

MIECHANICS SUB. CODE-BSPH1102

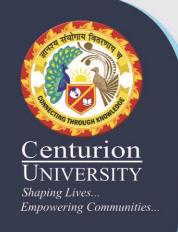
For B.Sc.(Physics & Chemistry)

(Rotational Dynamics)

Angular momentum of a particle and system of particles.

Torque

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Angular Momentum

Angular momentum is given by the moment of linear momentum about the fixed point

- It is also called moment of momentum.
- It is denoted by

•
$$\vec{J} = \vec{r} \times \vec{P}$$

- It is a vector quantity and its direction is perpendicular to both \vec{r} and \vec{P} .
- For a particle describing circular motion
- $J = mr^2\omega = mvr$



Angular Momentum of System of Particles

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If
$$J_1, J_2, J_3$$

Are the angular momentum of different particles of the system about a fixed axis, then the angular momentum of whole system is given by

$$\overrightarrow{J} = \overrightarrow{J_1} + \overrightarrow{J_2} + \overrightarrow{J_3} + .. = (\overrightarrow{r_1} \times \overrightarrow{P_1}) + (\overrightarrow{r_2} \times \overrightarrow{P_2}) + (\overrightarrow{r_3} \times \overrightarrow{P_3}) + \cdots$$

Or,
$$\vec{J} = \sum (\vec{r} \times \vec{P})$$

i.e. the angular momentum of the whole system about the point is the vector sum of the angular momentum of the individual particles about the point.



Torque

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$$\frac{d\vec{J}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{P}) = \frac{d\vec{r}}{dt} \times \vec{P} + \vec{r} \times \frac{d\vec{P}}{dt}$$

Now,
$$\frac{d\vec{r}}{dt} \times \vec{P} = \vec{v} \times m\vec{v} = 0$$

And $\frac{d\vec{P}}{dt}$ -rate of change of linear momentum= \vec{F}

So,
$$\frac{d\vec{J}}{dt} = \vec{r} \times \frac{d\vec{P}}{dt} = \vec{r} \times \vec{F}$$

The quantity $\vec{r} \times \vec{F}$ is called torque (=moment of force)



Thus the time rate of change of angular momentum gives the torque.

Hence torque
$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{J}}{dt}$$

For a system of particles,

$$\vec{\tau} = \sum (\vec{r} \times \vec{F})$$



Reference

Shaping Lives... Mechanics by D. S. Mathur, S. Chand, chp-13.17

