

# Tree Form

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- **Tree form**

- **Tree form theories**

- **Form ratios**

- **Taper equation**

# Tree Form

- Tree bole tapers generally from base to tip → dia ↓ with ht
- Taper varies rate-wise & shape-wise for different species
- Taper also get influenced by a wide range of environmental and contextual factors
- Different growth rate for different parts of the bole
- A complex interaction between the bole form & the tree crown, bole gets affected by the change in crown

# Tree Form

- Shape & branching habit vary with species
- Tree form is the shape of bole i.e. rate of diminution of diameter with height (base to tip)
- Required to know important aspects of form that may affect marketability

## **Stem or bole form**

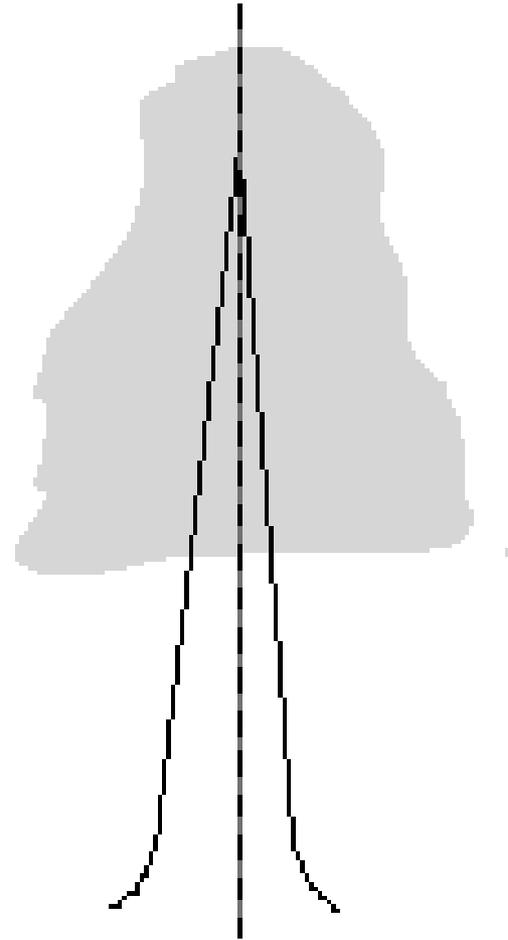
- rate of taper of a log or stem
- development of the form of stem depends on the mechanical stresses to which the tree is subjected
- stress come from dead weight of stem and crown and the wind force
- wind force causes the tree, to construct the stem in such a way that the relative resistance to shear is same at all the points on the longitudinal axis of the stem

# Tree Form

- Complex, not easy to approximate in known geometric shape in total
- Some general [**geometric shapes**] approximate portions of the tree bole, but there are many inflections and points of irregularity
- Knowledge of the tree form can help in
  - ❖ better estimation of bole volume or biomass
  - ❖ better understanding of the growth conditions

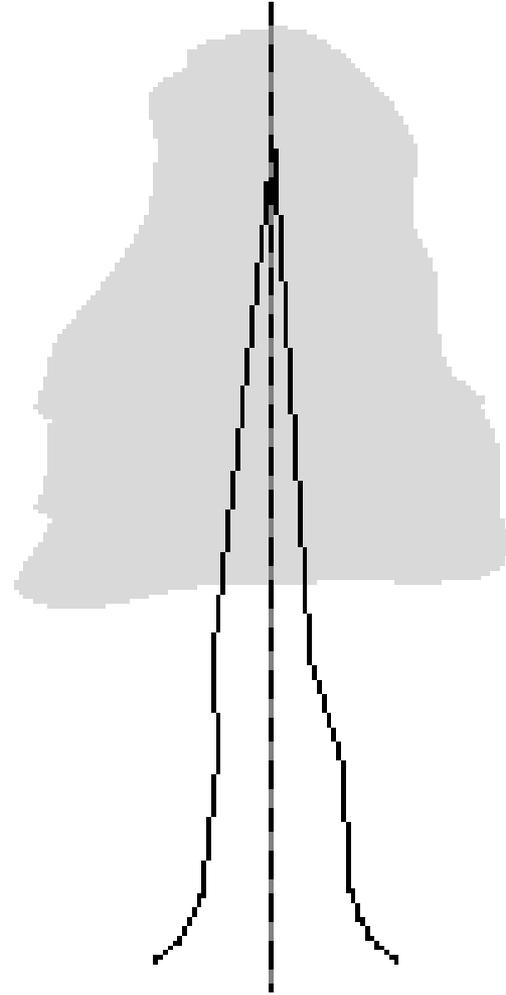
# Perfect tree form

- straight bole
- fine branches
- no apparent defects etc.



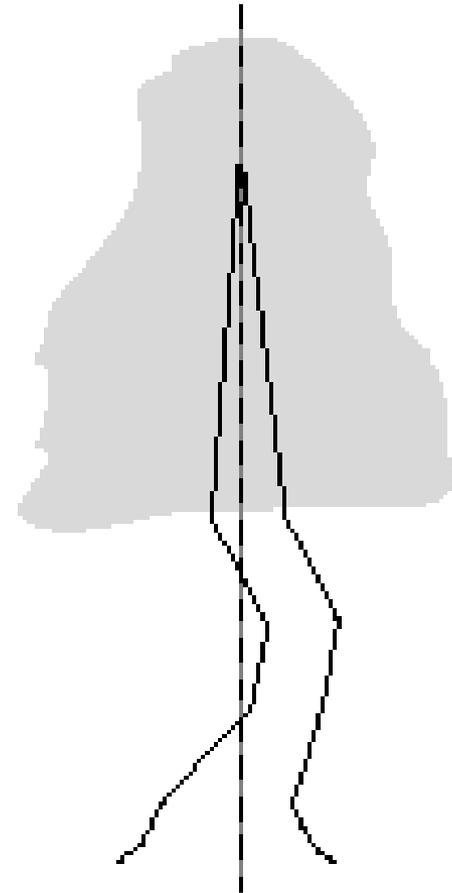
# Acceptable tree form

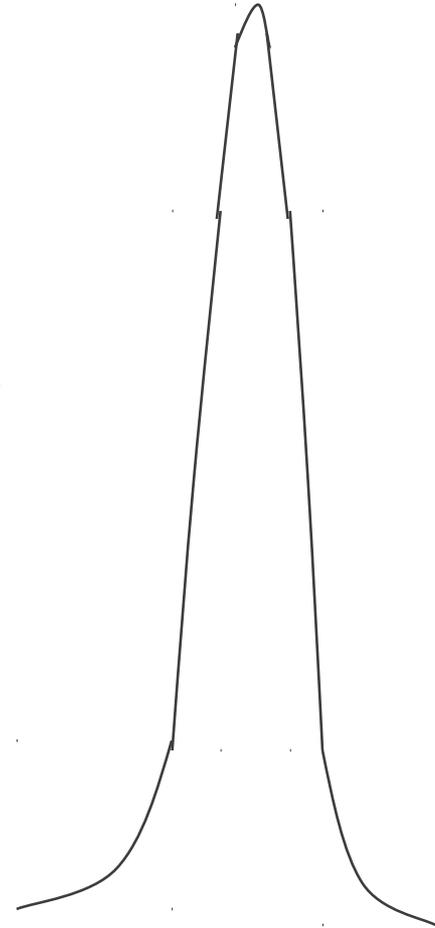
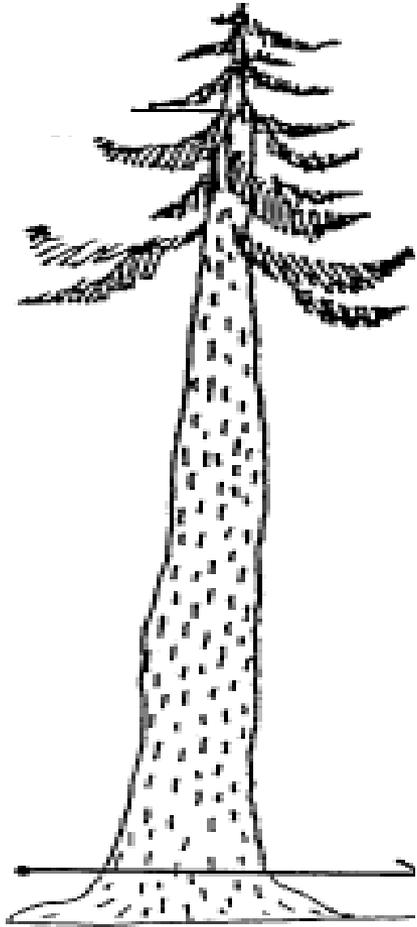
- not ideal
- some kinks in stem
- evidence of insect attack etc.



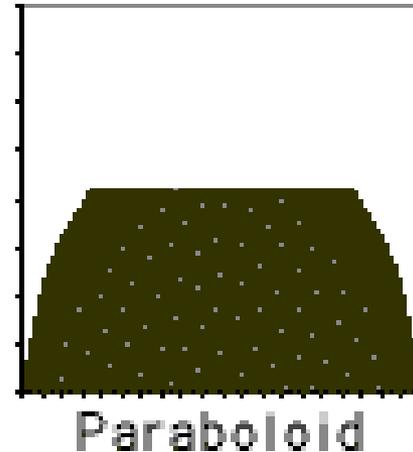
# Unacceptable tree form

- crooked bole
- severe butt sweep
- forked
- evidence of diseases e.g. rot



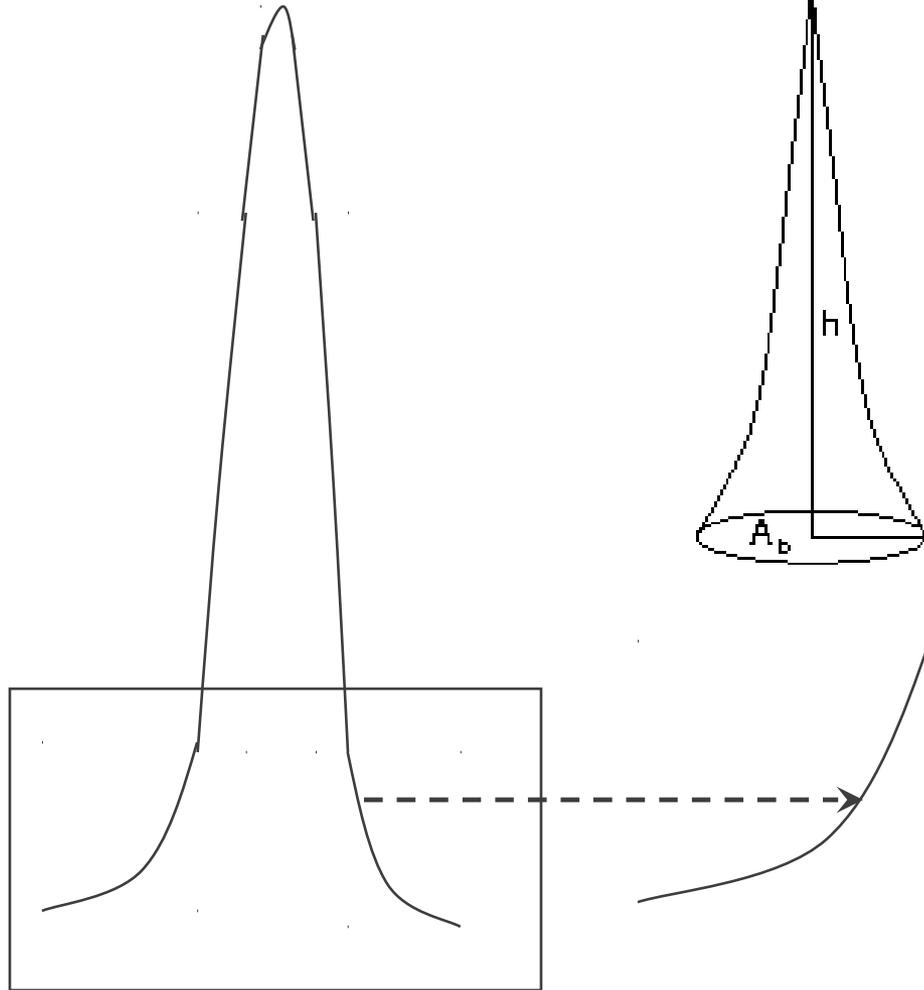


- Parts of a tree stem tend to approximate truncated parts of these common shapes.
  - base tends to be neiloid
  - tip tends to be conoid
  - main part of the bole tends to be paraboloid

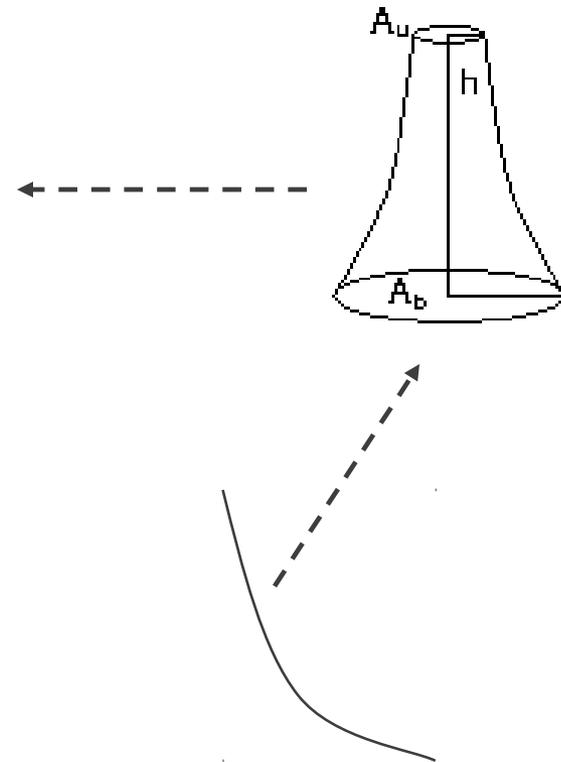


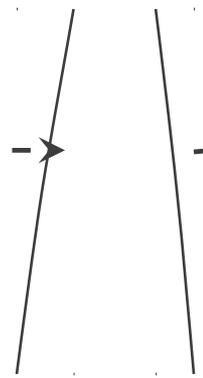
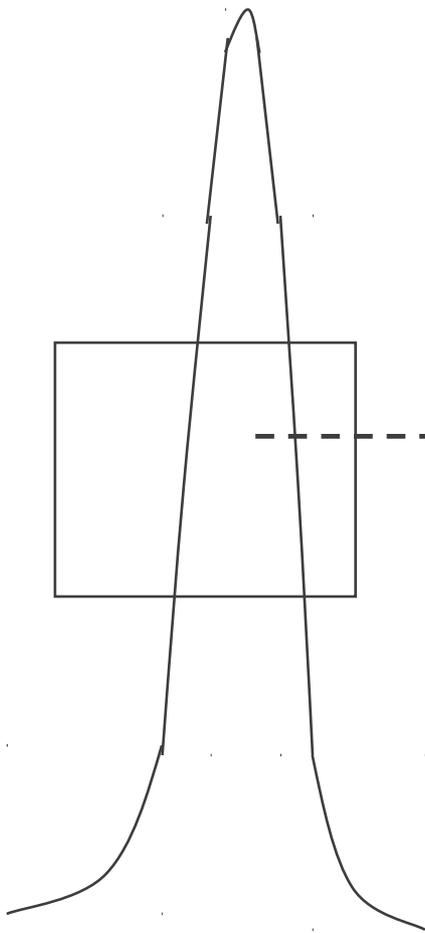
- The points of inflection between these shapes, however, are not constant.

Neiloid

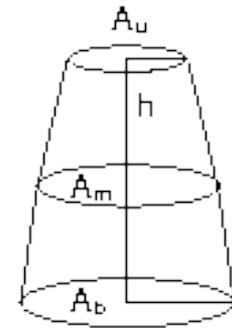


Frustrum of a Neiloid

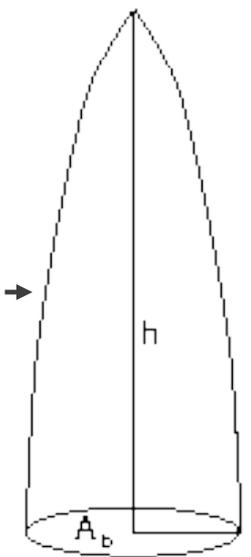




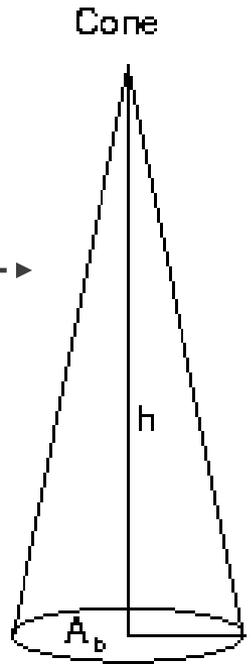
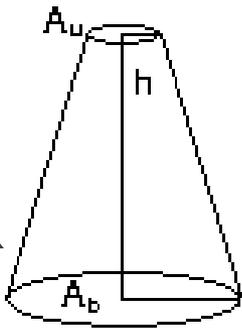
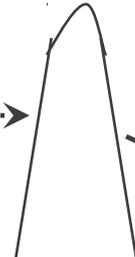
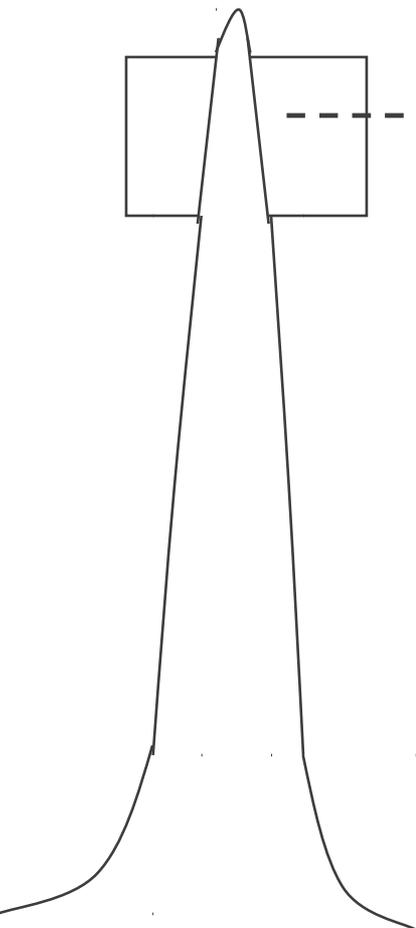
Frustrum of a Paraboloid

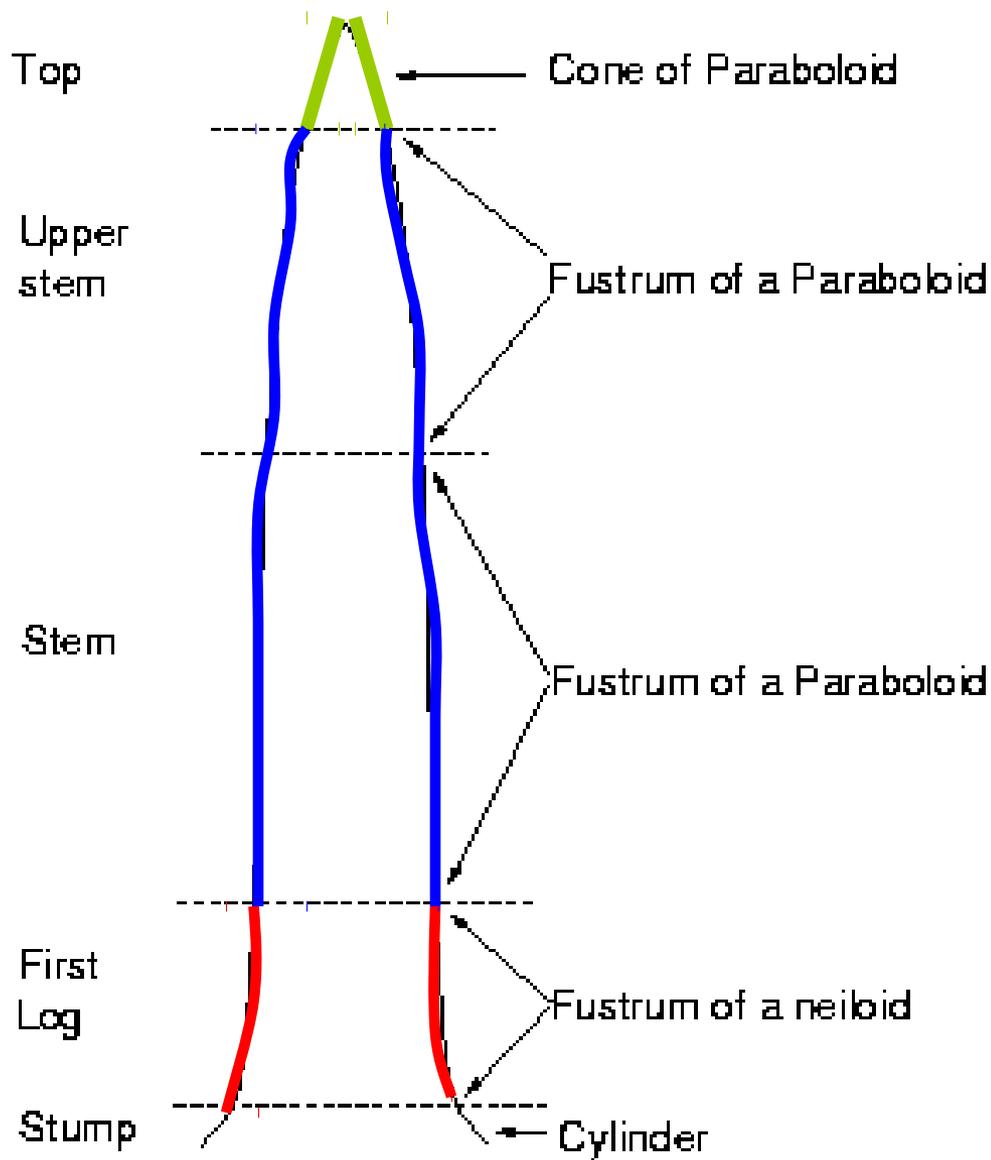


Paraboloid

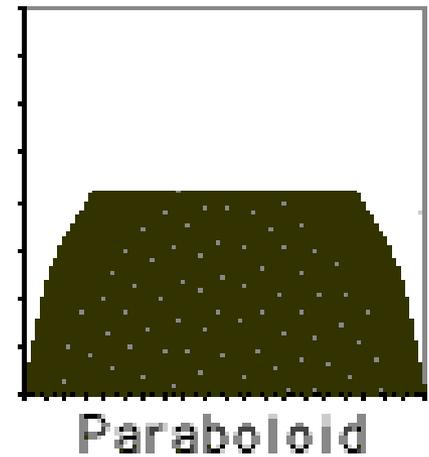


Frustrum of a  
Cone





## Tree form



The taper of a geometric shape  $y^2 = k * x^b$

where  $x$  = distance from the apex of the shape &  $y$  = dia

Specific values of  $b$  correspond to common shapes

1.  Paraboloid  $y^2 = k * x$
2.  Conoid  $y^2 = k * x^2$  *or*  $y = k' * x$
3.  Neiloid  $y^2 = k * x^3$

What about cylinder?

**Paraboloid:**  $y^2 = k * x^b$  *further divided into*

1. **Quadratic paraboloid** where  $b = 1 \rightarrow y^2 = k x$   
i.e. curve of ht vs.  $(\text{dia})^2$  is a straight line
2. **Cubic paraboloid** where  $b=0.66 \rightarrow y^2 = k x^{0.66}$  or  $y^3 = k' x$   
i.e. curve of ht vs.  $(\text{dia})^3$  is a straight line



- **Tree form**
- **Tree form theories**
- **Form ratios**
- **Taper equation**

# Tree form - theories

- Nutritional theory
- Water conducting theory
- Hormonal theory
- Mechanistic theory or Metzger's beam theory

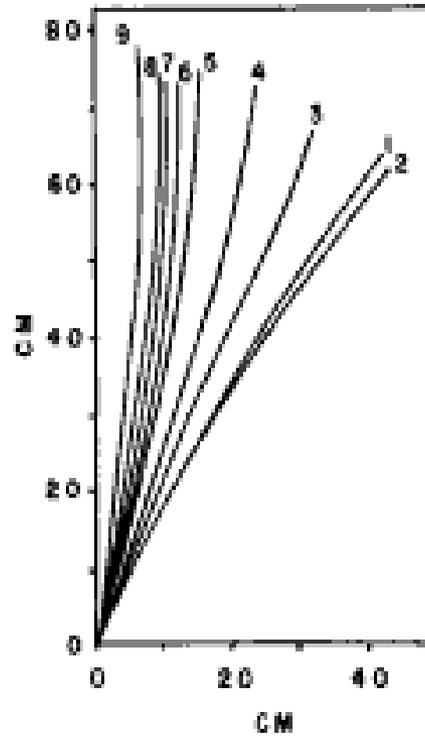
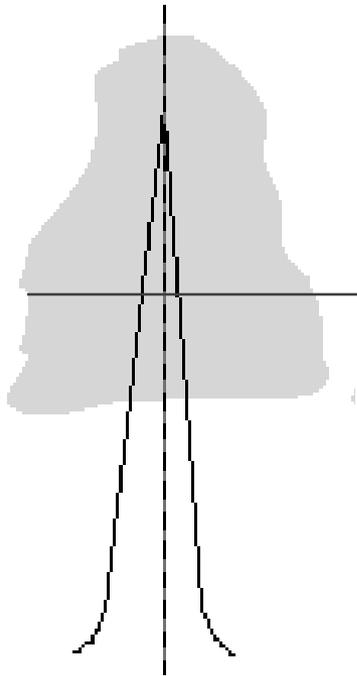
- **Nutritional theory** and **Water conducting theory** are based on ideas that deal with the movement of liquids through pipes. They relate tree bole shape to the need of the tree to transport water or nutrients within the tree
- **The hormonal theory** envisages that growth substances, originating in the crown, are distributed around and down the bole to control the activity of the cambium. These substances would reduce or enhance radial growth at specific locations on the bole and thus affect bole shape.

# Metzger's Theory

- Has received greatest acceptance so far
- **Tree stem** - a beam of uniform resistance to bending, anchored at the base and functioning as a lever → as a Cantilever beam of uniform size against the bending force of the wind
- Maximum pressure is on the base so the tree reinforces towards the base and more material deposited at lower ends

# Metzger's Theory

*contd...*



# Metzger's Theory

*contd...*

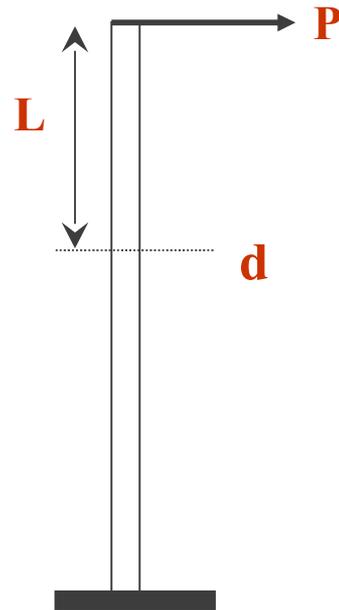
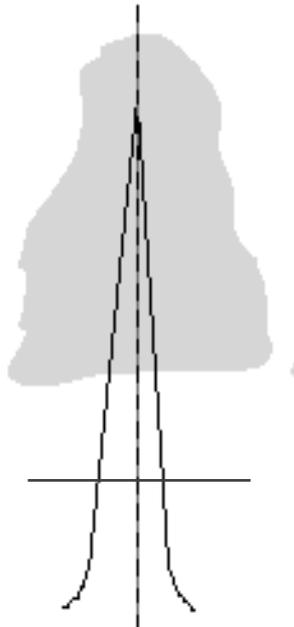
- A horizontal force will exert a strain on the beam that increases toward the point of anchorage, and if the beam is composed of homogeneous material the most economical shape would be a beam of uniform taper

Tree in open – short, with rapidly tapering boles

Tree in close canopy – long & nearly cylindrical boles

# Metzger's Theory

*contd...*



- $P$  = force applied at the free end
- $L$  = distance of a given cross section from the point of application of this force
- $d$  = diameter of the beam at this point

Then by simple rule of mechanics, the bending stress ( $\text{kg}/\text{cm}^2$ )

$$S = 32PL/\pi d^3$$

# Metzger's Theory

*contd...*

If  $\mathbf{W}$  = wind pressure per unit area

$\mathbf{A}$  = crown area

Then total pressure  $\mathbf{P} = \mathbf{W} \times \mathbf{A}$

$$S = 32\mathbf{P}L/\pi d^3$$

$$\Rightarrow S = (32/\pi) \mathbf{W} \mathbf{A} L /d^3$$

For a given tree  $\Rightarrow d^3 = (32/\pi) \mathbf{W} \mathbf{A} L/S \approx k L$

$$\Rightarrow d^3 \approx k L \quad (\mathbf{Cubic\ paraboloid})$$

Metzger demonstrated that this taper approximates the dimensions of a truncated cubic paraboloid (ht against dia<sup>3</sup> is linear) after confirming his theory for many stems, particularly conifers.

## Metzger theory

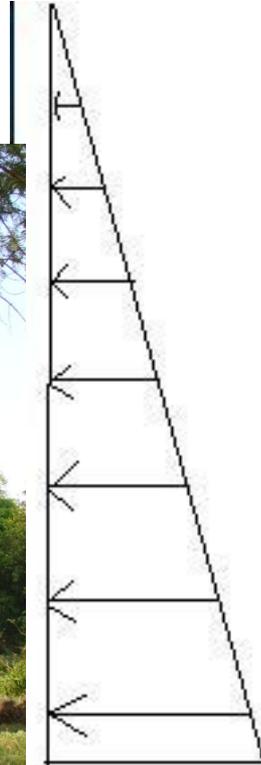
- Tree bole similar to a cubic paraboloid

$$d^3 \approx k L \quad (\text{Cubic paraboloid})$$

- Stem a beam of uniform resistance to bending anchored at the base, and functioning as a lever arm

# Metzger's Theory

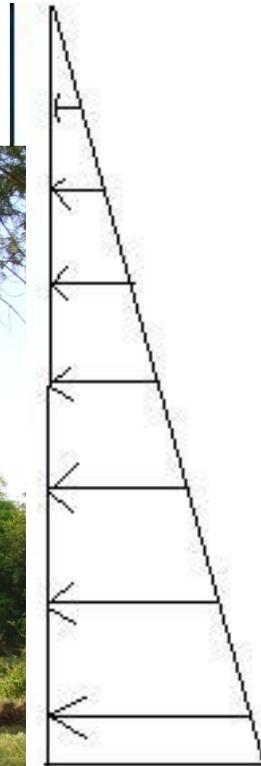
*contd...*



- Wind pressure, acting on crown is conveyed to the lower part of stem, in increasing measure with the length of bole
- Greatest pressure is exerted at the base of tree → danger of the tree snapping at base

# Metzger's Theory

*contd...*



- Tree reinforces itself towards the base to counteract this
- The limited growth material is so distributed that it affords uniform resistance all along its length to that pressure

# Metzger's Theory

*contd...*



- Trees growing in complete isolation have larger crowns so the pressure exerted on them is the greatest
- If such tree is to survive, it should allocate most of the growth material towards the base, even though it may have to be done at the expense of height

# Metzger's Theory

*contd...*

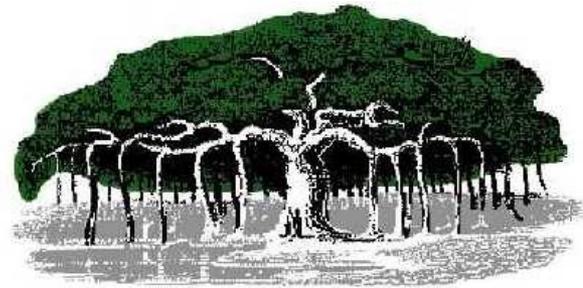


Trees growing in dense forest  
are subjected to lesser wind  
pressure

→ longer & cylindrical bole

# Methods of studying tree form

- By comparison of Standard Form ratios
- By classification of form on the basis of form ratios
- By compilation of taper tables



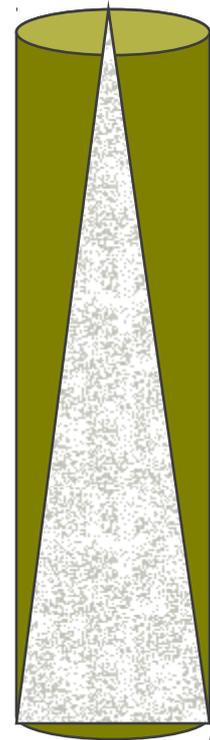
- **Tree form**
- **Tree form theories**
- **Form ratios**
- **Taper equation**

# Form Ratios — *form factor & form quotient*

## Form Factor

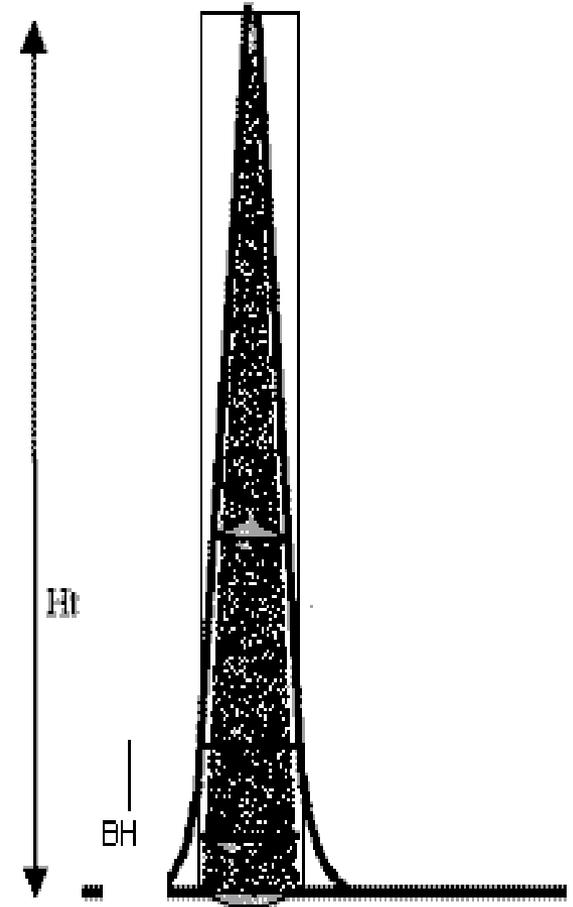
- ❖ summary of the overall stem shape
- ❖ ratio of its volume to the volume of a specified geometric solid of similar basal diameter and height
- ❖ Most commonly, the form factor of trees is based on a cylinder
  - **form factor = vol. (stem) / vol. (cylinder)**

or tree volume = form factor  $\times$  basal area  $\times$  height



# Form Factor

- The standard geometric shape for the bh form factor is a cylinder of the same height as the stem and with a sectional area equal to the sectional area of the stem at bh (i.e. basal area)
- Form factor is the ratio of the volume of the stem to the volume of the cylinder



# Classification of Form Factor

Depending upon the height of measurement of basal area

- **Artificial form factor** – vol. (whole tree) / vol. (cylinder with basal area at bh)
- **Absolute form factor** – vol. (tree above point of measurement) / vol. (cylinder at same level)
- **Normal form factor** – vol. (whole tree) / vol. (cylinder with measurement at some % of height)

# Artificial form factor or Breast height form factor

— *the most common*

- Standard geometric shape for this form factor is a cylinder
- Height of cylinder is same as the height of stem
- Sectional area of cylinder is equal to the sectional area of the stem at bh i.e. basal area

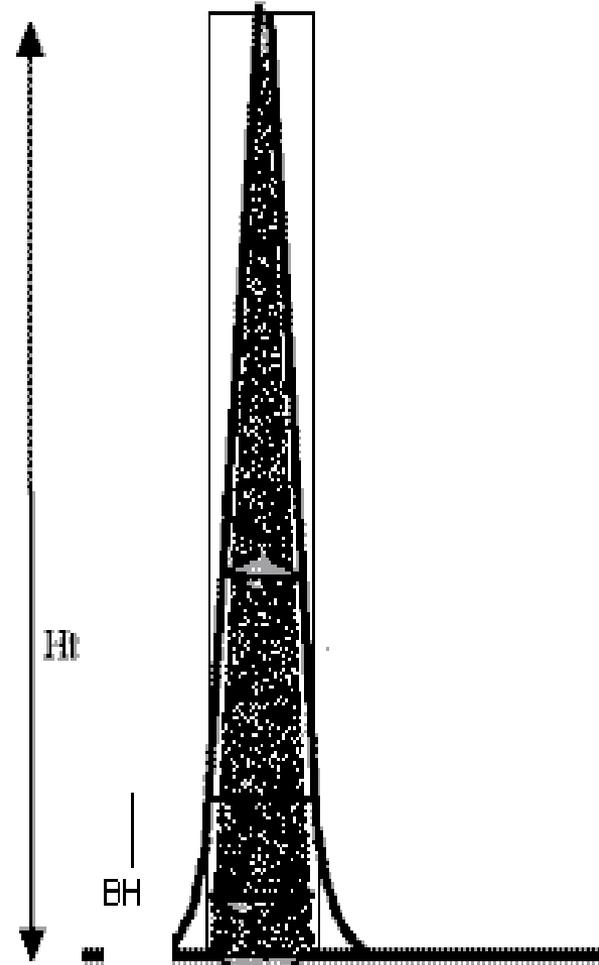
$$F = V/S \times h$$

F = form factor

V = volume of tree

S = basal area (at bh)

h = height of the tree



# Absolute form factor

- Defined as the ratio of the volume of the tree above point of measurement and the volume of a cylinder (of same dia or basal area) at same level
- Not used

# Normal form factor

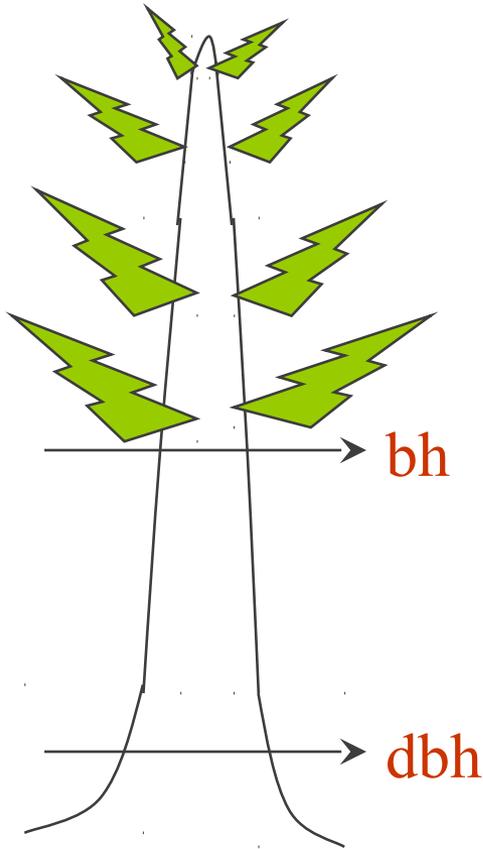
- True form factor
- Defined as the ratio of volume of whole tree and the volume of a cylinder with measurement at some % of height (generally  $1/10^{\text{th}}$ ,  $1/20^{\text{th}}$  .... of total height)
- Not very convenient, requires prior measurement of the height of tree before deciding the points of measurement
- Not much used

## Specific breast height form factors

Cylinder	1.00 (>0.9)
Neiloid	0.25 (0.2-0.3)
Conoid	0.33 (0.3-0.45)
Quadratic paraboloid	0.50 (0.45-0.55)
Cubic paraboloid	0.60 (0.55-0.65)

If the appropriate bh form factor for a tree of a given age, species and site can be determined, then the stem volume is easily calculated by multiplying the form factor by the tree height and basal area.

# Form quotient



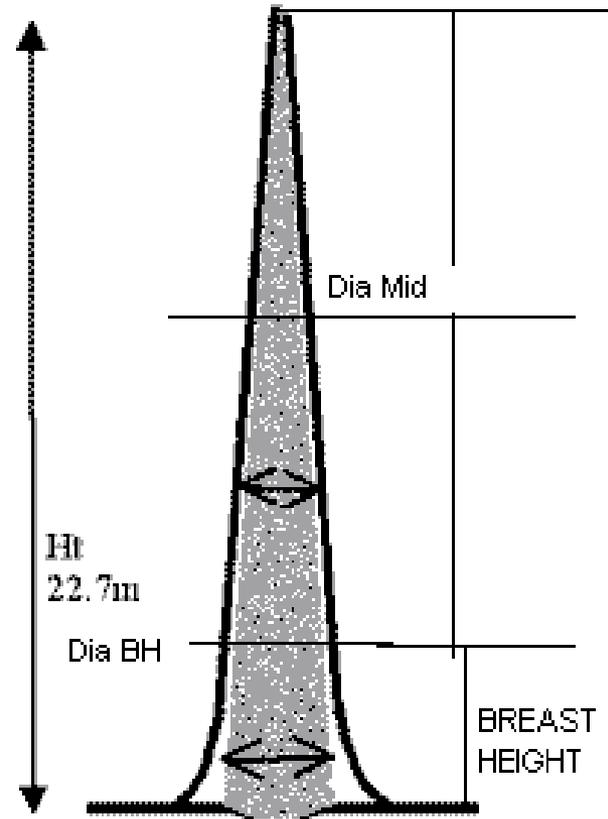
- Ratio of the diameter at two different places on the tree
- Generally calculated for some point above bh to the dbh
- Absolute form quotient – most common

# Form quotient

## Absolute form quotient

- Grouped into form classes
- Calculated by measuring the dia at a height halfway between bh and total tree height
- Absolute form quotient = dia at halfway/ dbh
- Commonly written as  $d_5/d_0$  and expressed as a decimal e.g. 0.70

*contd...*



# Form class

- Defined as one of the intervals in which the range of form quotients of trees is divided for classification and use
- Also applies to the class of trees which fall into such an interval
- Form quotient interval  
such as 0.50 to 0.55, 0.55 to 0.60 ...  
or mid-points of these intervals e.g. 0.525, 0.575 ...

# Form class & quotient

*contd...*

**Absolute form quotients also suggest general stem shapes:**

Neiloid	0.325 - 0.375	(FQ class 35)
Conoid	0.475 - 0.525	(FQ class 50)
Quadratic paraboloid	0.675 - 0.725	(FQ class 70)
Cubic paraboloid	0.775 - 0.825	(FQ class 80)

## **Form Point**

- Located approximately at the centre of gravity of crown since crown offers max. resistance. This point is the focal point of wind force.

## **Form Point Ratio**

- Percentage ratio of the height of the Form point to the total tree height.
- The greater the Form Point Ratio the more cylindrical tree
- However, this point is difficult to locate in crown  
*(Subjective)*

# Tree taper

- Defined as the change in stem dia between two measurement points divided by the length of the stem between these two points
- Taper equations attempt to describe it as a function of tree variables such as dbh, height, etc.

- **Tree form**
- **Tree form theories**
- **Form ratios**
- **Taper equation**

# Taper equations

- Try to fit the stem profile in model form and predict the dia at any point on the tree stem
- Based on simple input variables like dbh and total tree height, are unbiased
- Numerous approaches for the construction of these equations such as:
  - ✓ Use of computers to fit complex polynomial equations
  - ✓ Often the tree bole was segmented into 2 or 3 parts and separate equations fitted to each part
  - ✓ Development of a model that describes the continuous change in stem form from ground to tip

# Taper equations

*contd...*

Efforts have been made to discover a single, simple two-variable function involving only a few parameters which could be used to specify the entire tree profile but limitations are due to infinite variety of shapes of trees and other physical features

# Taper equations

*contd...*

Can be used to provide

- predictions of inside bark dia at any point on the stem
- estimates of total stem volume
- estimates of merchantable volume and merchantable height to any top dia and from any stump height
- estimates of individual log volumes

## Taper Table

- Taper Table portrays stem form in such a way that the data can be used in calculation of stem volume
- actual form is expressed by dia at fixed points from base to the tip of the tree
- if sufficient diameters are taken at successive pts. along stem, taper tables can be prepared

**Diameter Taper Table** : gives taper directly for dbh without referring to the tree form

**Form Class Taper Table** : Dia at different fixed points on the stem expressed as % of dbh (ub) for different form classes

## Equations of tree form

Despite the fact that trees don't conform to any geometric shape, some equations have been derived to describe tree form

$$\text{Diameter quotient} = (d \text{ at any given point}) / \text{dbh}$$

## Behre's Formula

$$d/d.b.h. = 1 / (a+b * l)$$

a & b are constants such that  $a+b = 1$

l is the distance from the tip of the tree (% of length of tree between bh & tip)

## Hojer's Formula

$$d/dbh = C \log [(c+1)/c]$$

d is Dia at '1' (same as previous)

C & c are constants for each form class

*“Trees assume infinite variety of shape”*-- explicit analytic definition of tree form requires considerable computational effort -- yet lacks generality”. Hence simple functions, graphical methods is adequate for most purposes.