

**Partial differential equations with variable coefficients reducible to Partial differential equations with constant coefficients**

Consider the PDE of the form

$$f(xD, yD')z = R(x, y) \text{ -----(1)}$$

This can be reduced to a Partial linear differential equation with constant coefficients by substituting  $x = e^X$ ,  $y = e^Y$  which implies  $X = \log x$ ,  $Y = \log y$

$$\frac{dx}{x} = \frac{dX}{1}$$

Now  $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial X} \frac{dX}{dx} = \frac{1}{x} \frac{\partial z}{\partial X}$  which implies  $x \frac{\partial z}{\partial x} = \frac{\partial z}{\partial X} = D_X$

Similarly we get  $x^2 \frac{\partial^2 z}{\partial x^2} = x^2 D^2 = D_X(D_X - 1)$

$$x^3 D^3 = D_X(D_X - 1)(D_X - 2)$$

...and so on

$$y \frac{\partial z}{\partial y} = y D' = D_Y$$

$$y^2 D'^2 = D_Y(D_Y - 1)$$

$$y^3 D'^3 = D_Y(D_Y - 1)(D_Y - 2) \text{ and so on.....}$$

Prob : 1. Solve the P D E  $x^2 D^2 - y^2 D'^2 + xD - y D' z = \log x$

The given P D E is  $x^2 D^2 - y^2 D'^2 + xD - y D' z = \log x$ -----(i)

Put  $x = e^X$ ,  $y = e^Y$  which implies  $X = \log x$ ,  $Y = \log y$

$$x \frac{\partial z}{\partial x} = \frac{\partial z}{\partial X} = D_X, \quad y \frac{\partial z}{\partial y} = \frac{\partial z}{\partial Y} = D_Y$$

$$x^2 D^2 = D_X(D_X - 1), \quad y^2 D'^2 = D_Y(D_Y - 1)$$

The pde (i) becomes

$$[D_X(D_X - 1) - D_Y(D_Y - 1) + D_X - D_Y]z = X$$

$$[D_X^2 - D_Y^2]z = X$$

To get Its Auxiliary Equation put  $D_X = m, D_Y = 1$

Therefore A E is  $m^2 - 1 = 0$ , the roots are  $m=1, m=-1$

$$C. F = \varphi_1(Y + X) + \varphi_2(Y - X)$$

$$\text{i.e., C. F} = \varphi_1(\log y + \log x) + \varphi_2(\log y - \log x)$$

$$\varphi_1(\log(xy)) + \varphi_2(\log(y/x))$$

$$P.I = \frac{1}{D_X^2 - D_Y^2} X$$

**By taking**  $D_X^2$  common

$$\frac{1}{D_X^2 \left[1 - \frac{D_Y^2}{D_X^2}\right]} X$$

$$P.I = \frac{1}{D_X^2} \left[1 - \frac{D_Y^2}{D_X^2}\right]^{-1} (X)$$

$$= \frac{1}{D_X} \int X dX = \frac{X^3}{6} = \frac{\log x^3}{6} = \frac{3 \log x}{6}$$

$$P.I = \frac{3 \log x}{6}$$

The solution is  $z = C.F + P.I =$

$$\varphi_1(\log(xy)) + \varphi_2(\log(y/x)) + \frac{3 \log x}{6}.$$

HW: 2. Solve the p d e  $x^2 D^2 + 2xy D D' + y^2 D'^2 )z = x^3 y^2$

HW : 3. Solve the P D E  $x^2 D^2 - y^2 D'^2 )z = x^2 y$

### Assignment –III

1. Solve the differential equation  $(D^2 + 3^2)y = \sec 3x$  by the method of variation parameter.

2. Solve the Cauchy homogeneous differential equation  $x^2y'' + 4xy' + 2y = e^x$

3. Form the differential equation by eliminating arbitrary function  $xyz = f(x+y+z)$

Solve the partial differential equation  $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

4. Solve the partial differential equation  $(D_x^2 + 5D_xD_y + 6D_y^2)z = e^{x-y}$

5. Solve the p d e  $(D_x^2 + 4D_xD_y - 5D_y^2)z = \sin(2x - 3y)$

6. Solve the p d e  $(x^2D^2 + 2xyD D' + y^2D'^2)z = x^3y^2$

7. Solve the P D E  $(x^2D^2 - y^2D'^2)z = x^2y$

8. Solve the p d e  $(D^2 + DD' - 6D'^2)z = x^2y$

9. Form the differential Equation by eliminate  $\varphi$  from  $z = (x+y)\varphi(x^2 - y^2)$ .