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# DEMODULATION OF FM SIGNALS

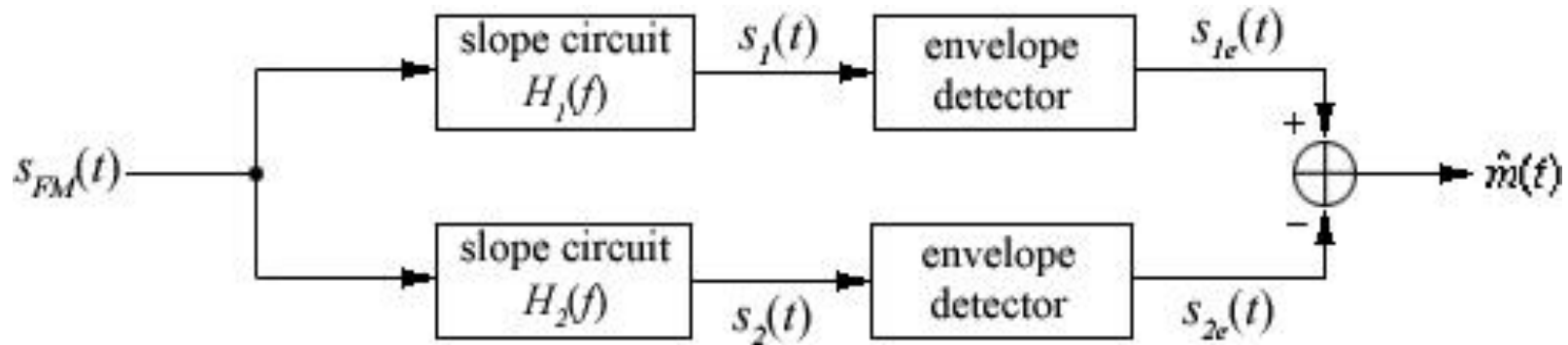
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## Demodulation of FM signals

Consider the following receiver architecture



Frequency discriminator implementation of an FM demodulator

The slope circuit is characterized by a purely imaginary transfer function

$$H_i(s), i = 1, 2.$$

Let  $H_1(f)$  be described by

$$H_1(f) = \begin{cases} j2\pi\alpha(f - f_c + B_T/2), & f_c - B_T/2 \leq f \leq f_c + B_T/2 \\ j2\pi\alpha(f + f_c - B_T/2), & -f_c - B_T/2 \leq f \leq -f_c + B_T/2 \\ 0, & \text{elsewhere} \end{cases}$$

where  $a > 0$  is a constant that determines the slope of  $H_1(s)$ .

Define  $G_1(f) \equiv H_1(f)/j$ , then  $g_1(t)$  is the impulse response of a real bandpass system described by  $G_1(f)$ .

In the time domain,

$$g_1(t) = g_{1,I}(t) \cos(2\pi f_c t) - g_{1,Q}(t) \sin(2\pi f_c t)$$

where  $g_{1,I}(t)$  and  $g_{1,Q}(t)$  are the in-phase and quadrature components of  $g_1(t)$ .

Therefore, the complex envelope of  $g_1(t)$  is described by

$$\tilde{g}_1(t) = g_{1,I}(t) + jg_{1,Q}(t)$$

which implies that

$$g_1(t) = \text{Re} \left\{ \tilde{g}_1(t) e^{j2\pi f_c t} \right\}$$

Using this information, we get

$$\begin{aligned} \tilde{g}_1(t) e^{j2\pi f_c t} + \tilde{g}_1^*(t) e^{-j2\pi f_c t} &= (g_{1,I}(t) + jg_{1,Q}(t)) e^{j2\pi f_c t} + (g_{1,I}(t) - jg_{1,Q}(t)) e^{-j2\pi f_c t} \\ &= 2g_{1,I}(t) \cos(2\pi f_c t) - 2g_{1,Q}(t) \sin(2\pi f_c t) \\ &= 2g_1(t) \end{aligned}$$

But,

$$F\{2g_1(t)\} = F\{\tilde{g}_1(t)e^{j2\pi f_c t} + \tilde{g}_1^*(t)e^{-2j\pi f_c t}\}$$

or

$$2G_1(f) = \tilde{G}_1(f - f_c) + \tilde{G}_1^*(-(f + f_c))$$

which implies that  $\tilde{G}_1(f)$  has a lowpass frequency response limited to

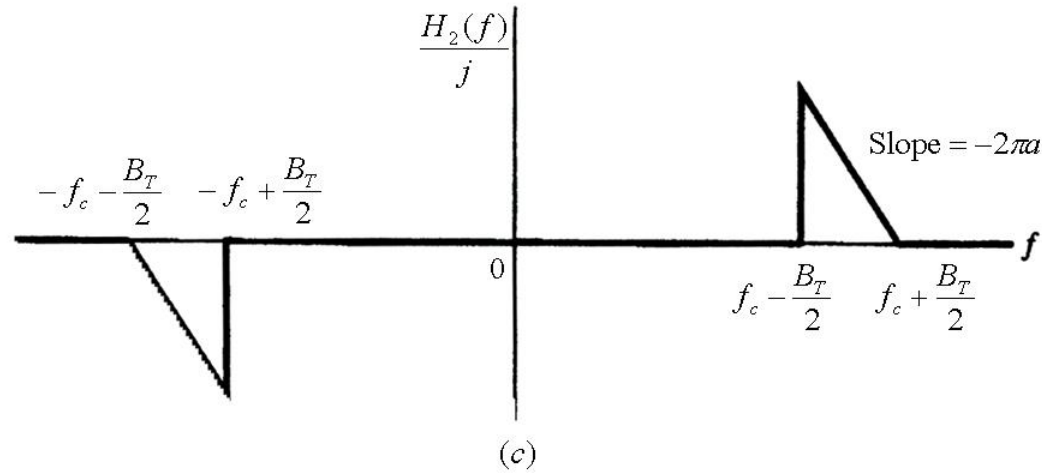
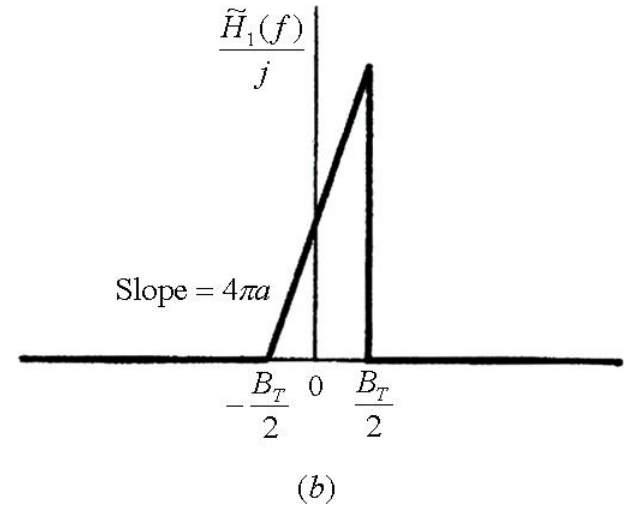
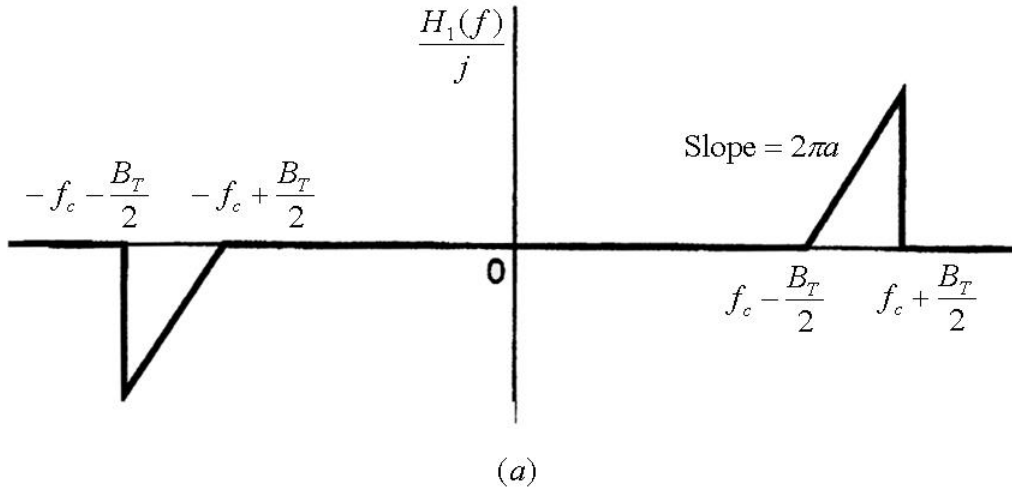
$$|f| \leq B_T / 2$$

and that

$$\tilde{G}_1(f - f_c) = G_1(f), f > 0.$$

Observations:  $\tilde{G}_1(f)$  can be obtained by taking the part of  $G_1(f)$  that corresponds to positive frequencies, shifting it to the origin and then scaling it by a factor of 2.

In the next figure  $G_1(f)$  is replaced by  $H_1(f)/j$  and  $\tilde{G}_1(f)$  replaced by  $\tilde{H}_1(f)/j$ .



Frequency responses of  $H_1(f)$ , and  $H_2(f)$

$\tilde{H}_1(f)$ ,

From the previous derivation,

$$\tilde{H}_1(f) = \begin{cases} j4\pi a(f + B_T / 2), & |f| \leq B_T / 2 \\ 0, & \text{elsewhere} \end{cases}$$

But,

$$\begin{aligned} s_{FM}(t) &= A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right) \\ &= \operatorname{Re}\left\{A_c e^{j2\pi k_f \int_0^t m(\tau) d\tau} \cdot e^{j2\pi f_c t}\right\} \\ &= \operatorname{Re}\left\{\tilde{s}_{FM}(t) \cdot e^{j2\pi f_c t}\right\}. \end{aligned}$$

If  $s_1(t)$  is the output of the slope filter  $H_1(f)$  when the input is  $s_{FM}(t)$  then the complex envelope of the output is

$$\tilde{s}_1(t) = \frac{1}{2} \tilde{h}_1(t) * \tilde{s}_{FM}(t)$$

In the frequency domain,

$$\begin{aligned}\tilde{S}_1(f) &= \frac{1}{2} \tilde{H}_1(f) \tilde{S}_{FM}(f) \\ &= \begin{cases} j2\pi a(f + B_T/2) \tilde{S}_{FM}(f), & |f| \leq B_T/2 \\ 0, & \textit{elsewhere} \end{cases}\end{aligned}$$

But,

$$F\left\{\frac{dx(t)}{dt}\right\} = j2\pi fX(f)$$

which implies that

$$\begin{aligned}\tilde{s}_1(t) &= a \frac{d\tilde{s}_{FM}(t)}{dt} + ja\pi B_T \tilde{s}_{FM}(t) \\ &= j2\pi a A_c k_f m(t) e^{j2\pi k_f \int_0^t m(\tau) d\tau} + 2\pi a A_c B_T e^{j2\pi k_f \int_0^t m(\tau) d\tau} \\ &= j\pi a A_c B_T \left[ 1 + \frac{2k_f}{B_T} m(t) \right] e^{j2\pi k_f \int_0^t m(\tau) d\tau}\end{aligned}$$

Therefore,

$$\begin{aligned} s_1(t) &= \operatorname{Re} \left\{ \tilde{s}_1(t) e^{j2\pi f_c t} \right\} \\ &= \operatorname{Re} \left\{ j\pi a A_c B_T \left[ 1 + \frac{2k_f}{B_T} m(t) \right] e^{j2\pi f_c t + j2\pi k_f \int_0^t m(\tau) d\tau} \right\} \\ &= -\pi a A_c B_T \left[ 1 + \frac{2k_f}{B_T} m(t) \right] \sin \left( 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right) \\ &= \pi a A_c B_T \left[ 1 + \frac{2k_f}{B_T} m(t) \right] \cos \left( 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau + \frac{\pi}{2} \right) \end{aligned}$$

Observation:  $s_1(t)$  contains both AM and FM.

However, if

$$\left| 2 \frac{k_f}{B_T} m(t) \right| < 1, \forall t$$



Then a distortionless envelope detector can extract  $m(t)$  plus a bias, i.e.,

$$s_{1e}(t) = \pi a A_c B_T \left[ 1 + \frac{2k_f}{B_T} m(t) \right].$$

Finally, if

$$\tilde{H}_2(f) = \tilde{H}_1(-f)$$

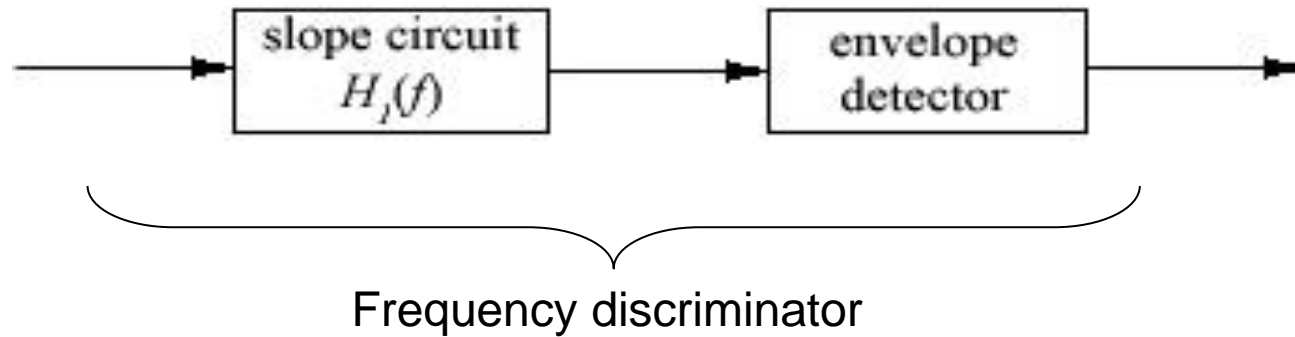
then

$$s_{2e}(t) = \pi a A_c B_T \left[ 1 - \frac{2k_f}{B_T} m(t) \right].$$

Moreover,

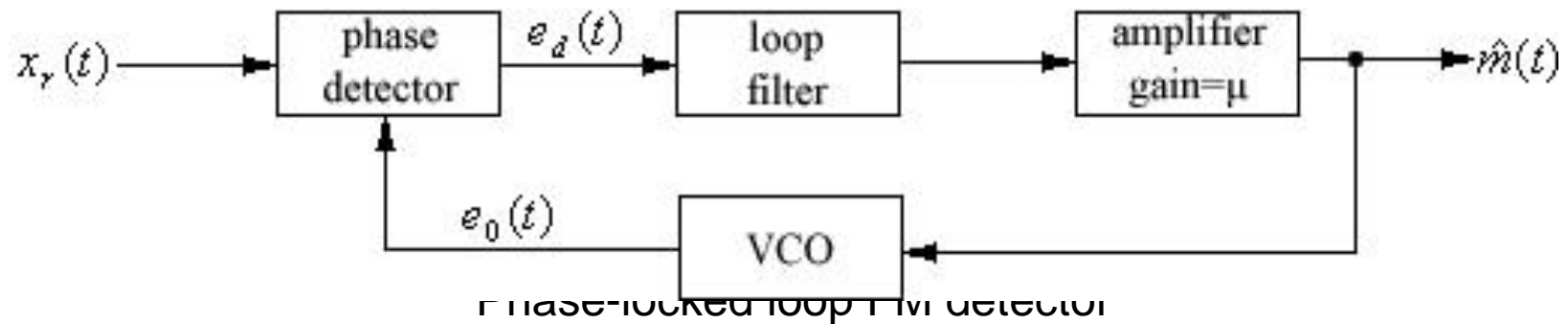
$$\begin{aligned} \hat{m}(t) &= s_{1e}(t) - s_{2e}(t) \\ &= 4\pi a A_c m(t) \end{aligned}$$

The cascade of a slope circuit and an envelope detector is known as a frequency discriminator.



## FM demodulation via phase-locked loops

We consider the Phase-locked loop (PLL) FM detector shown below

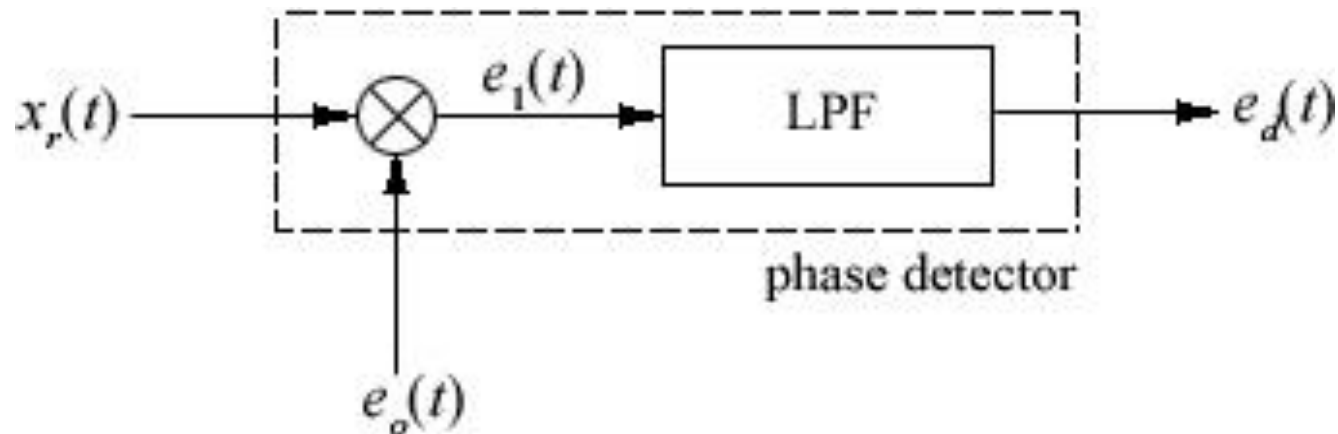


From the previous block diagram,

$$x_r(t) = A_c \cos(\omega_c t + \phi(t))$$

$$e_o(t) = A_v \sin(\omega_c t + \theta(t))$$

Let the phase detector be described by



then,

$$\begin{aligned} e_1(t) &= A_c \cos(\omega_c t + \phi(t)) \cdot A_v \sin(\omega_c t + \theta(t)) \cdot k_d \\ &= \frac{k_d A_c A_v}{2} [\sin[2\omega_c t + \phi(t) + \theta(t)] + \sin[\phi(t) - \theta(t)]] \end{aligned}$$

with the proper choice of lowpass filter, the output of the phase detector is

$$e_d(t) = \frac{k_d A_c A_v}{2} \sin[\phi(t) - \theta(t)]$$

A VCO is an FM modulator with peak frequency deviation

$$\Delta f = \max_t \left| \frac{d\theta(t)}{dt} \right|$$

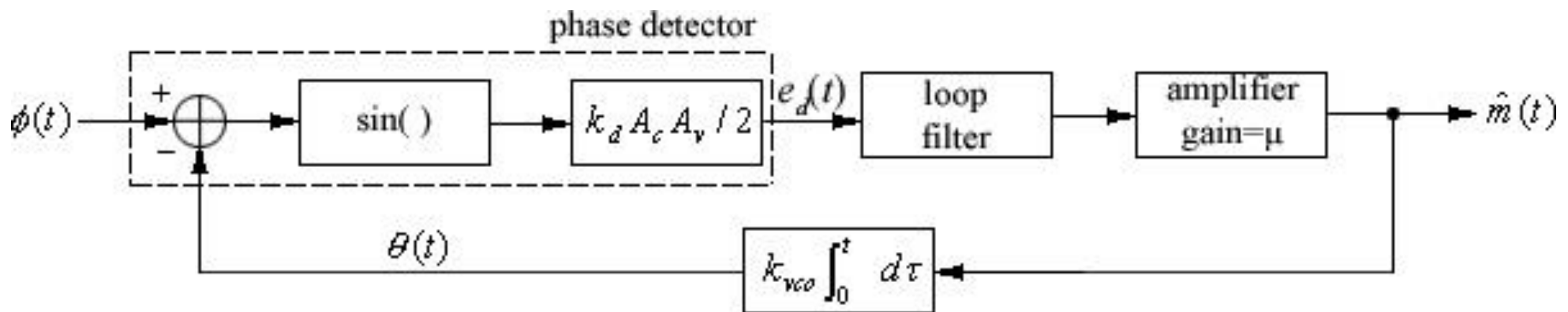
where

$$\frac{d\theta(t)}{dt} = k_{vco} \hat{m}(t)$$

implies that

$$\theta(t) = k_{vco} \int_0^t \hat{m}(\tau) d\tau.$$

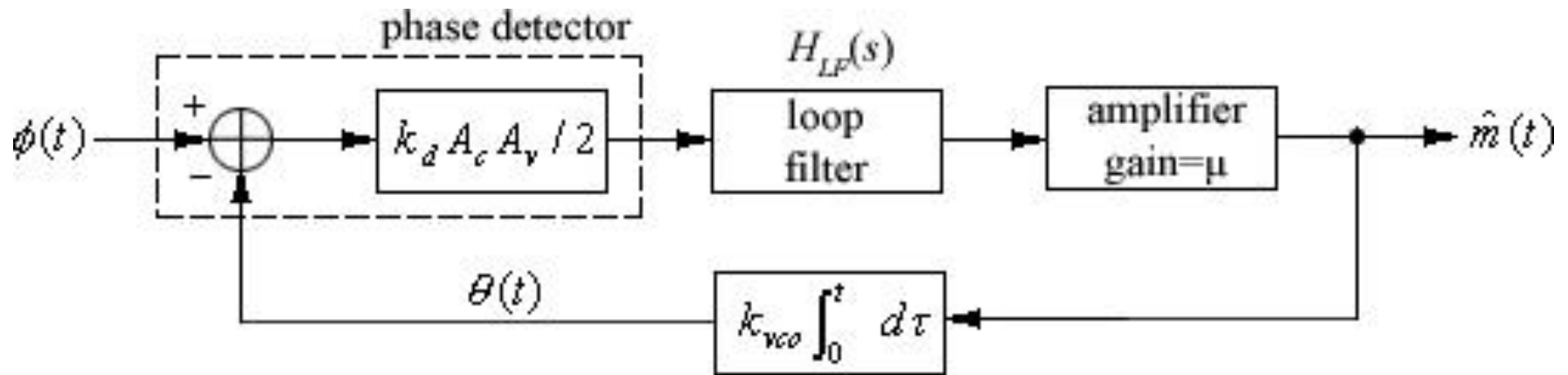
An equivalent nonlinear model is now shown



Let the PLL operate in the lock condition, i.e.,  $\theta(t) \cong \phi(t)$  or that  $|\theta(t) - \phi(t)|$  is small. Then,

$$\sin[\phi(t) - \theta(t)] \cong \phi(t) - \theta(t)$$

and the linear approximation of the PLL is given by



Linear model of the phase-locked loop

Let the loop filter have the transfer function  $H_{LF}(s) = 1$ , then

$$\begin{aligned} \hat{m}(t) &\cong \frac{k_d A_c A_v \mu}{2} [\phi(t) - \theta(t)] \\ &= k_t [\phi(t) - \theta(t)] \end{aligned}$$

Thus, the output of the vco is given by

$$\theta(t) = k_{vco} \int_0^t \hat{m}(\tau) d\tau = k_t k_{vco} \int_0^t [\phi(\tau) - \theta(\tau)] d\tau$$

Let  $k_0 = k_t k_{vco}$ , then

$$\frac{d\theta(t)}{dt} = k_0 [-\theta(t) + \phi(t)]$$

or

$$\frac{d\theta(t)}{dt} + k_0 \theta(t) = k_0 \phi(t),$$

In the s-domain, assuming zero initial conditions,

$$(s + k_0)\Theta(s) = k_0 \Phi(s),$$

The closed-loop transfer function is therefore given by

$$H_{cl}(s) = \frac{\Theta(s)}{\Phi(s)} = \frac{k_0}{s + k_0}$$

The corresponding impulse response is

$$h_{cl}(t) = k_0 e^{-k_0 t} u(t)$$

Let us now find out what happens when the loop gain  $k_0$  is increased, i.e.,

$$\lim_{k_0 \rightarrow \infty} H_{cl}(s) = \lim_{k_0 \rightarrow \infty} \frac{k_0}{s + k_0} = 1$$

Clearly,

or  $\Theta(s) \rightarrow \Phi(s)$ , for large  $k_0$ .

$$\theta(t) \rightarrow \phi(t)$$

Example: Let the message be a step function, i.e.,  $m(t) = Au(t)$ , then

$$x_{FM}(t) = A_c \cos \left( \omega_c t + k_\omega \int_0^t Au(\tau) d\tau \right)$$

In this case,

$$\phi(t) = k_\omega A \int_0^t u(\tau) d\tau = k_\omega A t$$

In the s-domain, and

$$\Phi(s) = \frac{Ak_\omega}{s^2} \quad \Theta(s) = \frac{Ak_\omega k_0}{s^2(s + k_0)}$$

The Laplace transform of  $\hat{m}(t)$  is then given by

$$\hat{M}(s) = k_t [\Phi(s) - \Theta(s)] = \frac{Ak_\omega k_t}{s(s+k_0)} = Ak_\omega k_0 \left[ \frac{1}{k_0} \right] \left[ \frac{1}{s} - \frac{1}{s+k_0} \right] = \frac{Ak_\omega}{k_{vco}} \left[ \frac{1}{s} - \frac{1}{s+k_0} \right].$$

Let  $k_1 = k_\omega/k_{vco}$ , then in the time domain,

$$\hat{m}(t) = Ak_1 u(t) - Ak_1 e^{-k_0 t} u(t) = k_1 m(t) - Ak_1 e^{-k_0 t} u(t)$$

Clearly, as  $t \rightarrow \infty$ , the estimate

$$\hat{m}(t) \rightarrow k_1 m(t).$$

Observation: This result is valid when the initial phase error is small.

Remark: A large loop gain  $k_0$  results in practical difficulties, hence, a different loop filter has to be used.

Consider the loop filter described by

$$H_{LF}(s) = (s+a)/s, \quad a > 0$$

Then the output of the vco is given by

$$\Theta(s) = \frac{k_0 H_{LF}(s)}{s} [\Phi(s) - \Theta(s)]$$



Let  $|\phi(t) - \theta(t)|$  be small, then the closed-loop transfer function is

$$H_{cl}(s) = \frac{\Theta(s)}{\Phi(s)} = \frac{k_0 H_{LF}(s)}{s + k_0 H_{LF}(s)} = \frac{k_0(s+a)}{s^2 + k_0 s + k_0 a}$$

Define  $\Psi(s) \hat{=} \Phi(s) - \Theta(s)$ , then

where

$$\Psi(s) = \Phi(s) - \frac{k_0 H_{LF}(s)}{s + k_0 H_{LF}(s)} \Phi(s) = \frac{s^2}{s^2 + k_0 s + k_0 a} \Phi(s) = \frac{s^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \Phi(s)$$

$$\omega_n = \sqrt{k_0 a}$$

$$2\xi\omega_n = k_0$$

$$\xi = \frac{k_0}{2\omega_n} = \frac{k_0}{2\sqrt{k_0 a}} = \frac{1}{2} \sqrt{\frac{k_0}{a}}$$

Consider again the step function message  $m(t) = Au(t)$ . Then

$$\phi(t) = k_\omega \int_0^t Au(\tau) d\tau = Ak_\omega \int_0^t d\tau = Ak_\omega t$$

In the complex frequency domain,

$$\Phi(s) = \frac{k_{\omega} A}{s^2}$$

Let  $\Delta\omega \equiv k_{\omega} A$ , then  $\Phi(s) = \Delta\omega/s^2$

and

$$\Psi(s) = \frac{\Delta\omega}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

If  $0 < \xi < 1$ , then

$$\psi(t) = \frac{\Delta\omega}{\omega_n \sqrt{1 - \xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_n \sqrt{1 - \xi^2} t\right)$$

and

Hence the  $\lim_{t \rightarrow \infty} \psi(t) = 0$  steady-state phase error is zero.

A typical value of  $\xi$  is 0.707.



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