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MECHANICS

SUB. CODE-BSPH1102

For B.Sc.(Physics & Chemistry)

Module-VII (Oscillations)

Lesson-40

SHM: Simple Harmonic Oscillations. Differential equation of SHM and its solution

By

Mr. G. K. Sahu

Assistant Professor,

*CENTURION UNIVERSITY OF TECHNOLOGY AND MANAGEMENT,
ODISHA*



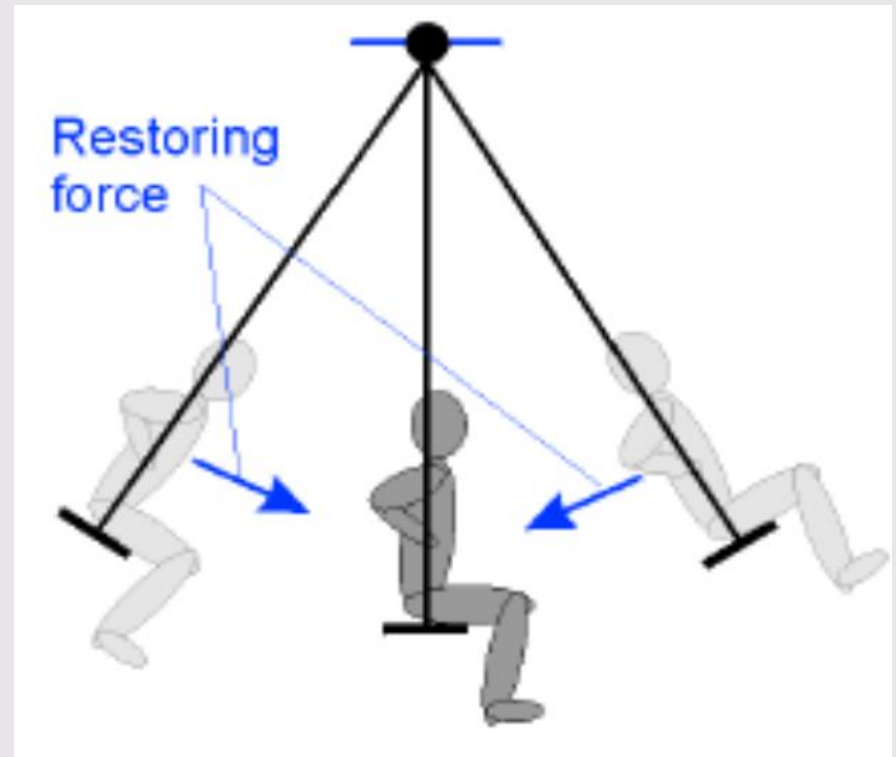
INTRODUCTION

Periodic motion – Motion which repeats itself after a regular interval of time is called periodic motion.

Oscillation (Vibration)- The motion in which a particle moves in a to and fro motion about a mean position over the same path is called oscillation or vibration.

Restoring force – The force which brings oscillator back to its original position is called restoring force.

Damping force – The opposing force which reduces the oscillation of the oscillator in a system is called damping force.





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SIMPLE HARMONIC OSCILLATIONS

Statement – The motion in which acceleration of the body is directly proportional to its displacement from its mean point is called simple harmonic oscillation

In S.H.O.,

RESTORING FORCE \propto DISPLACEMENT

$$F \propto x$$

$$F = -kx \text{ -----(1)}$$

Where F = restoring force

x = displacement of oscillator

k = force constant or spring constant (unit: N/m)

Negative sign indicates that the direction of the restoring force and displacement of the oscillator are opposite to each-other.



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SIMPLE HARMONIC OSCILLATIONS

From Newton's 2nd law of motion $F = ma = m \frac{d^2x}{dt^2}$
.....(2)

From equation (1) and (2) we can write

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Or, $\frac{d^2x}{dt^2} + \omega_0^2x = 0$

Where $\omega_0^2 = \frac{k}{m}$

$\omega_0 = \sqrt{\frac{k}{m}}$ = angular frequency of the body



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SIMPLE HARMONIC OSCILLATIONS

Solution – After solving the above equation by trial solution method we will get

$$x = A \cos(\omega_0 t + \theta) \text{ -----(3a)}$$

OR $x = A \sin(\omega_0 t + \theta) \text{ -----(3b)}$

Where x = displacement of oscillator

A = Amplitude of oscillator

ω_0 = angular frequency of the oscillator

θ = epoch (initial phase)

$\omega_0 t + \theta$ = phase



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SIMPLE HARMONIC OSCILLATIONS

Time period (T): Time taken to complete one oscillation

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi \sqrt{\frac{m}{k}}$$

Frequency (f): Number of oscillations per unit time.

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$



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SIMPLE HARMONIC OSCILLATIONS

Displacement, Velocity and Acceleration of the S.H.O.

Displacement of S.H.O. at $\theta = 0$ is

$$x = A \cos(\omega_0 t)$$

Velocity of of S.H.O. at $\theta = 0$ is

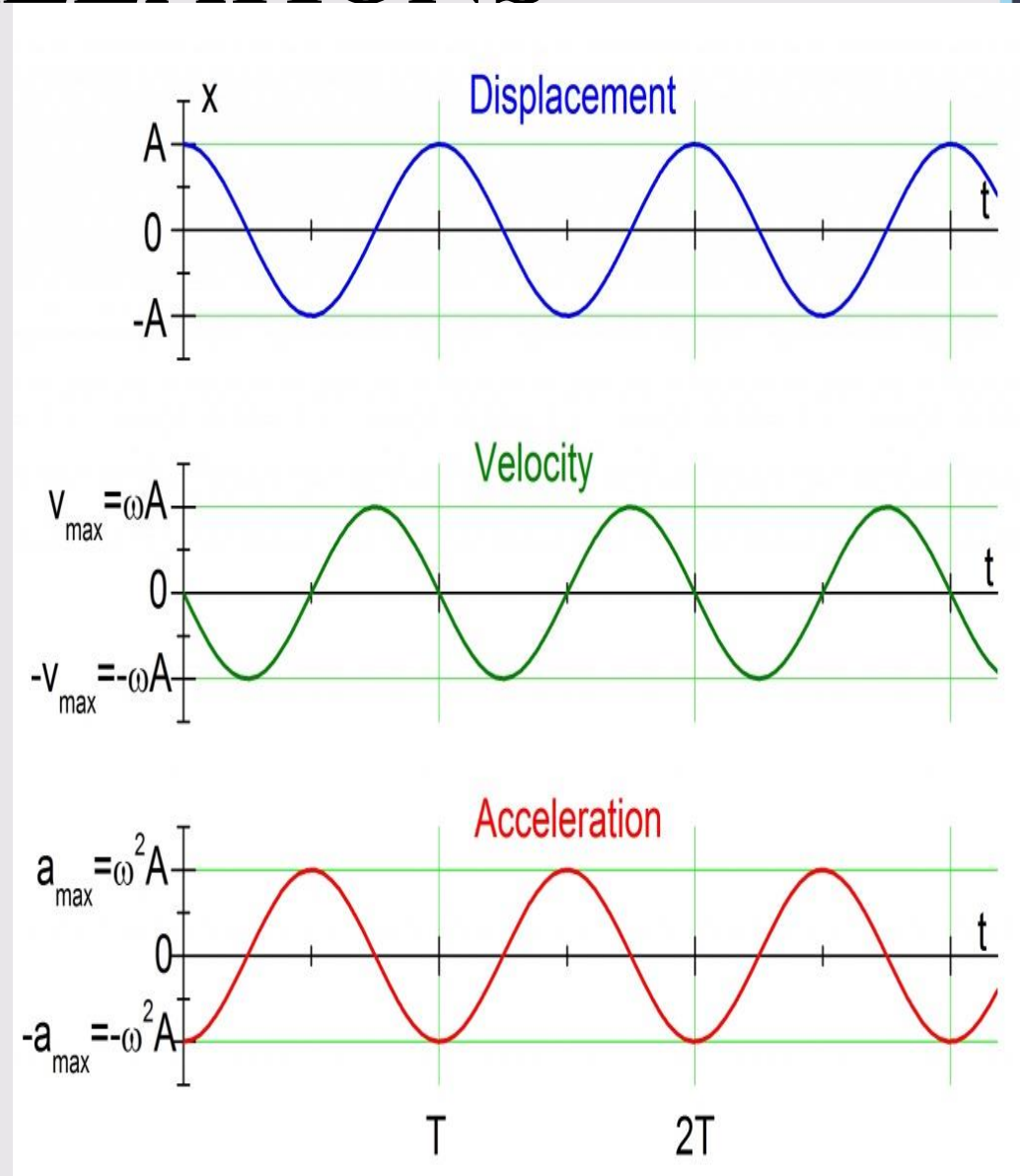
$$V = \frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t) \text{-----}$$

(6)

Acceleration of S.H.O. at $\theta = 0$ is

$$a = \frac{d^2x}{dt^2} = -A\omega_0^2 \cos(\omega_0 t) \text{---}$$

(7)





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SIMPLE HARMONIC OSCILLATIONS

Displacement, Velocity and Acceleration of the S.H.O.

Displacement

$$x = A \cos(\omega_0 t + \theta)$$

Velocity $V = \frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t + \theta) =$
 $\omega_0 \sqrt{A^2 - x^2}$

Acceleration of S.H.O. at $\theta = 0$ is

$$a = \frac{d^2x}{dt^2} = -A\omega_0^2 \cos(\omega_0 t) = -\omega_0^2 x$$



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SIMPLE HARMONIC OSCILLATIONS

From the above figure it is clear that

- Amplitude of velocity is ω_0 times the displacement.
- Phase difference between displacement and velocity graph is $\pi/2$.
- Amplitude of acceleration is ω_0 times the velocity.
- Phase difference between acceleration and velocity graph is $\pi/2$.



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Energy in SHM

Total Energy = Potential Energy + Kinetic Energy

Potential Energy

Let F be the restoring force at displacement x

When the particle is further displaced by dx , the work done against the force is $dW = \vec{F} \cdot \vec{dx} = F dx \cos 180^\circ = -F dx$
 $= -(-kx)dx = kx dx$

So, the total work done in displacing the particle from mean position to a distance x is given by

$$W = \int_0^W dW = k \int_0^x x dx = \frac{1}{2} kx^2 = \frac{1}{2} m\omega_0^2 x^2$$

Hence Potential Energy $E = \frac{1}{2} m\omega_0^2 x^2$ (1)



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Energy in SHM

$$\text{Kinetic Energy} = E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega_0^2(A^2 - x^2)$$

So, Total Energy (E) is

$$E = E_p + E_k$$

$$E = \frac{1}{2}m\omega_0^2 x^2 + \frac{1}{2}m\omega_0^2(A^2 - x^2)$$

$$E = \frac{1}{2}m\omega_0^2 A^2$$

Hence the total energy is independent of position as well as time. So, it remains constant



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Energy In S.H.O.(Alternate Method)

We know the potential energy of S.H.O. is

$$\text{P.E.} = \frac{1}{2} kx^2 = \frac{1}{2} k[A \cos(\omega_0 t)]^2 = \frac{1}{2} m\omega_0^2 A^2 \cos^2(\omega_0 t)$$

Kinetic energy of S.H.O. is

$$\text{K.E.} = \frac{1}{2} mv^2 = \frac{1}{2} m\omega_0^2 A^2 \sin^2(\omega_0 t)$$

So total energy is

$$\begin{aligned} \text{T.E.} &= \text{P.E.} + \text{K.E.} = \frac{1}{2} m\omega_0^2 A^2 \cos^2(\omega_0 t) + \frac{1}{2} m\omega_0^2 A^2 \sin^2(\omega_0 t) \\ &= \frac{1}{2} m\omega_0^2 A^2 \end{aligned}$$

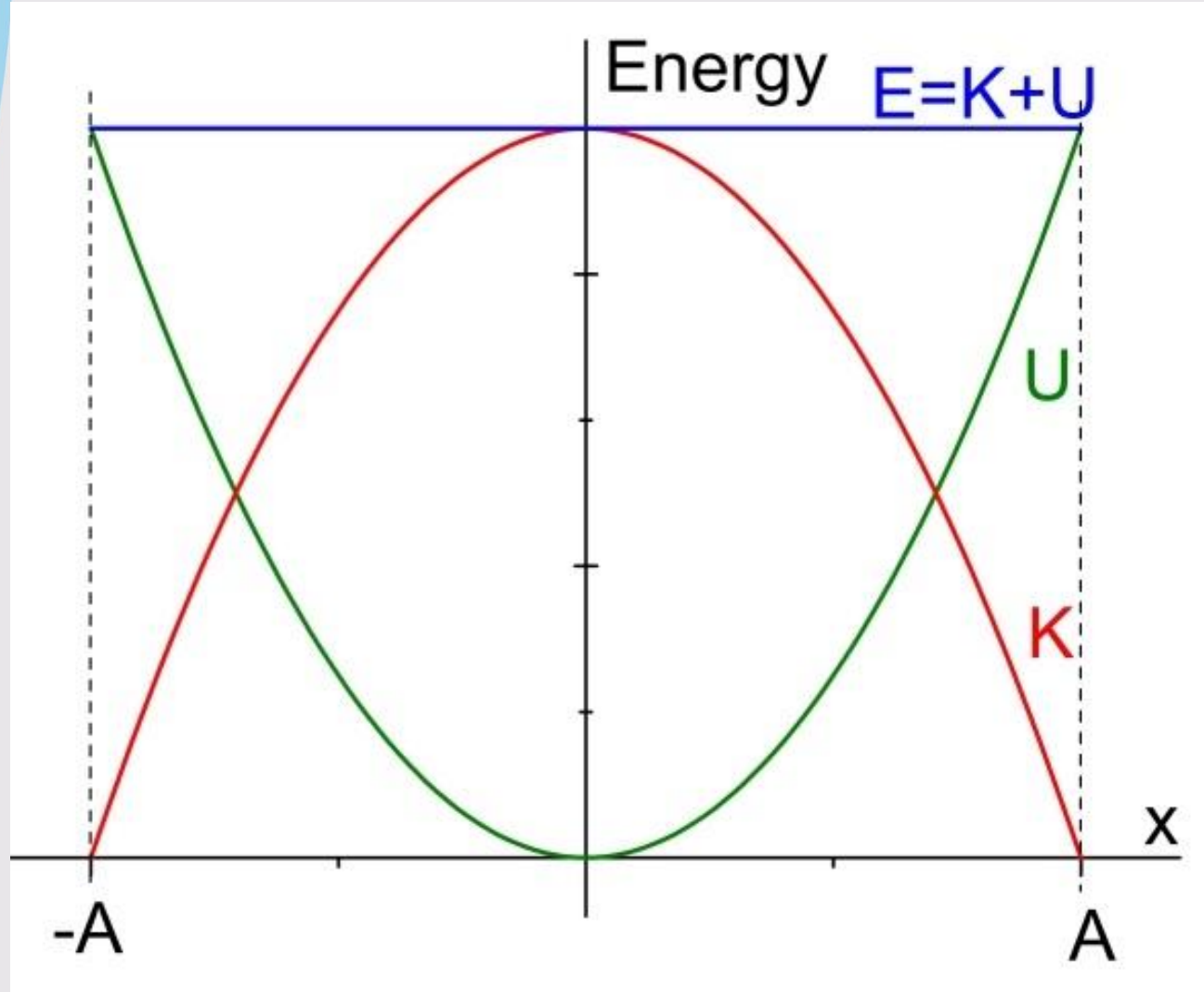
From the above equation it is cleared that total energy of S.H.O. remains constant w.r.t. time . Since M , ω_0 and A are constant w.r.t. time.



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Energy In S.H.O.





SIMPLE PENDULUM

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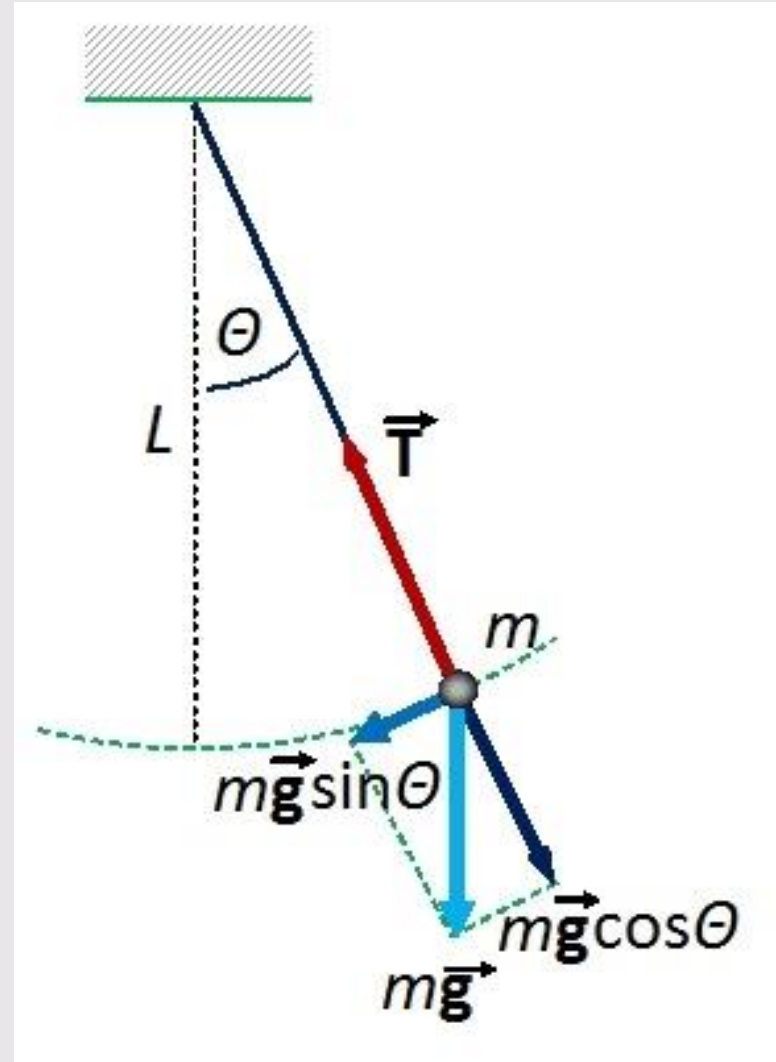
A point mass suspended from a rigid support with the help of massless, flexible and inelastic string. When the bob of the simple pendulum is displaced through a small angle from its mean position, it will execute SHM.

Here, angular frequency $\omega = \sqrt{\frac{g}{l}}$

Time Period, $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{l}{g}}$

Frequency, $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$

Examples of S.H.O.





Examples of S.H.O.

2. Oscillation of the loaded vertical spring

When the mass m is displaced from the mean position and released, it starts executing S.H.M.

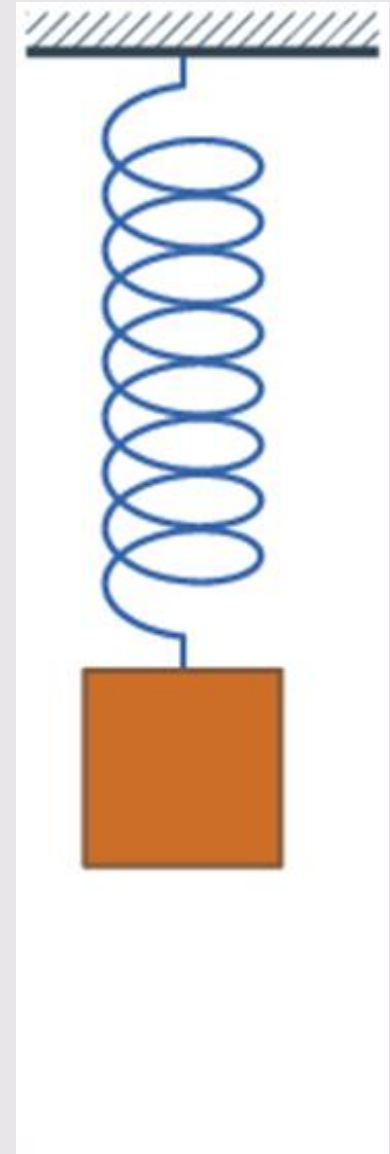
Here, angular frequency $\omega = \sqrt{\frac{k}{m}}$

Time Period, $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$

Frequency, $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

If l is the extension of spring due to the load, then the time period of oscillation of the spring is given by

$$T = 2\pi \sqrt{\frac{l}{g}}$$





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Problems

1. The differential equation of motion of freely oscillating body is given by $2 \frac{d^2x}{dt^2} + 18\pi^2 x = 0$. Calculate the natural frequency of the body.
2. The total energy of a simple harmonic oscillator is 0.8 erg. What is its kinetic energy when it is midway between the mean position and an extreme position?
3. The displacement of a one dimensional simple harmonic oscillator of mass 5gram is $y(t) = 2\cos(0.6t + \theta)$ where y & t are in cm & second respectively. Find the maximum kinetic energy of the oscillator.



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Reference

Mechanics by *D. S. Mathur*, S. Chand, chp-7.1 to 7.3

