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MECHANICS
SUB. CODE-BSPH1102
For B.Sc.(Physics & Chemistry)
(Rotational Dynamics)

Angular momentum of a particle and system of particles.
Torque

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Angular Momentum

Angular momentum is given by the moment of linear momentum about the fixed point

- It is also called moment of momentum.
- It is denoted by
- $\vec{J} = \vec{r} \times \vec{P}$
- It is a vector quantity and its direction is perpendicular to both \vec{r} and \vec{P} .
- For a particle describing circular motion
- $J = mr^2\omega = mvr$



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Angular Momentum of System of Particles

If $\vec{J}_1, \vec{J}_2, \vec{J}_3$

Are the angular momentum of different particles of the system about a fixed axis, then the angular momentum of whole system is given by

$$\vec{J} = \vec{J}_1 + \vec{J}_2 + \vec{J}_3 + \dots = (\vec{r}_1 \times \vec{P}_1) + (\vec{r}_2 \times \vec{P}_2) + (\vec{r}_3 \times \vec{P}_3) + \dots$$

Or, $\vec{J} = \sum (\vec{r} \times \vec{P})$

i.e. the angular momentum of the whole system about the point is the vector sum of the angular momentum of the individual particles about the point.



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Torque

Differentiating the equation $\vec{J} = \vec{r} \times \vec{P}$, we get

$$\frac{d\vec{J}}{dt} = \frac{d}{dt} (\vec{r} \times \vec{P}) = \frac{d\vec{r}}{dt} \times \vec{P} + \vec{r} \times \frac{d\vec{P}}{dt}$$

Now, $\frac{d\vec{r}}{dt} \times \vec{P} = \vec{v} \times m\vec{v} = 0$

And $\frac{d\vec{P}}{dt}$ -rate of change of linear momentum= \vec{F}

So, $\frac{d\vec{J}}{dt} = \vec{r} \times \frac{d\vec{P}}{dt} = \vec{r} \times \vec{F}$

The quantity $\vec{r} \times \vec{F}$ is called torque (=moment of force)



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Thus the time rate of change of angular momentum gives the torque.

$$\text{Hence torque } \vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{J}}{dt}$$

For a system of particles,

$$\vec{\tau} = \sum (\vec{r} \times \vec{F})$$



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Reference

Mechanics by *D. S. Mathur*, S. Chand, chp-13.17

