

DAMPED HARMONIC OSCILLATIONS

Mr. Gouri Kumar Sahu
Senior Lecturer in Physics
CUTM, Paralakhemundi

DAMPED HARMONIC OSCILLATIONS

- **Damped oscillation** – The oscillation which takes place in the presence of dissipative force are known as damped oscillation
 - Here amplitude of oscillation decreases w.r.t. time
 - Damping force always acts in a opposite direction to that of motion and is velocity dependence.
 - For small velocity the damping force is directly proportional to the velocity
- Mathematically

$$F_d \propto v$$

$$F_d = -bv \text{ -----(1)}$$

DAMPED HARMONIC OSCILLATIONS

From Newton's 2nd law of motion $F_{\text{net}} = ma = m \frac{d^2x}{dt^2}$ -----
 -----(4)

After solving the equation (3) and (4) we can write

$$m \frac{d^2x}{dt^2} = -b \frac{dx}{dt} - kx$$

Or, $\frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + \omega_0^2 x = 0$ -----
 -----(5)

Where $b = 2m\beta$ and $\omega_0^2 = \frac{k}{m}$

Eq(5) is a homogeneous, 2nd order differential equation.

The general solution of eq(5) for $\beta \neq \omega_0$ is

$$x = e^{-\beta t} \left[A_1 e^{\left(\sqrt{\beta^2 - \omega_0^2}\right)t} + A_2 e^{\left(-\sqrt{\beta^2 - \omega_0^2}\right)t} \right] \text{ -----(6)}$$

DAMPED HARMONIC OSCILLATIONS

A_1 and A_2 are constants depend on the initial position and velocity of the oscillator.

Depending on the values of β and ω_o , three types of motion are possible.

- Such as
 1. Under damped ($\omega_0^2 > \beta^2$)
 2. Over damped ($\omega_0^2 < \beta^2$)
 3. Critical damped ($\omega_0^2 = \beta^2$)

Case-1: Under damped

Condition: $\beta^2 < \omega_o^2$

So, $\beta^2 - \omega_o^2 = -ve$, Hence $\sqrt{\beta^2 - \omega_o^2} = \sqrt{-(\omega_o^2 - \beta^2)} = i\omega$

Where $\omega = \sqrt{(\omega_o^2 - \beta^2)}$

Hence the solution becomes

$$\begin{aligned} x(t) &= e^{-\beta t} (A_1 e^{i\omega t} + A_2 e^{-i\omega t}) \\ &= e^{-\beta t} [(A_1 \cos \omega t + iA_1 \sin \omega t) + (A_2 \cos \omega t - iA_2 \sin \omega t)] \\ &= e^{-\beta t} [(A_1 + A_2) \cos \omega t + i(A_1 - A_2) \sin \omega t] \\ &= e^{-\beta t} [(A \sin \varphi) \cos \omega t + (A \cos \varphi) \sin \omega t] \end{aligned}$$

$$x(t) = Ae^{-\beta t} \sin(\omega t + \varphi)$$

Or, $x(t) = Ae^{-\beta t} \sin\left((\sqrt{\omega_o^2 - \beta^2})t + \varphi\right)$ -----[

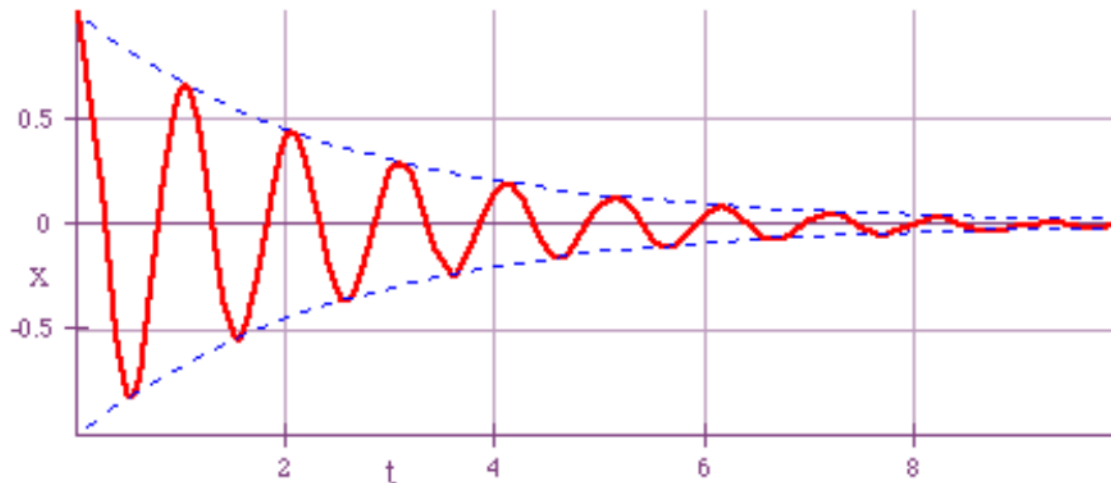
7]

Case-1: Under damped

Where $A_1 + A_2 = A \sin \varphi$ and $i(A_1 - A_2) = A \cos \varphi$

Equation (7) represents damped harmonic oscillation with amplitude $Ae^{-\beta t}$ which decreases exponentially with time and the time period of vibration is $T = \frac{2\pi}{\sqrt{(\omega_o^2 - \beta^2)}}$ which is greater than that in the absence of damping.

Example: Motion of Simple pendulum in air medium.



Decrement

- Decrement: The ratio between amplitudes of two successive maxima.

Let A_1, A_2, A_3 ---- are the amplitudes at time $t=t, t+T, t+2T, \dots$ respectively where T is time period of damped oscillation. Then

$$\begin{aligned} A_1 &= Ae^{-\beta t} \\ A_2 &= Ae^{-\beta(t+T)} \\ A_3 &= Ae^{-\beta(t+2T)} \end{aligned}$$

Hence decrement $d = \frac{A_1}{A_2} = \frac{A_2}{A_3} = e^{\beta T}$

Hence **logarithmic decrement** is given by

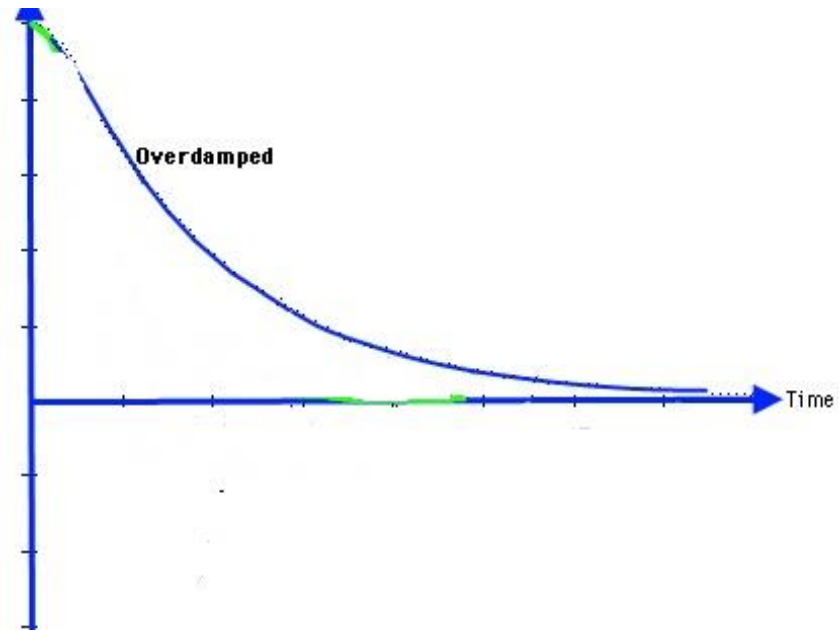
$$\lambda = \log_e e^{\beta T} = \beta T = \frac{2\pi\beta}{\sqrt{(\omega_o^2 - \beta^2)}}$$

Case-2: Over Damped

- Condition: $\beta^2 > \omega_0^2$
- $\beta^2 - \omega_0^2 = \text{positive}$
- Let $\sqrt{\beta^2 - \omega_0^2} = \alpha$, so from eq(6), we have

$$x = e^{-\beta t} [A_1 e^{\alpha t} + A_2 e^{(-\alpha)t}] = A_1 e^{-(\beta-\alpha)t} + A_2 e^{-(\beta+\alpha)t} \text{ -----(8)}$$

Since both the powers are negative, the body once displaced comes to the equilibrium position slowly without performing oscillations



Case-3: CRITICAL DAMPING

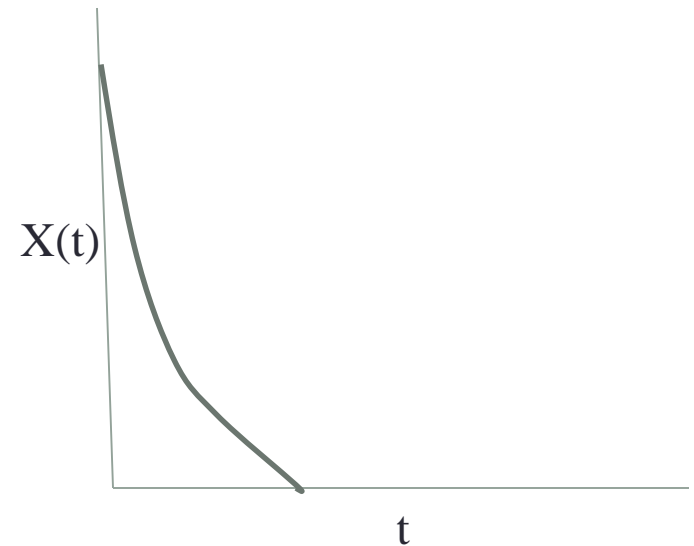
Condition: $\beta^2 = \omega_o^2$

Solution: $x(t) = (C + Dt)e^{-\beta t}$

The motion is non oscillatory and the displacement approaches zero asymptotically.

The rate of decrease of displacement in this case is much faster than that of over damped case.

- **Example** – suspension of spring of automobile.

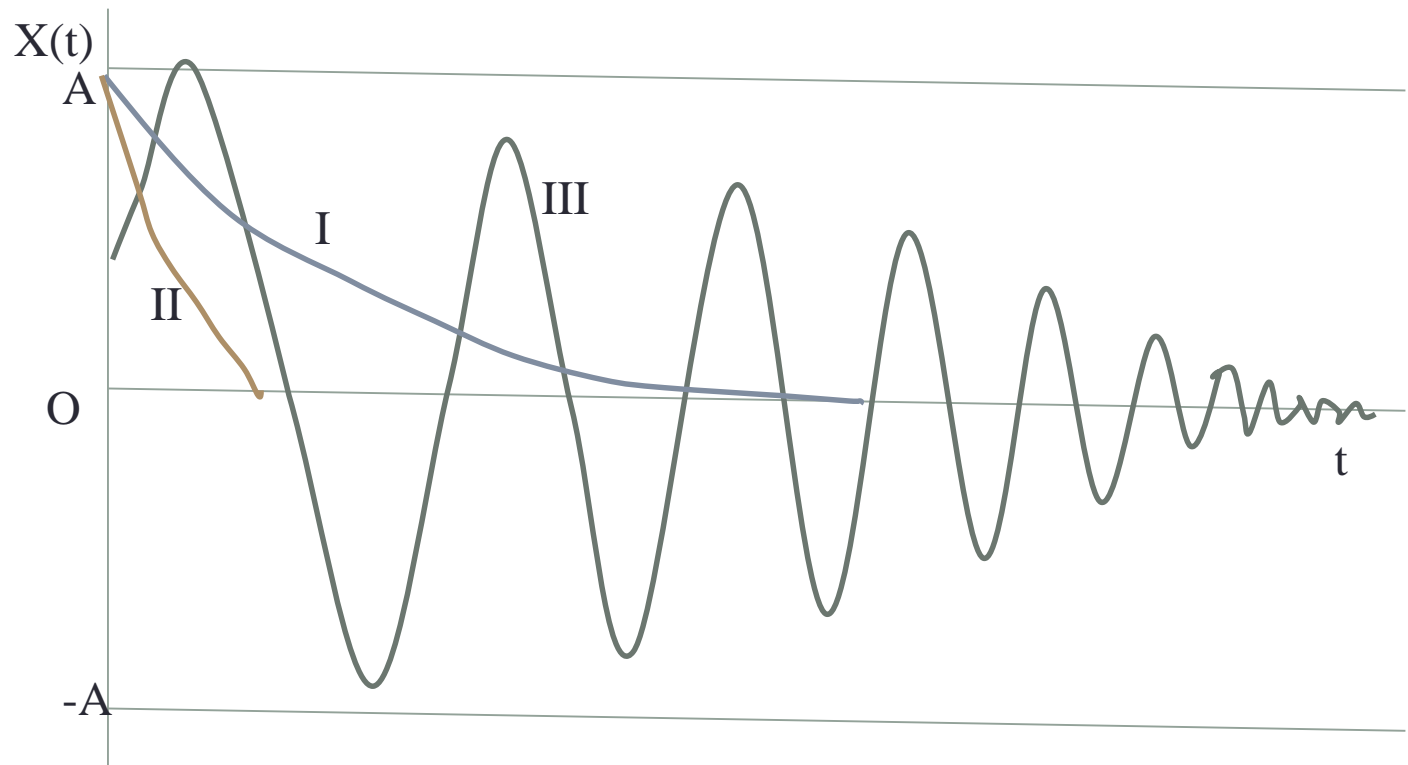


Damped Harmonic Oscillations

I: UNDER DAMPED

II: OVER DAMPED

III: CRITICAL DAMPED



Problems

1. What is the physical significance of damping coefficient? What is its unit (2marks)
2. Give the graphical comparison among the following three types of harmonic motion:
 - a) Under damped harmonic motion
 - b) Over damped harmonic motion
 - c) Critically damped harmonic motion
3. What is logarithmic decrement? Find the ratio of n th amplitude with 1st amplitude in case of under damped oscillation.(2 mark)
4. The natural angular frequency of a simple harmonic oscillator of mass 2gm is 0.8rad/sec. It undergoes critically damped motion when taken to a viscous medium. Find the damping force on the oscillator when its speed is 0.2cm/sec. (2marks)(Ans: 0.64dyne)