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# **MECHANICS**

## **SUB. CODE-BSPH1102**

*For B.Sc.(Physics & Chemistry)*

### **Collisions**

Relation between Elastic constants. ( $Y$ ,  $\eta$  &  $\sigma$ )

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# 17:-RELATION BETWEEN $Y, \eta$ and $\sigma$

If a tangential force  $F$  is applied to the upper face ABEF of a cube ABCDEFGH, then the face ABCD is displaced to new position A'B'CD and diagonal DB increases to DB' while diagonal CA decreases to CA'.

$$\text{Shearing stress} = \frac{F}{\text{Area } ABGH} = T$$

Now the shearing stress along AB is equivalent to the tensile stress along DB and an equal compressive stress along CA at right angles.

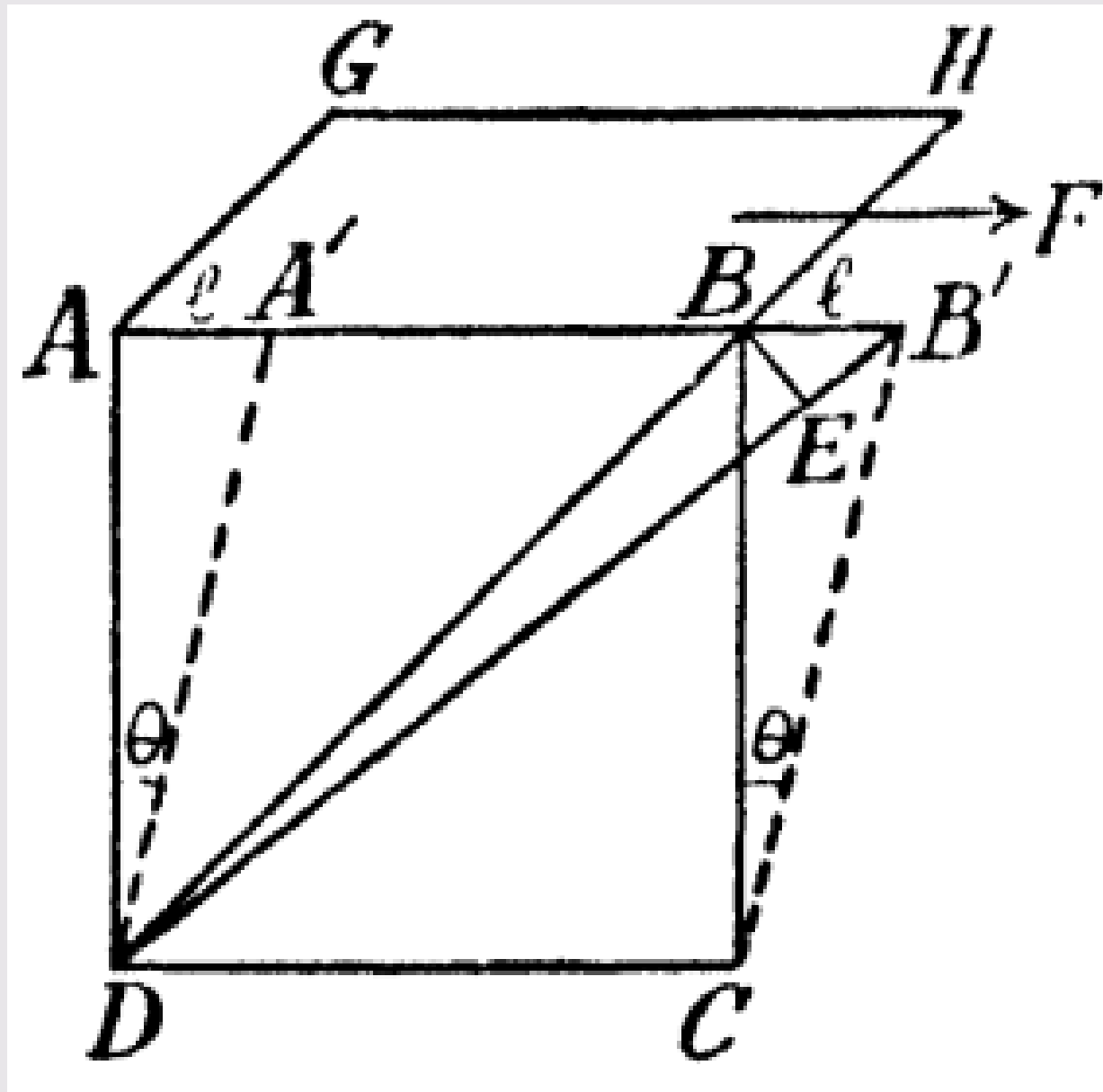
Let  $\alpha$ =longitudinal strain per unit stress

$\beta$  =lateral strain per unit stress



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Now the increase of diagonal DB due to tensile stress =  $DB \cdot T \cdot \alpha$  and increase in diagonal DB due to compressive stress along AC =  $DB \cdot T \cdot \beta$

$$\therefore \text{Total increase along DB} = DB \cdot T \cdot (\alpha + \beta) \\ = \sqrt{2}LT(\alpha + \beta) \quad \text{eq(1)}$$

Where L is the length of a side of the cube.

Now,  $BE \perp DB'$ . So, increase in length  $\approx B'E$

$$\text{Since } \theta \text{ is very small, } B'E = BB' \cos 45^\circ = \frac{BB'}{\sqrt{2}} = \frac{l}{\sqrt{2}}$$

Where  $l$  is the change in length of the side of the cube. Putting in eq(1)

$$\frac{l}{\sqrt{2}} = \sqrt{2}LT(\alpha + \beta)$$



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$$\text{or, } T \cdot \frac{L}{l} = \frac{1}{2(\alpha + \beta)}$$

$$\text{Also } \eta = \frac{T}{l/L} = \frac{TL}{l}$$

$$\therefore \eta = \frac{1}{2(\alpha + \beta)} = \frac{1}{2\alpha(1 + \beta/\alpha)} \quad \text{eq(2)}$$

Now by definition, Poisson's ratio  $\sigma = \frac{\beta}{\alpha}$

and Young's modulus  $Y = \frac{1}{\alpha}$

So,

$$\eta = \frac{Y}{2(1 + \sigma)}$$

$$\text{or, } Y = 2\eta(1 + \sigma) \quad \text{eq(3)}$$



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# Relation between $Y$ , $K$ and $\sigma$

Let us consider a unit cube ABCDEFGH and let forces  $T_x$ ,  $T_y$  and  $T_z$  act to the face BEHC and AFGD, EFGH and ABCD, ABEF and CDGH respectively as shown in the figure.

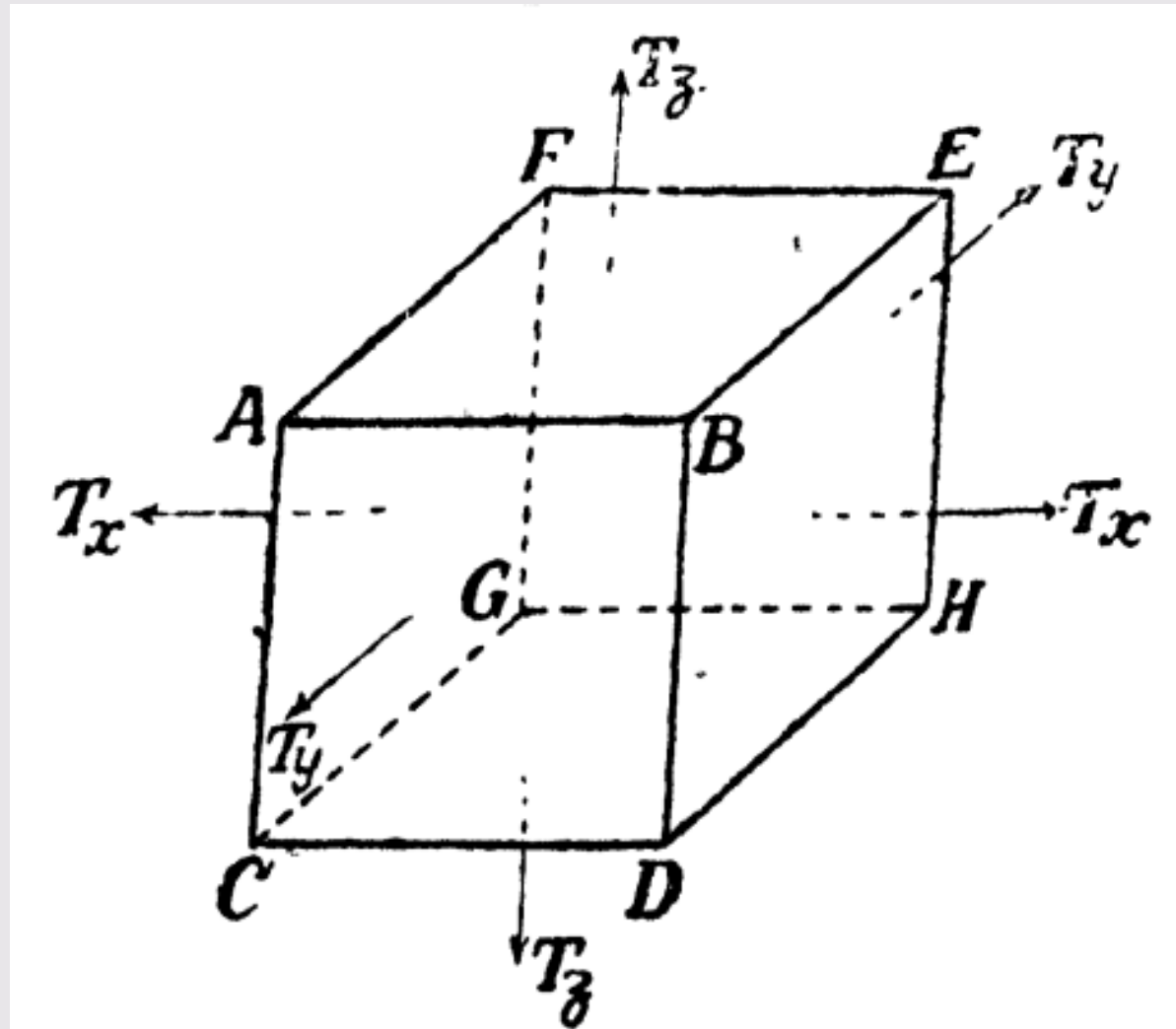
If  $\alpha$  corresponds to the increase per unit length per unit tension along the direction of force, then the increase in the edge AB, BE and BC will be  $T_x \cdot \alpha$ ,  $T_y \cdot \alpha$  and  $T_z \cdot \alpha$  respectively.

If  $\beta$  corresponds to the contraction per unit length per unit tension in a direction  $\perp$  to the force, then the contraction produced  $\perp$  to the edge AB, BE and BC will be  $T_x \cdot \beta$ ,  $T_y \cdot \beta$  and  $T_z \cdot \beta$  respectively.



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So, the length's of the edges will be :

$$AB = 1 + T_x \cdot \alpha - T_y \cdot \beta - T_z \cdot \beta$$

$$BE = 1 + T_y \cdot \alpha - T_x \cdot \beta - T_z \cdot \beta$$

$$BC = 1 + T_z \cdot \alpha - T_x \cdot \beta - T_y \cdot \beta$$

Hence the new volume of the cube= $AB \times BE \times BC$

$$= (1 + T_x \cdot \alpha - T_y \cdot \beta - T_z \cdot \beta)$$

$$\times (1 + T_y \cdot \alpha - T_x \cdot \beta - T_z \cdot \beta)$$

$$\times (1 + T_z \cdot \alpha - T_x \cdot \beta - T_y \cdot \beta)$$

Since  $\alpha$  and  $\beta$  are very small, higher powers are neglected.





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$$\therefore \text{Volume} = 1 + \alpha(T_x + T_y + T_z) - 2\beta(T_x + T_y +$$



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Now the bulk modulus,

$$K = \frac{\text{Stress}}{P \text{ Volume Strain}_1}$$
$$= \frac{3P(\alpha - 2\beta)}{3(\alpha - 2\beta)}$$

$$\text{or, } K = \frac{1/\alpha}{3(1-2\beta/\alpha)}$$

$$\therefore K = \frac{Y}{3(1-2\sigma)}$$



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# Relation between $Y$ , $\eta$ and $K$

We have

$$\eta = \frac{Y}{2(1 + \sigma)} \quad [1]$$

$$K = \frac{Y}{3(1 - 2\sigma)} \quad [2]$$

From eq(1), we have

$$2 + 2\sigma = \frac{Y}{\eta} \quad [3]$$

and from eq(2), we have

$$1 - 2\sigma = \frac{Y}{3K} \quad [4]$$



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Adding eq(3) and (4), we have

$$3 = \frac{Y}{3K} + \frac{Y}{\eta} = Y \left[ \frac{1}{3K} + \frac{1}{\eta} \right]$$

or  $\frac{3}{Y} = \frac{(\eta+3K)}{3\eta K}$

$$\therefore Y = \frac{9\eta K}{\eta+3K} \quad [5]$$



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# Reference

**Mechanics by *D. S. Mathur*, S. Chand, chp-13.12**

