

Performance

12. Gliding Flight (Steady State)

If the engine is turned off, ($T = 0$), and one desires to maintain airspeed, it is necessary to put the vehicle at such an attitude that the component of the gravity force in the direction of the velocity vector balances the drag. The equations of motion are given by:

$$\begin{aligned} 0 - D - W \sin \gamma &= m \dot{V} = 0 \\ L - W \cos \gamma &= m V \dot{\gamma} = 0 \end{aligned} \quad (1)$$

where γ is the flight path angle (the angle the velocity makes with the horizontal).

If we divide one equation by the other, we get:

$$\tan \gamma = -\frac{D}{L} = -\frac{1}{\frac{L}{D}} \quad (2)$$

We see from Eq. (2) that the flight path angle is negative, as expected! We can then define the glide angle as the negative of the flight path angle and write:

$$\tan \gamma_1 = \frac{1}{\left(\frac{L}{D}\right)} \quad (3)$$

where $\gamma_1 =$ glide angle (and is positive).

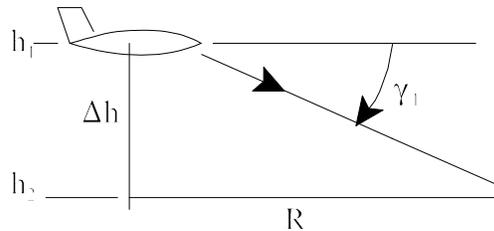
We can observe the following: 1) the glide angle depends only on L/D and is independent of the weight of the vehicle!, 2) the flattest glide angle occurs at the maximum L/D .

Glide Range

The glide range is how far it travels along the ground during the glide descent. It is easy to see from the figure that

$$\tan \gamma_1 = \frac{h_1 - h_2}{R} = \frac{-\Delta h}{R}$$

or



$$R = \frac{h_1 - h_2}{\tan \gamma_1} = \frac{L}{D} (h_1 - h_2) \quad (4)$$

Hence the range for gliding flight depends on the L/D and Δh . It is clear that the maximum range occurs when L/D is maximum. Therefore the maximum range glide is flown at the minimum drag airspeed, V_{md} .

Small Glide Angle Assumption

In most cases, the glide angle will be small for an equilibrium glide. Under these circumstances, we can make the following approximations ($\gamma_1 \ll \pi$):

$$\cos \gamma_1 \approx 1 \quad \sin \gamma_1 \approx \tan \gamma_1 \approx \gamma_1 \approx \frac{1}{(L/D)}$$

The most important result of this assumption is that we can make the approximation that

$$L = W \cos \gamma \approx W \quad \Rightarrow \quad V = \sqrt{\frac{W}{1/2 \rho S C_L}} \quad (5)$$

Hence we can use the weight in order to compute the airspeed. Without this assumption the calculations can become more difficult.

Rate of Climb (Sink)

The rate of climb is given by

$$\dot{h} = V \sin \gamma \quad (6)$$

From Eq. (1) we can eliminate $\sin \gamma$ to get

$$\dot{h} = -V \frac{D}{W} \approx -V \frac{D}{L} = -V \frac{C_D}{C_L} = -\sqrt{\frac{W}{1/2 \rho S C_L}} \frac{C_D}{C_L}$$

or

$$\dot{h} = -\sqrt{\frac{W}{1/2 \rho S}} \frac{C_D}{C_L^{3/2}} \quad (7)$$

We can note the rate of climb is negative (hence a sink rate), and that it is directly related to the quantity $C_D / C_L^{3/2}$. Therefore, if we want to minimize the sink rate, we must minimize the quantity $C_D / C_L^{3/2}$. Recall, however that this is the same requirement that we had for minimizing the power required. Note that if we minimize the sink rate, we maximize the time to descend or maximize the time aloft, or endurance.

Summary

- 1) For maximum range, we must operate at the maximum L/D condition (minimum drag)
- 2) For maximum endurance (minimum sink rate) we must operate at the minimum power require condition.

Time to descend

From Eq. (7) we can see that the rate of descent depends on the altitude (through the density, ρ). So to get an exact solution for the time to descend we would need to include density variations in our calculations. However, if the change in altitude is relatively small (whatever that means?) we can approximate the time to descend by assuming the density is constant and, in addition, assume we fly at constant angle-of-attack (constant C_L and C_D). Under these circumstances and assumptions the rate of descent is constant. Thus we have:

$$\text{time of flight} = TOF = \frac{-\Delta h}{\dot{h}} \tag{8}$$

where \dot{h} is assumed constant. Generally the value of \dot{h} used is that calculated for an **altitude halfway between the initial and final altitudes**. If large altitude changes are involved, the above equation can be used for several smaller increments in altitude and the results summed.

Example:

A sailplane weighs 1000 lbs with a wing loading $W/S = 12.5 \text{ lbs/ft}^2$. The drag polar is given by $C_L = 0.010 + 0.022 C_L^2$. Find the time to glide from 1000 ft to sea-level flying at minimum sink rate conditions.

Minimum sink rate occurs at the minimum power required flight condition.

$$C_{L_{mp}} = \sqrt{\frac{3 C_{D_{0L}}}{K}} = \sqrt{\frac{3 (0.010)}{0.022}} = 1.17 \qquad C_{L_{mp}} = 4 C_{D_{0L}} = 4 (0.010) = 0.04$$

$$\left(\frac{L}{D}\right)_{mp} = \frac{1.17}{0.04} = 29.2 \quad \Rightarrow \quad \tan \gamma_1 = \frac{1}{\left(\frac{L}{D}\right)} \quad \Leftrightarrow \quad \gamma_1 = 1.96 \text{ deg}$$

Note that $(L/D)_{\max} = 33.7$

$$V_{mp} = \sqrt{\frac{W}{\rho S C_{L_{mp}}}} \Big|_{h=500} = \sqrt{\frac{12.5}{0.002343(1.17)}} = 95.50 \text{ ft/sec}$$

$$\dot{h} = V \sin \gamma = (95.50) \sin(-1.96) = -3.27 \text{ ft/sec}$$

$$TOF = \frac{-\Delta h}{\dot{h}} = \frac{0 - 1000}{-3.27} = 306 \text{ sec}$$

The range of the glide is: $R = \frac{L}{D} (-\Delta h) = 29.2(1000) = 29,200 \text{ ft}$

Example

Consider our executive jet that has a weight of 10,000 lbs, a wing area of 200 ft², and a parabolic drag polar $C_D = 0.02 + 0.05 C_L^2$. We would like to calculate the glide range and endurance from 20,000 ft. We would like to compare the range and endurance for a max range flight condition with that for a max endurance flight condition.

Max Range Flight Condition

The max range condition occurs at max L/D or the min drag condition:

$$C_{L_{md}} = \sqrt{\frac{C_{D_{0L}}}{K}} = \sqrt{\frac{0.02}{0.05}} = 0.632 \quad C_{D_{md}} = 2 C_{D_{0L}} = 2(0.02) = 0.04$$

and $\left(\frac{L}{D}\right)_{md} = \frac{1}{2\sqrt{C_{D_{0L}}K}} = \frac{C_{L_{md}}}{C_{D_{md}}} = \frac{0.632}{0.04} = 15.81$

Range (max)

$$R = \Delta h \frac{L}{D} \Rightarrow R_{\max} = \Delta h \left(\frac{L}{D} \right)_{\max} = 20,000 (15.81) = 316227 \text{ ft} = 59.9 \text{ miles}$$

The endurance for this flight is given by: $TOF = \frac{\Delta h}{\dot{h}_{avg}}$, where $\dot{h}_{avg} = V_{10K} \sin \gamma = V_{10K} \frac{1}{L/D}$

$$V_{10K} = \sqrt{\frac{W}{1/2 \rho_{10K} S C_L}} = \sqrt{\frac{10000}{1/2 (0.001756) (200) (0.632)}} = 300.21 \text{ ft/sec}$$

(Could also use $\dot{h} = \sqrt{\frac{W}{1/2 \rho S} \frac{C_D}{C_L^{3/2}}}$)

then

$$TOF = \frac{20000}{300.21 (1/15.81)} = 1053 \text{ sec} = 17.55 \text{ min (Endurance for max range conditions)}$$

Max Endurance Flight Condition

Maximum endurance glide occurs at the minimum power required flight condition.

$$C_{L_{mp}} = \sqrt{\frac{3 C_{D_{0L}}}{K}} = \sqrt{\frac{3 (0.02)}{0.05}} = 1.095 \quad C_{D_{mp}} = 4 C_{D_{0L}} = 4 (0.02) = 0.08$$

$$\left(\frac{L}{D} \right)_{mp} = \frac{V_{L_{mp}}}{C_{D_{mp}}} = \frac{1.095}{0.08} = 13.693 \quad (\text{Less than } L/D|_{\max})$$

Range:

$$\text{Glide range: } R_{mp} = \Delta h \frac{L}{D} = 20000 (13.693) = 273861 \text{ ft} = 51.87 \text{ miles}$$

Endurance;

$$V_{10K} = \sqrt{\frac{W}{1/2 \rho_{10K} S C_L}} = \sqrt{\frac{10000}{1/2 (0.001756) 200 (1.095)}} = 228.1 \text{ ft/sec}$$

$$TOF_{\max} = \frac{\Delta h}{\dot{h}} \frac{\Delta h}{V \sin \gamma} = \frac{\Delta h}{V 1/(L/D)} = \frac{20000}{(228.1)(1/13.693)} = 1200.7 \text{ sec} = 20.0 \text{ min}$$

Summary:

Flight condition →	Max Range	Max Endurance
Range	59.9 mi	51.87 mi
Endurance (TOF)	17.55 min	20.0 min

As expected, max range conditions give biggest range, and max endurance conditions give max TOF.

In addition we could have selected a specified L/D. For the case of L/D = 10 we obtain the following results:

$$L/D = 10, \quad \text{Range} = 37.88 \text{ mi} \quad \text{Endurance: } TOF_1 = 6.63 \text{ min, or } TOF_2 = 18.60 \text{ min}$$

Note that there are two flight conditions at L/D = 10 ! The range is the same, but the endurance is not.

An Exact Solution for Glide Endurance

It turns out that we can get an exact solution for the glide time if we assume a standard atmosphere. The equation we developed for sink rate is:

$$\dot{h} = -\sqrt{\frac{W}{1/2 \rho S}} \frac{C_D}{C_L^{3/2}} \quad \text{Or} \quad dt = -\sqrt{\frac{S}{2W}} \frac{C_L^{3/2}}{C_D} \sqrt{\rho(h)} dh$$

In the troposphere:

$$\rho(h) = \rho_{SL} \left(1 + \frac{K}{T_{SL}} h \right)^{-\left(\frac{g}{RK} + 1\right)}$$

where K is the temperature gradient in the troposphere, and R is the gas constant. If we integrate the time equation above from altitude to sea-level, we get:

$$TOF = \sqrt{\frac{S}{2W}} \frac{C_L^{3/2}}{C_D} \left\{ \frac{\sqrt{\rho_{SL}} \frac{T_{SL}}{K}}{-1/2 \left(\frac{g}{RK} + 1 \right) + 1} \left[\left(1 + \frac{K}{T_{SL}} h \right)^{-1/2 \left(\frac{g}{RK} + 1 \right) + 1} - 1 \right] \right\} \quad (9)$$

If we substitute minimum power conditions into Eq. (9), with $h = 20,000$ ft, for the same aircraft as in the example problems, we get the time of flight, $TOF = 20.0$ min!

For other altitudes to sea-level:

h(ft)	0	5K	10K	15K	20K	25K	30K	35K	40K
TOF(sec)	0	337	649	938	1203	1448	1671	1875	2061