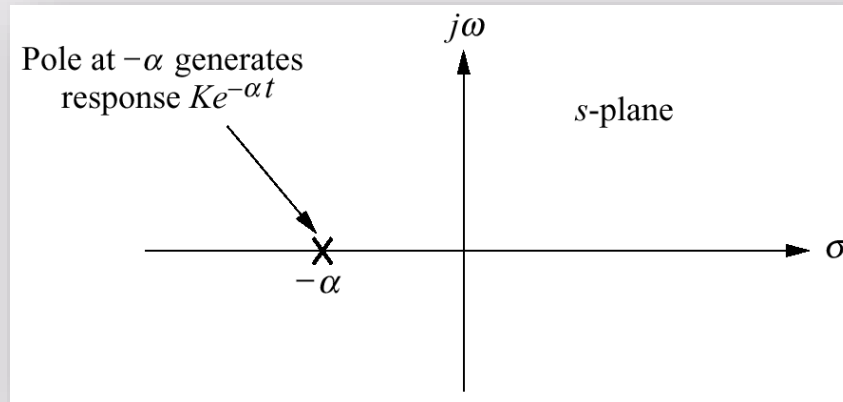
- c) Evolution of a system response. Follow the **blue arrows** to see the evolution of system component generated by the pole or zero



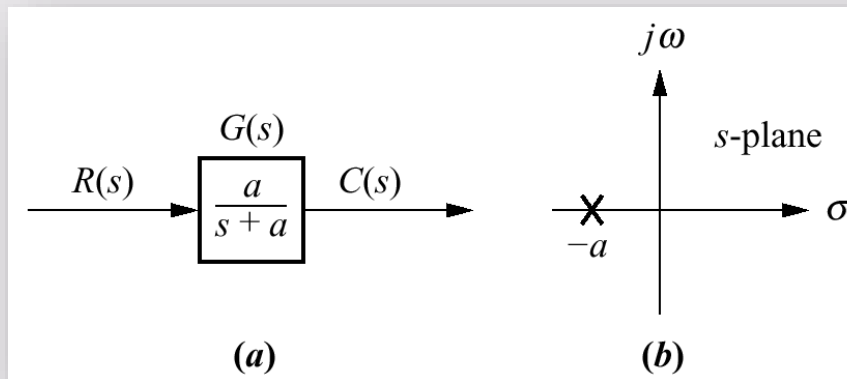
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# INFLUENCE OF POLES ON TIME RESPONSE



Effect of a real-axis pole upon transient response



- First-order system
- Pole plot of the system



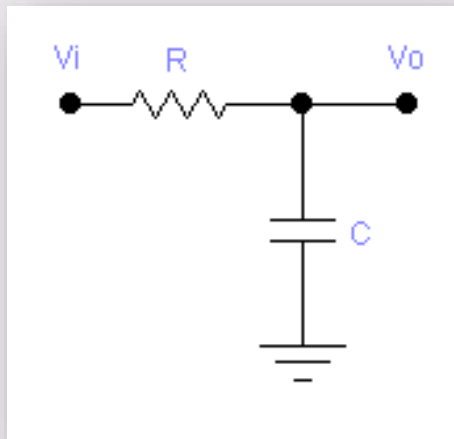
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# FIRST-ORDER SYSTEM

General form:

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{\tau s + 1}$$

Problem: **Derive the transfer function for the following circuit**

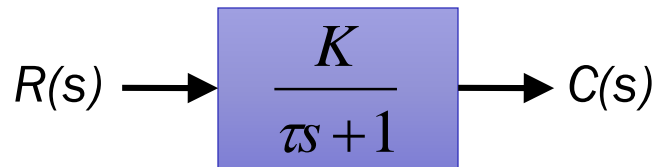


$$G(s) = \frac{1}{RCs + 1}$$

# FIRST-ORDER SYSTEM

Transient Response: Gradual change of output from initial to the desired condition.

Block diagram representation:



Where,

$K$  : Gain

$\tau$  : Time constant

By definition itself, the input to the system should be a step function which is given by the following:

$$R(s) = \frac{1}{s}$$





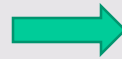
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# FIRST-ORDER SYSTEM

General form:

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{\tau s + 1}$$



$$C(s) = G(s)R(s)$$

Output response:

$$\begin{aligned} C(s) &= \left(\frac{1}{s}\right) \left(\frac{K}{\tau s + 1}\right) \\ &= \frac{A}{s} + \frac{B}{\tau s + 1} \end{aligned}$$

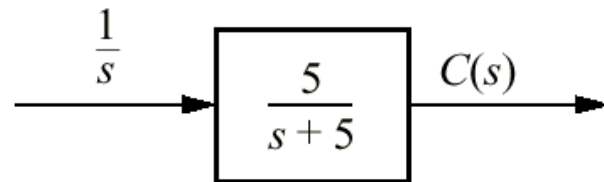
$$c(t) = A + \frac{B}{\tau} e^{-t/\tau}$$



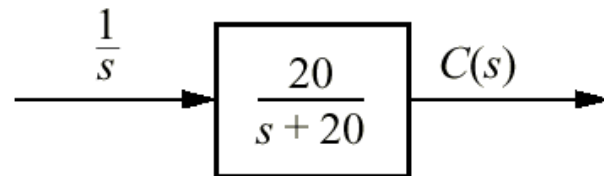
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# FIRST-ORDER SYSTEM

Problem: Find the forced and natural responses for the following systems



(a)



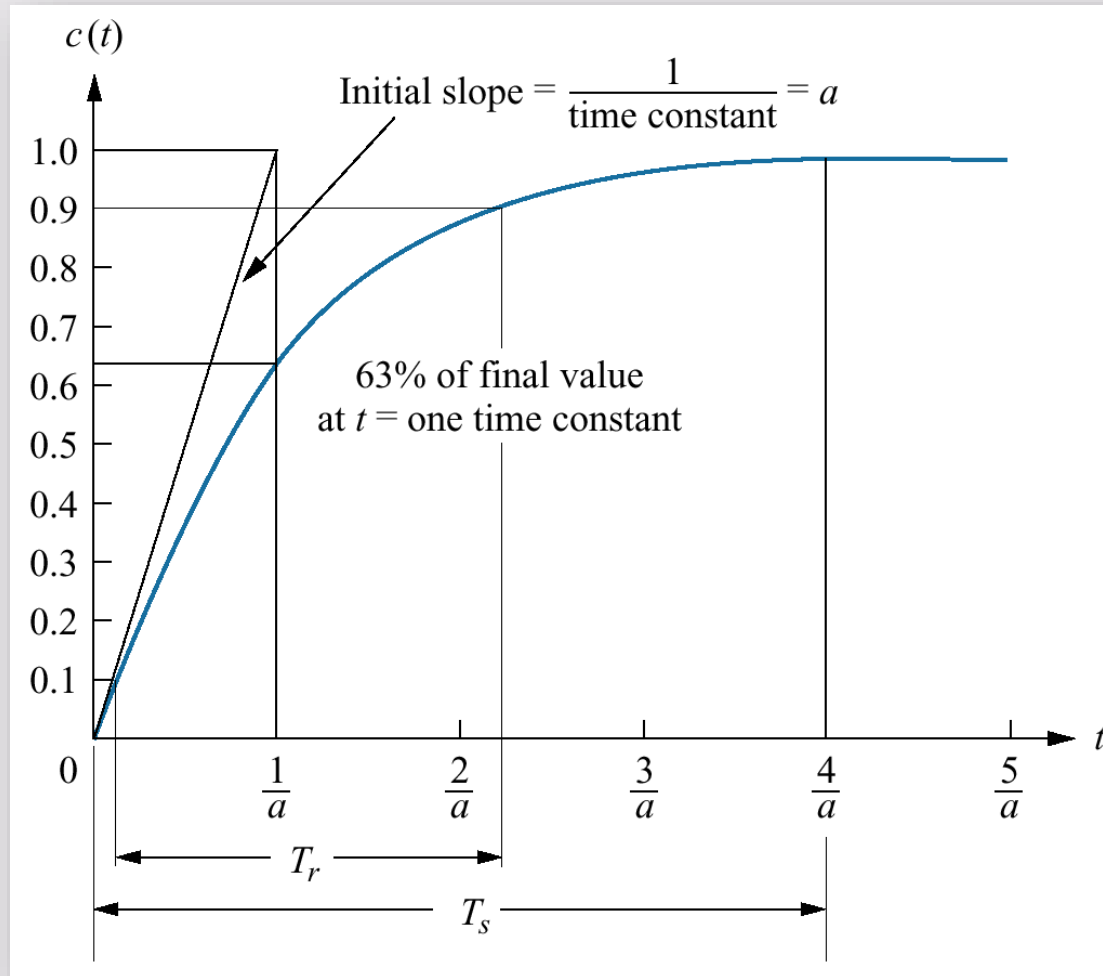
(b)



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# FIRST ORDER SYSTEM



First-order system response to a unit step



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# SECOND-ORDER SYSTEM

General form:

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Where,

K : Gain

$\zeta$  : Damping ratio

$\omega_n$  : Undamped natural frequency

Roots of denominator:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$



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# SECOND-ORDER SYSTEM

Natural frequency,  $\omega_n$

Frequency of oscillation of the system without damping.

Damping ratio,  $\zeta$

Quantity that compares the exponential decay frequency of the envelope to the natural frequency.

$$\zeta = \frac{\text{Exponential decay frequency}}{\text{Natural frequency (rad/s)}}$$



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# SECOND-ORDER SYSTEM

Problem: Find the step response for the following transfer function

$$G(s) = \frac{225}{s^2 + 30s + 225}$$

Answer:

$$c(t) = 1 - e^{-15t} - 15te^{-15t}$$



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# SECOND-ORDER SYSTEM

**Problem: For each of the transfer function, find the values of  $\zeta$  and  $\omega_n$ , as well as characterize the nature of the response.**

a) 
$$G(s) = \frac{400}{s^2 + 12s + 400}$$

b) 
$$G(s) = \frac{900}{s^2 + 90s + 900}$$

c) 
$$G(s) = \frac{225}{s^2 + 30s + 225}$$

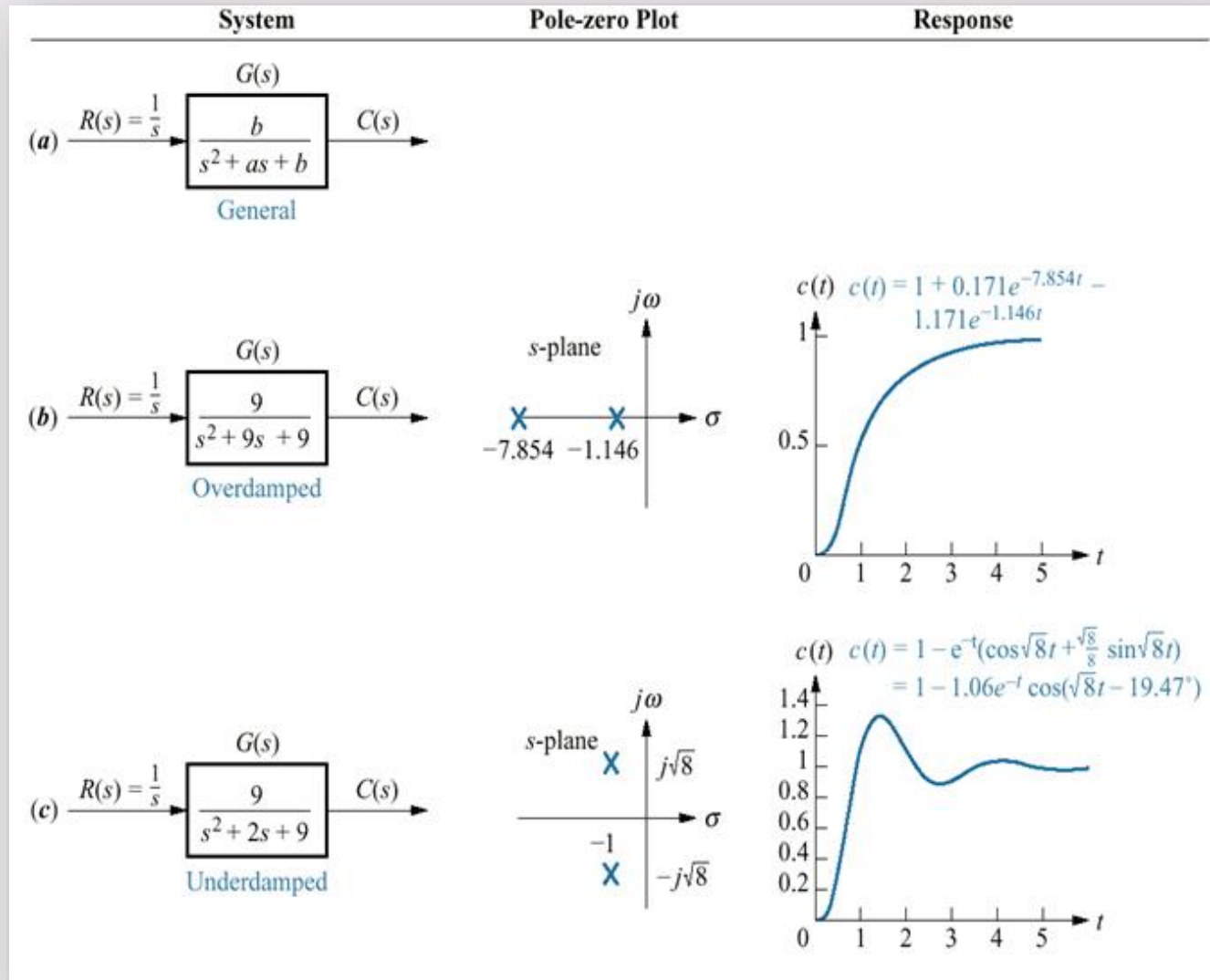
d) 
$$G(s) = \frac{625}{s^2 + 625}$$



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# SECOND-ORDER SYSTEM







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# SECOND-ORDER SYSTEM

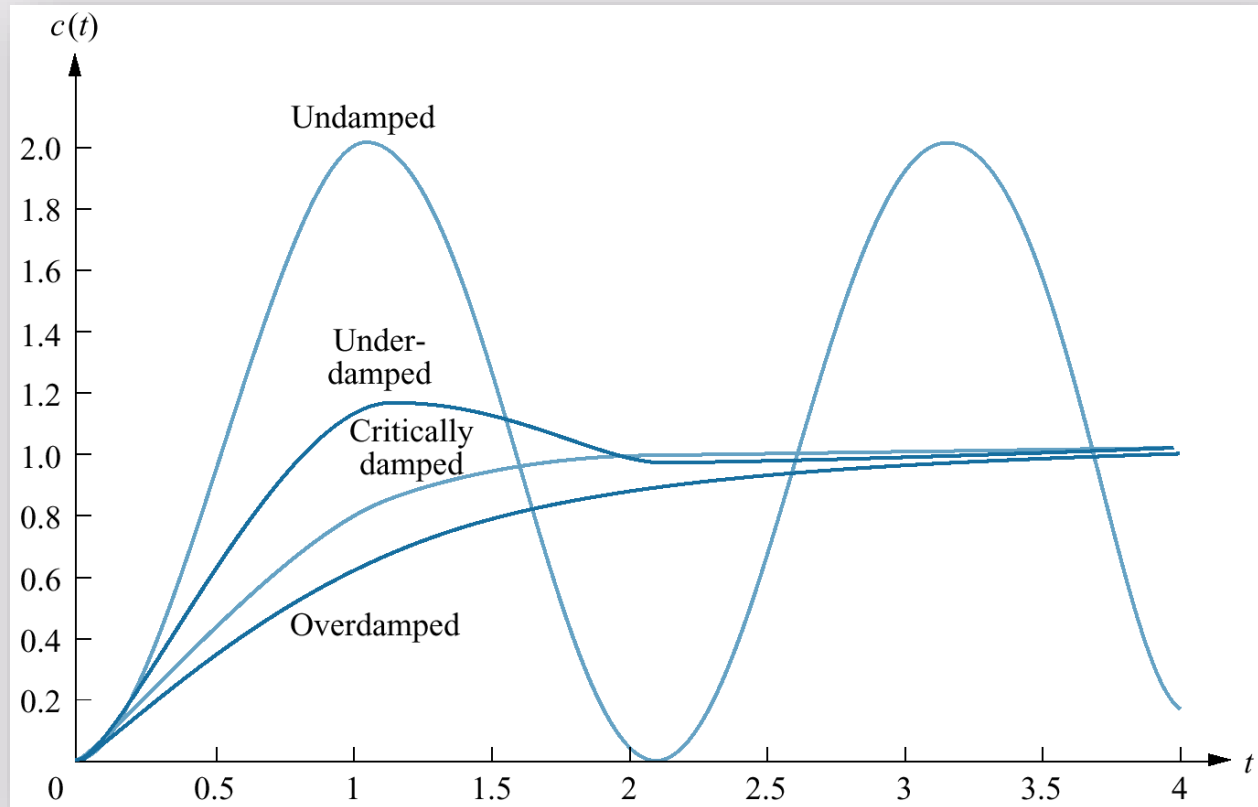
System	Pole-zero Plot	Response
<p>(d)</p> <p style="text-align: center;"><math>G(s) = \frac{9}{s^2 + 9}</math></p> <p style="text-align: center;">Undamped</p>	<p>s-plane</p>	<p><math>c(t)</math></p> <p style="text-align: center;"><math>c(t) = 1 - \cos 3t</math></p>
<p>(e)</p> <p style="text-align: center;"><math>G(s) = \frac{9}{s^2 + 6s + 9}</math></p> <p style="text-align: center;">Critically damped</p>	<p>s-plane</p>	<p><math>c(t)</math></p> <p style="text-align: center;"><math>c(t) = 1 - 3te^{-3t} - e^{-3t}</math></p>



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# SECOND-ORDER SYSTEM

## Step responses for second-order system damping cases

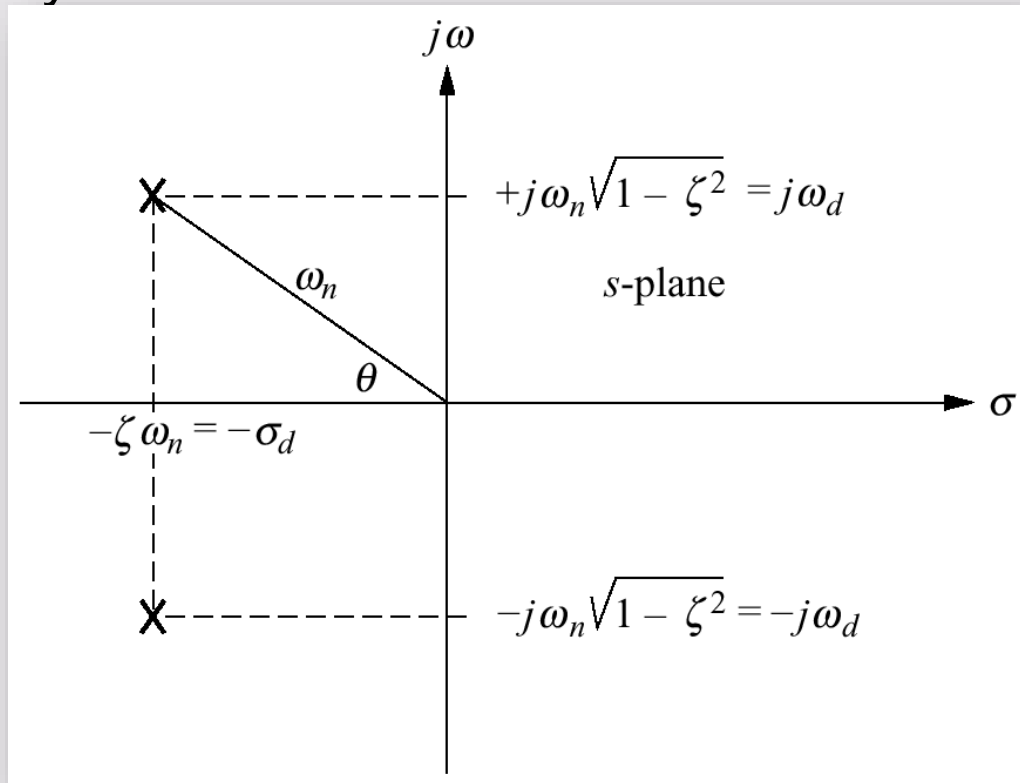




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# SECOND-ORDER SYSTEM

Pole plot for the underdamped second-order system

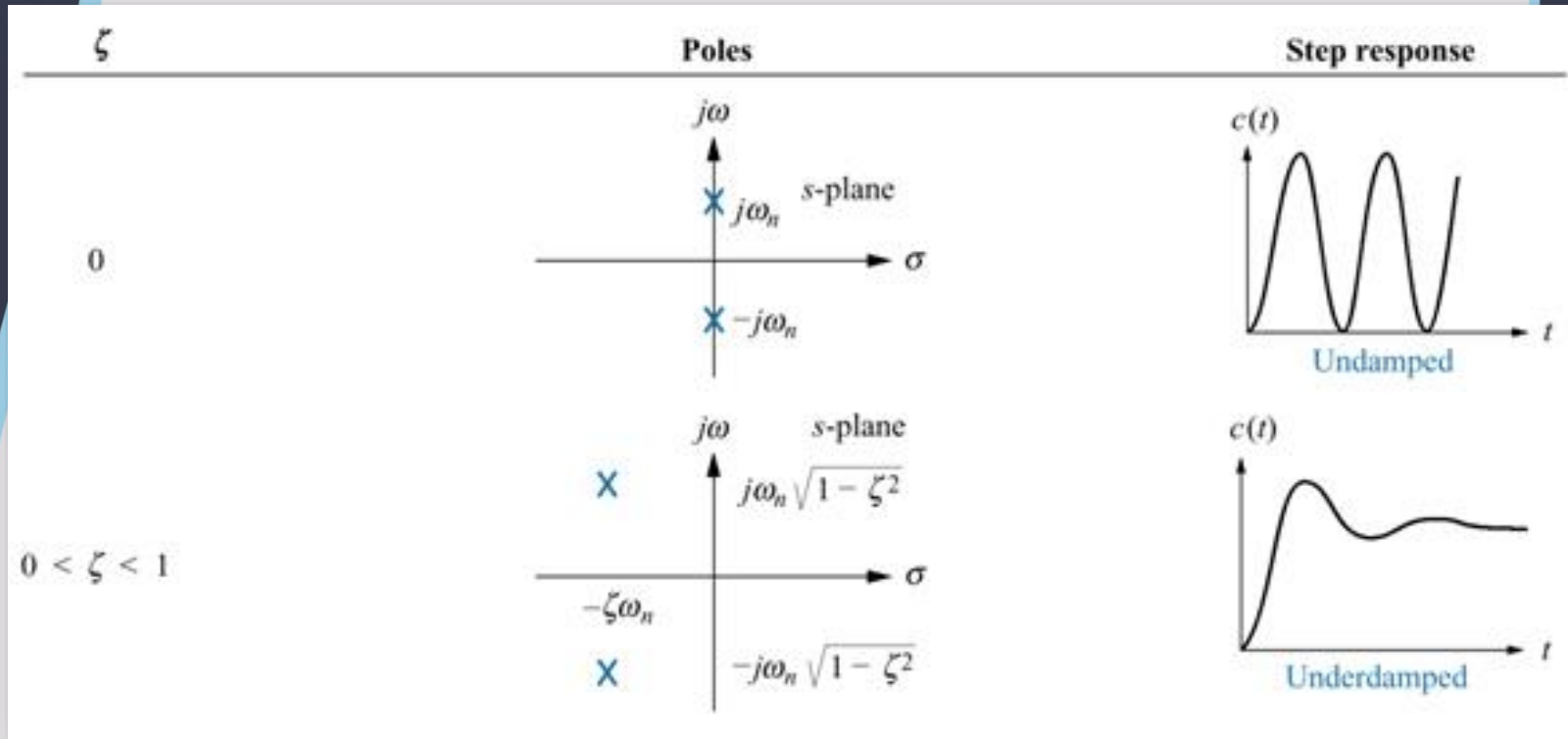




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# SECOND-ORDER SYSTEM

Second-order response as a function of damping ratio

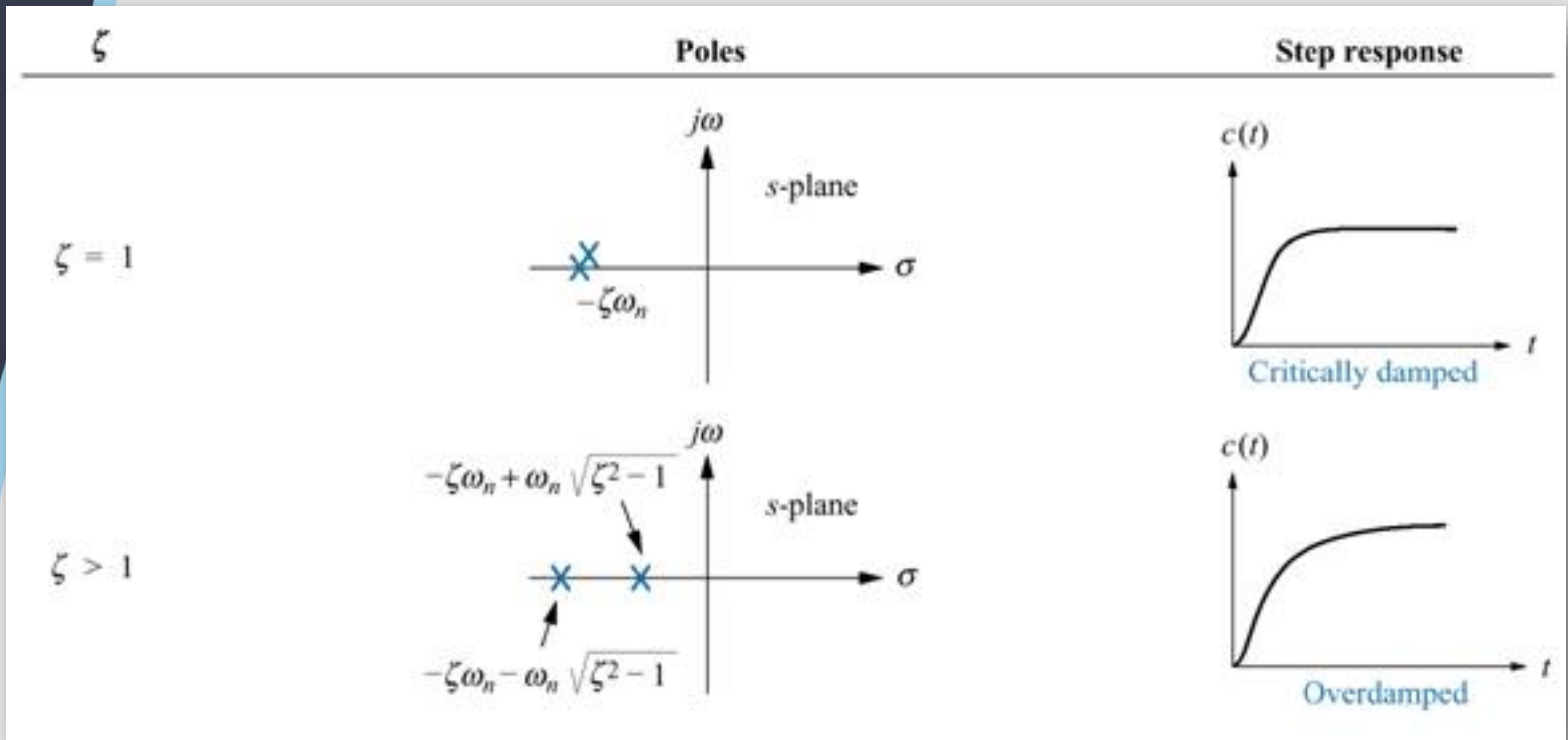




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# SECOND-ORDER SYSTEM

## Second-order response as a function of damping ratio





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# SECOND-ORDER SYSTEM

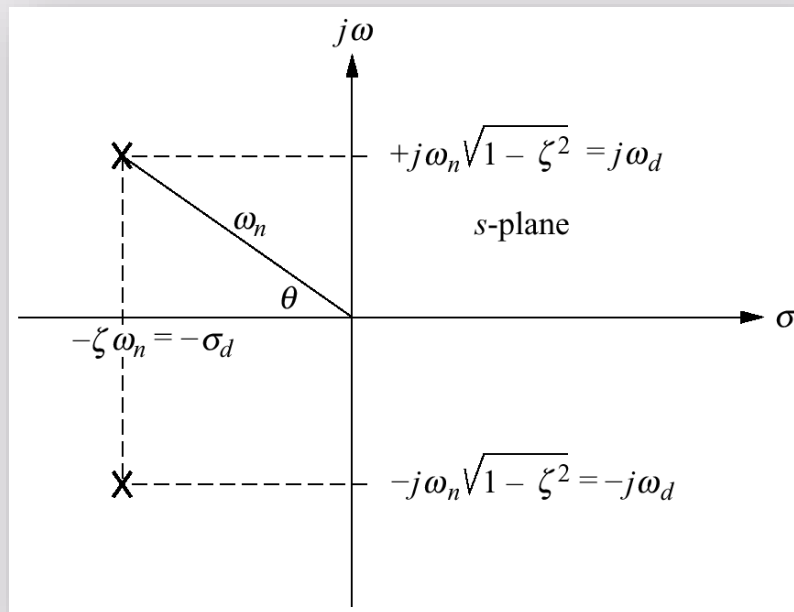
When  $0 < \zeta < 1$ , the transfer function is given by the following.

$$G(s) = \frac{K\omega_n^2}{(s + \zeta\omega_n + j\omega_d)(s + \zeta\omega_n - j\omega_d)}$$

Where,

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

Pole position:

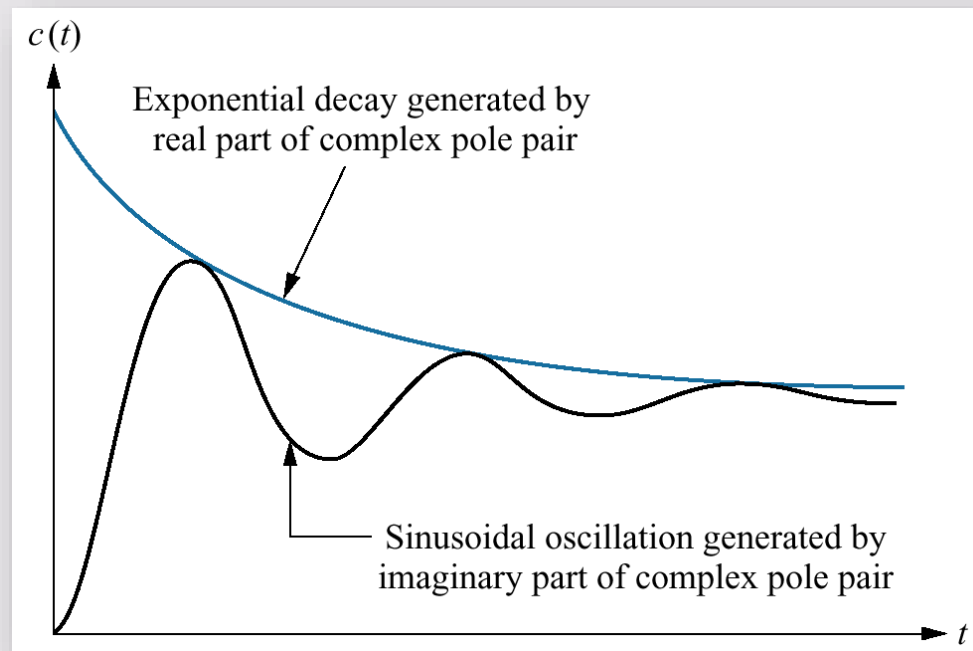




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# SECOND-ORDER SYSTEM

Second-order response components generated by complex poles





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# SECOND-ORDER SYSTEM

Second-order underdamped responses for damping ratio value

