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MECHANICS

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For B.Sc.(Physics & Chemistry)

Twisting torque on a Cylinder or Wire

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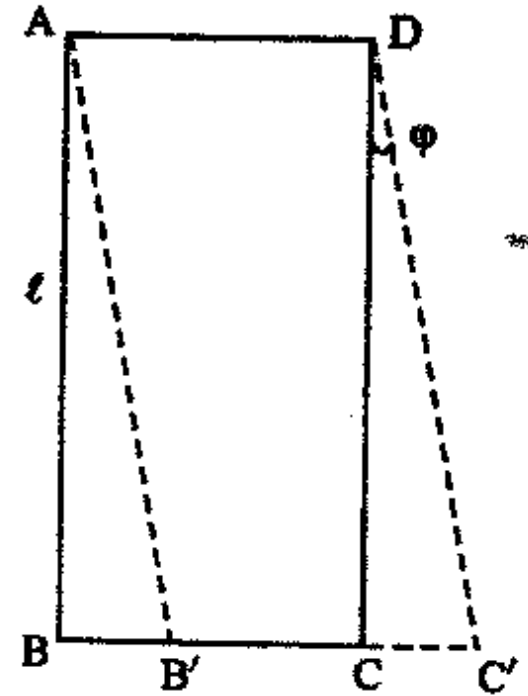
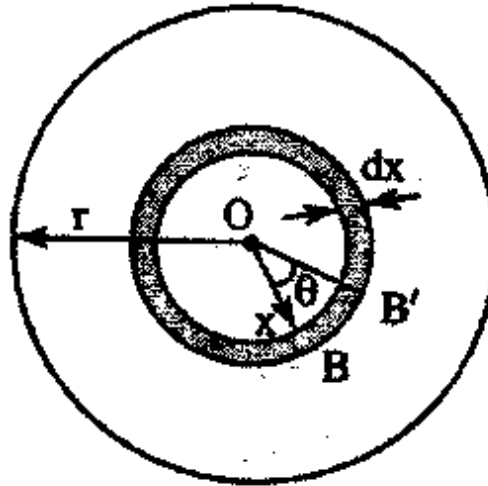
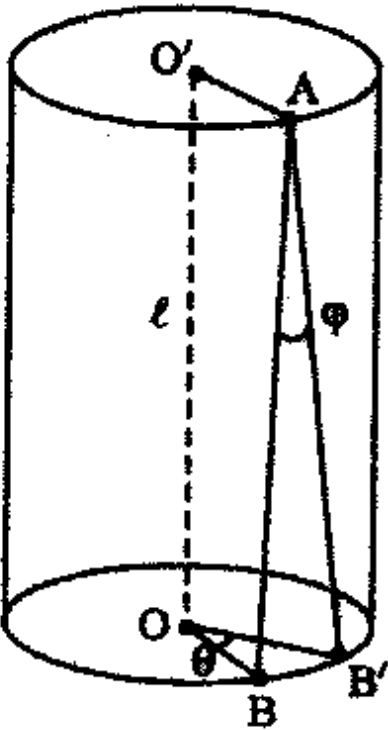
Twisting Couple on a Cylinder (or Wire)

- The twisting of a structural member about its longitudinal axis by two equal and opposite forces is expressed through a certain angle.
- The stress seen in this situation is not tensile or compressive, it is said to be shearing or shear stress.
- The strain in this case is measured by an angle in unit of radians



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- Let us consider a cylindrical rod of length l and radius r with its upper end fixed.
- Let a twisting couple be applied to the lower end of the rod in a plane perpendicular to its length and let the rod twist through an angle θ (radians).
- While the rod is twisted restoring couple acts in the opposite direction and in the position of equilibrium, the twisting couple is equal and opposite to the restoring couple.



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- To calculate this couple, let us consider the solid cylinder to be made up of a larger number of concentric thin walled cylinders.
- Let us consider one such hollow cylinder of radius x , and radial thickness dx .
- When the rod is twisted through an angle θ , the angle through which the rim of the cylinder is sheared is ϕ .



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- i.e. $BB' = l \phi$ Also $BB' = x \theta$
- $\phi = \frac{\theta}{l} x$
- From this it is clear that with x , ϕ varies. ϕ has the maximum value when x is the greatest. i.e., the strain is maximum on the outermost part of the cylinder and minimum on the innermost.
- In other words, the shearing stress is not uniform through out the material.
- If η is the rigidity modulus,
- $\eta = \frac{\text{Shearing stress}}{\text{Angle of shear}} = \frac{F}{\phi}$



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Hence

$$\text{Shearing stress } F = \eta\phi = \eta \frac{\theta}{l} x$$

Now the face area of the hollow cylinder = $2\pi x dx$

$$\begin{aligned} \therefore \text{Total shearing force on this area} &= F \times 2\pi x dx \\ &= \eta \frac{\theta}{l} x \times 2\pi x dx = \left(\frac{2\pi\eta\theta}{l} \right) x^2 dx \end{aligned}$$



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- Therefore, moment of this force about the axis $00'$
- $$= \left(\frac{2\pi\eta\theta}{l} \right) x^2 dx \cdot x = \left(\frac{2\pi\eta\theta}{l} \right) x^3 dx$$
- Total twisting couple of the cylinder can be obtained by integrating this expression between limits $x = 0$ and $x = r$.
- \therefore Total twisting couple on the cylinder =
- $$\tau = \int_0^r \left(\frac{2\pi\eta\theta}{l} \right) x^3 dx$$
- $$\Rightarrow \tau = \left(\frac{2\pi\eta\theta}{l} \right) \int_0^r x^3 dx$$
- $$\Rightarrow \tau = \left(\frac{2\pi\eta\theta}{l} \right) \frac{r^4}{4} = \frac{\pi\eta\theta r^4}{2l}$$



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- If $\theta = 1$ radian, we have
- Twisting couple per unit twist $C = \frac{\pi\eta r^4}{2l}$
- This twisting couple per unit twist of the wire is called the *torsional rigidity* or *modulus of torsion* of the cylinder of wire.
- It is evident from this relation that the couple required is proportional to the fourth power of the radius.



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Reference

Mechanics by *D. S. Mathur*, S. Chand, chp-13.12

