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NOISE

Electronics & Communication Engg. Dept.

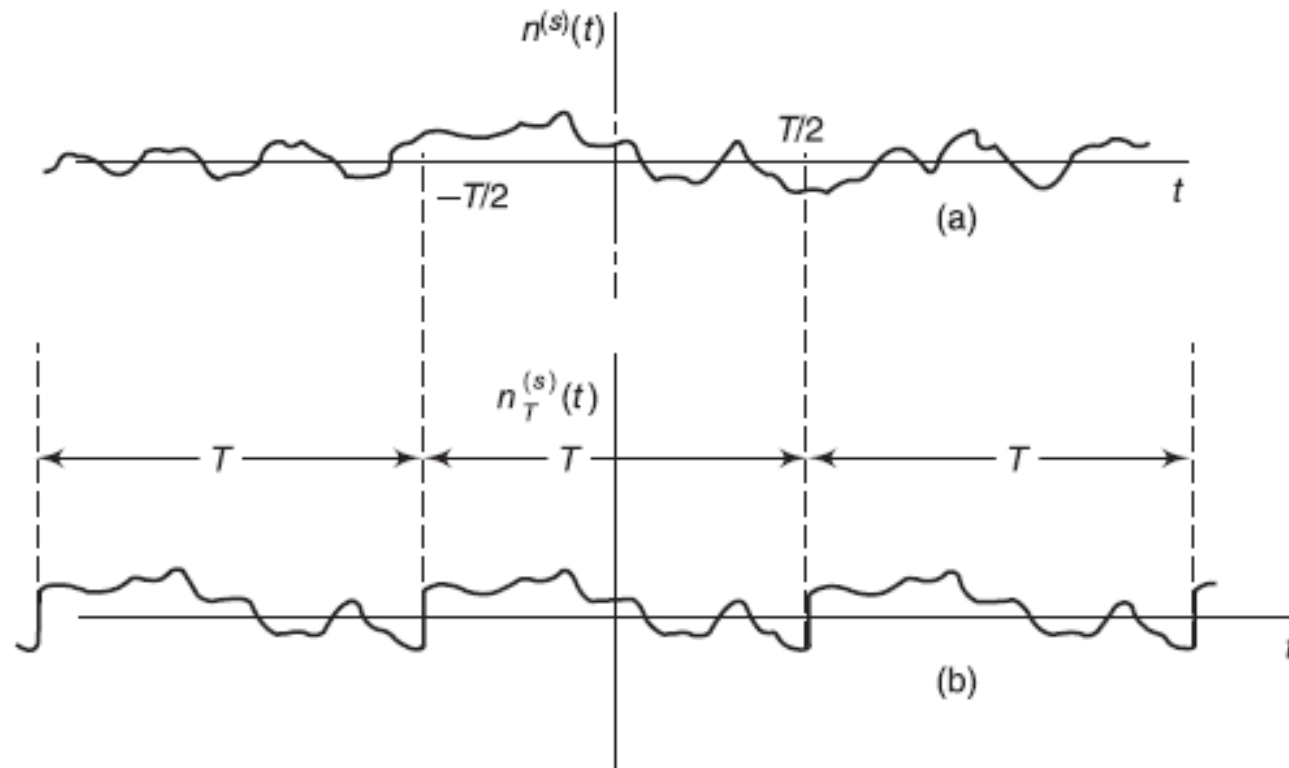
School of Engineering & Technology

CUTM

Sources of Noise

- Thermal Noise
- Shot Noise
- Additive Noise
- Multiplicative Noise (fading)
- Gaussian Noise

Noise: Frequency Domain Representation



(a) A sample noise waveform. (b) A periodic waveform is generated by repeating the interval in (a) from $-T/2$ to $T/2$.

$$n_T^{(s)}(t) = \sum_{k=1}^{\infty} (a_k \cos 2\pi k \Delta f t + b_k \sin 2\pi k \Delta f t)$$

$$n_T^{(s)}(t) = \sum_{k=1}^{\infty} c_k \cos(2\pi k \Delta f t + \theta_k) \quad c_k^2 = a_k^2 + b_k^2 \quad \theta_k = -\tan^{-1} \frac{b_k}{a_k}$$

Noise : Power Spectrum

$$G_n(k \Delta f) \equiv G_n(-k \Delta f) \equiv \frac{c_k^2}{4 \Delta f} = \frac{a_k^2 + b_k^2}{4 \Delta f}$$

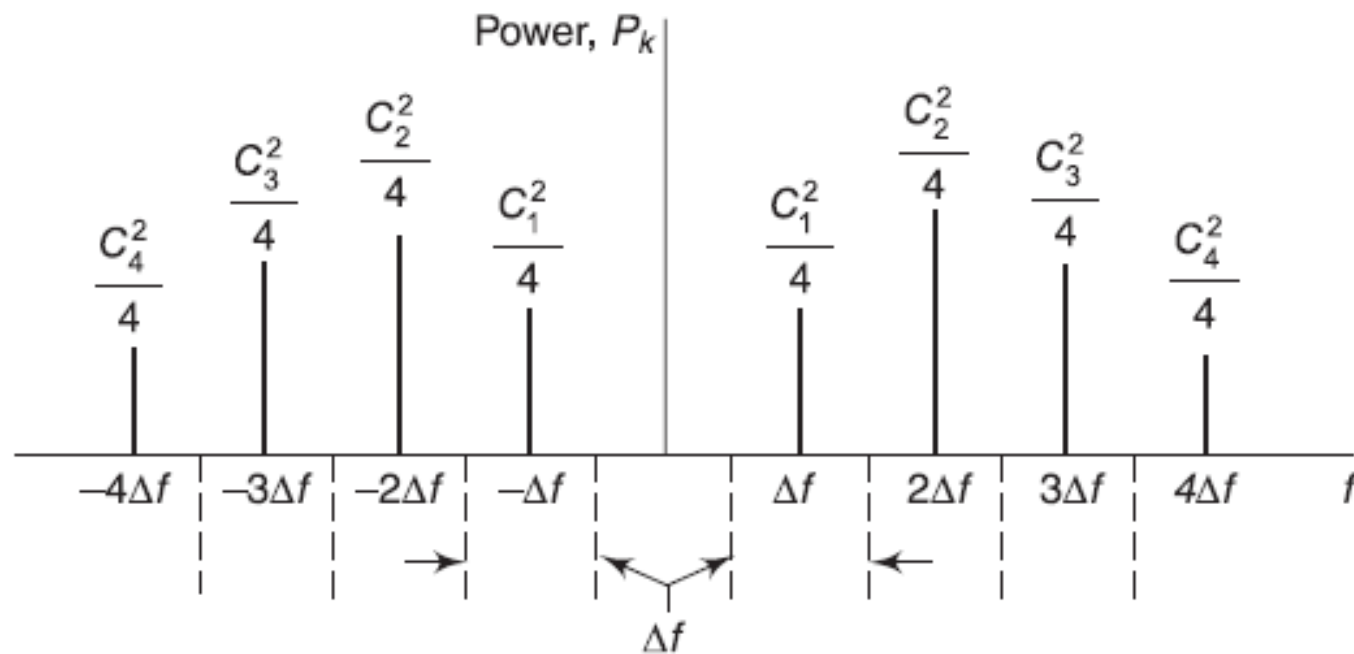


Fig. 7.2 The power spectrum of the waveform $n_T^{(s)}$.

mean power spectral density $G_n(k \Delta f) \equiv G_n(-k \Delta f)$

total power $P_k = 2G_n(k \Delta f) \Delta f$

Noise Representation

$$n(t) = \lim_{\Delta f \rightarrow 0} \sum_{k=1}^{\infty} (a_k \cos 2\pi k \Delta f t + b_k \sin 2\pi k \Delta f t)$$

$$n(t) = \lim_{\Delta f \rightarrow 0} \sum_{k=1}^{\infty} c_k \cos (2\pi k \Delta f t + \theta_k) \quad \overline{c_k^2} = \overline{a_k^2} + \overline{b_k^2}$$

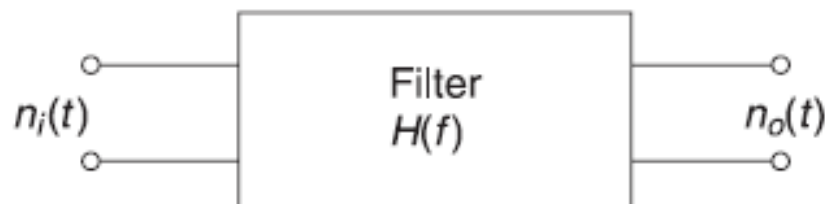
$$G_n(f) = \lim_{\Delta f \rightarrow 0} \frac{\overline{c_k^2}}{4 \Delta f} = \lim_{\Delta f \rightarrow 0} \frac{\overline{a_k^2} + \overline{b_k^2}}{4 \Delta f}$$

$$P(f_1 \rightarrow f_2) = \int_{-f_2}^{-f_1} G_n(f) df + \int_{f_1}^{f_2} G_n(f) df = 2 \int_{f_1}^{f_2} G_n(f) df$$

total power P_T is

$$P_T = \int_{-\infty}^{\infty} G_n(f) df = 2 \int_0^{\infty} G_n(f) df$$

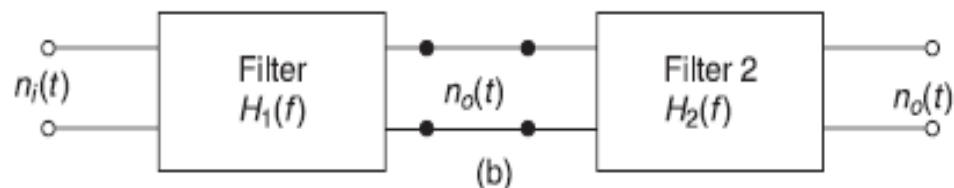
Filtering Gaussian Noise



$$n_o(t) = \int_{-\infty}^{\infty} n_i(\tau) h(t - \tau) d\tau = \int_{-\infty}^t n_i(\tau) h(t - \tau) d\tau$$

$$n_o(t) = \lim_{\Delta\tau \rightarrow 0} \sum_{k=-\infty}^{k=t/\Delta\tau} n_i(k \Delta\tau) h(t - k \Delta\tau) \Delta\tau$$

Thus, at any time, with $t = t_0$, $n_o(t) = n_o(t_0)$, as given by Eq. (7.14), is a linear superposition of independent gaussian random variables. Hence, as discussed in Sec. 2.4.2, $n_o(t_0)$ is a gaussian random variable, and $n_o(t)$ is a gaussian random process.



$$H(f) = H_1(f)H_2(f)$$

a linear superposition of *dependent* gaussian random variables is still gaussian.

Noise : Spectral Component

$$n_k(t) = a_k \cos 2\pi k \Delta f t + b_k \sin 2\pi k \Delta f t$$

$$n_k(t) = c_k \cos (2\pi k \Delta f t + \theta_k)$$

$$P_k = \overline{[n_k(t)]^2} = \overline{a_k^2} \cos^2 2\pi k \Delta f t + \overline{b_k^2} \sin^2 2\pi k \Delta f t \\ + \overline{2a_k b_k} \sin 2\pi k \Delta f t \cos 2\pi k \Delta f t$$

$$P_k = 2G_n(k \Delta f) \Delta f = 2G_n(-k \Delta f) \Delta f = \overline{a_k^2} = \overline{b_k^2} = \frac{\overline{a_k^2}}{2} + \frac{\overline{b_k^2}}{2} = \frac{\overline{c_k^2}}{2}$$

$$\text{Since } \overline{a_k^2} = \overline{b_k^2} \quad P_k = \overline{a_k^2} + \overline{2a_k b_k} \sin 2\pi k \Delta f t \cos 2\pi k \Delta f t$$

$$P_k = \overline{a_k^2} \text{ independently of time, } \overline{a_k b_k} = 0 \quad n_k(t_1) = a_k$$

$$n_k(t) = a_k \cos 2\pi k \Delta f t + b_k \sin 2\pi k \Delta f t$$

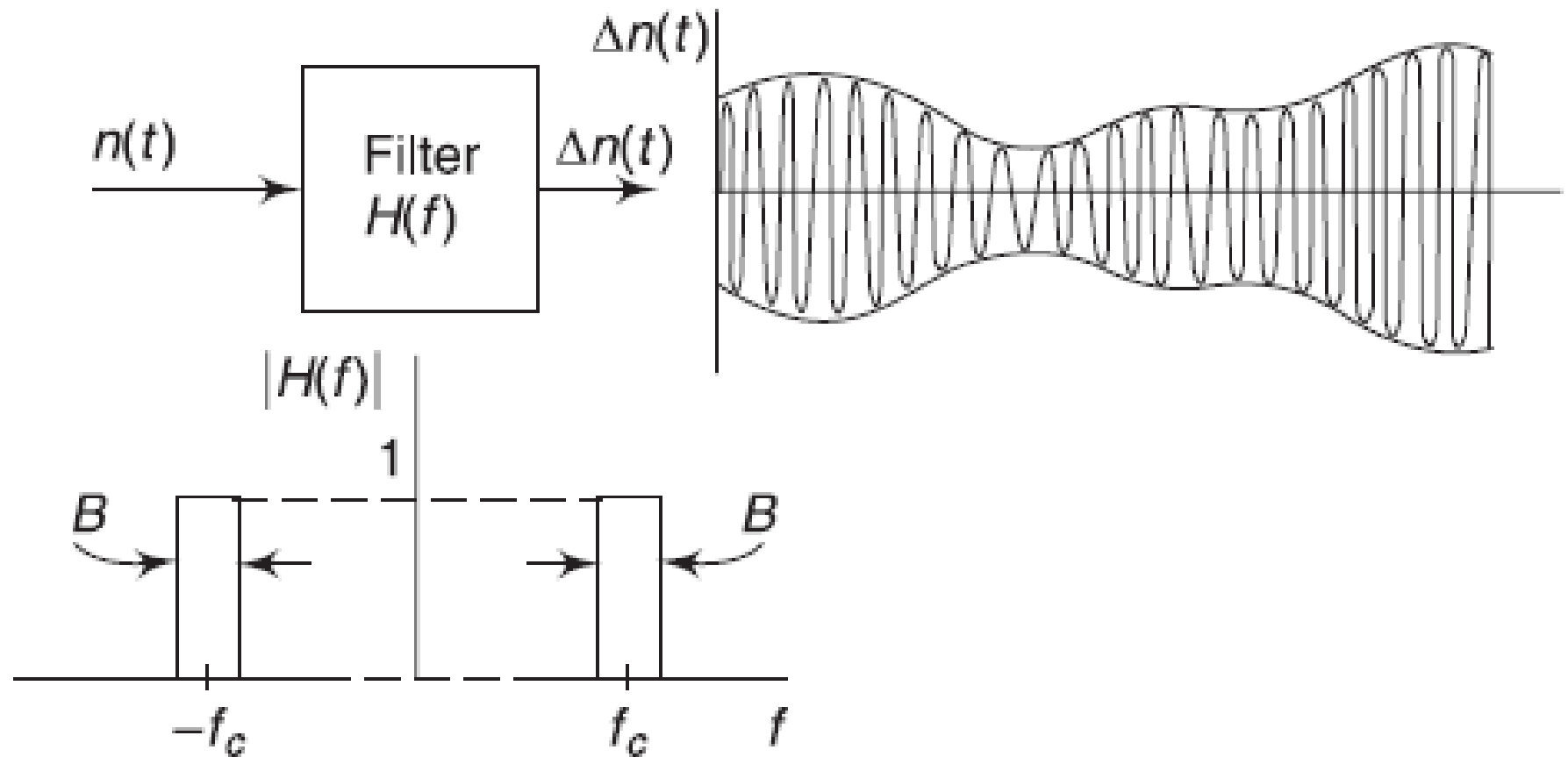
$$n_l(t) = a_l \cos 2\pi l \Delta f t + b_l \sin 2\pi l \Delta f t$$

$$\overline{a_k a_l} = \overline{a_k b_l} = \overline{b_k a_l} = \overline{b_k b_l} = 0$$

$$f(c_k) = \frac{c_k}{P_k} e^{-ck^2/2P_k} \quad c_k \geq 0$$

$$f(\theta_k) = \frac{1}{2\pi} \quad -\pi \leq \theta_k \leq \pi$$

Noise : Narrowband Filter Response



Noise PSD : Effect of Filter

$$H(k \Delta f) = |H(k \Delta f)| e^{j\varphi_k} = |H(k \Delta f)| / \varphi_k$$

$$n_{k_o}(t) = |H(k \Delta f)| a_k \cos(2\pi k \Delta f t + \varphi_k) + |H(k \Delta f)| b_k \sin(2\pi k \Delta f t + \varphi_k)$$

Since $|H(k \Delta f)|$ is a deterministic function,

$$\overline{[|H(k \Delta f)| a_k]^2} = |H(k \Delta f)|^2 \overline{a_k^2} \quad \text{and} \quad \overline{[|H(k \Delta f)| b_k]^2} = |H(k \Delta f)|^2 \overline{b_k^2}$$

$$\text{power } P_{k_o} \text{ associated with } n_{k_o}(t) \text{ is } P_{k_o} = |H(k \Delta f)|^2 \frac{\overline{a_k^2} + \overline{b_k^2}}{2}$$

$$G_{n_o}(k \Delta f) = |H(k \Delta f)|^2 G_{n_i}(k \Delta f)$$

$$G_{n_o}(f) = |H(f)|^2 G_{n_i}(f)$$

Mixing Noise with Sinusoid

$$n_k(t) \cos 2\pi f_0 t = \frac{a_k}{2} \cos 2\pi(k \Delta f + f_0)t + \frac{b_k}{2} \sin 2\pi(k \Delta f + f_0)t \\ + \frac{a_k}{2} \cos 2\pi(k \Delta f - f_0)t + \frac{b_k}{2} \sin 2\pi(k \Delta f - f_0)t$$

$$G_n(k \Delta f + f_0) = G_n(k \Delta f - f_0) = \frac{G_n(k \Delta f)}{4}$$

$$G_n(f + f_0) = G_n(f - f_0) = \frac{G_n(f)}{4}$$

$$n_{p1}(t) = \frac{a_k}{2} \cos 2\pi p \Delta f t - \frac{b_k}{2} \sin 2\pi p \Delta f t$$

$$n_{p2}(t) = \frac{a_l}{2} \cos 2\pi p \Delta f t + \frac{b_l}{2} \sin 2\pi p \Delta f t$$

$$\overline{a_k a_l} = \overline{a_k b_l} = \overline{b_k a_l} = \overline{b_k b_l} = 0, \quad E[n_{p1}(t)n_{p2}(t)] = 0$$

$$E\{[n_{p1}(t) + n_{p2}(t)]^2\} = E\{[n_{p1}(t)]^2\} + E\{[n_{p2}(t)]^2\}$$

Mixing Noise with Noise

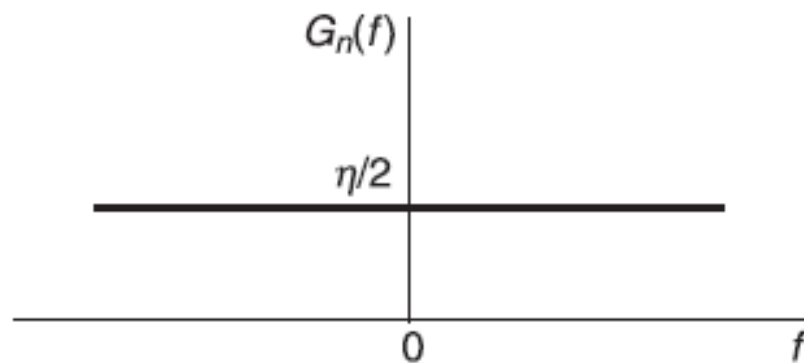
$$\begin{aligned} n_k(t)n_l(t) &= \frac{1}{2} c_k c_l \cos [2\pi(k+l) \Delta ft + \theta_k + \theta_l] \\ &+ \frac{1}{2} c_k c_l \cos [2\pi(k-l) \Delta ft + \theta_k - \theta_l] \end{aligned}$$

$$P_{k+l} = P_{k-l} = \overline{\left(\frac{1}{2} c_k c_l \right)^2}$$

Since c_k and c_l are independent random variables,

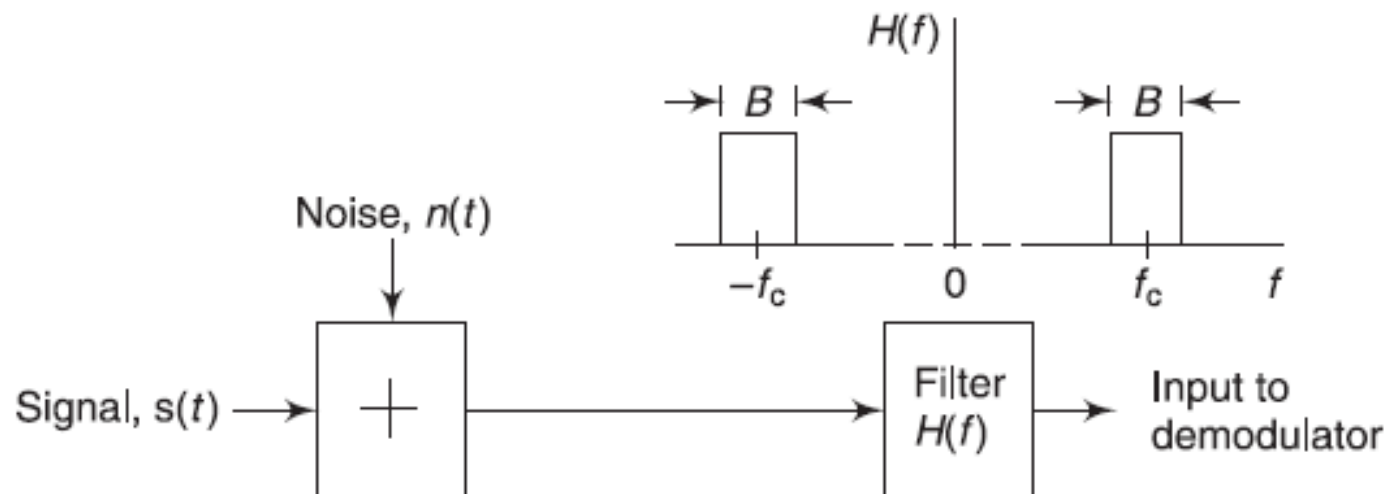
$$P_{k+l} = P_{K-l} = \frac{1}{8} \overline{c_k^2} \overline{c_l^2} = \frac{1}{2} P_k P_l$$

Linear Filtering of Noise



$$G_n(f) = \frac{\eta}{2}$$

Fig. 7.6 Power spectral density of white noise.



A filter is placed before a demodulator to limit the noise power input to the demodulator.

Noise and Low Pass Filter

RC Low Pass Filter: $H(f) = \frac{1}{1 + jf/f_c}$

$$G_{n_o}(f) = G_{n_i}(f)|H(f)|^2 \qquad G_{n_o}(f) = \frac{\eta}{2} \frac{1}{1 + (f/f_c)^2}$$

$$N_o = \int_{-\infty}^{\infty} G_{n_o}(f) df = \frac{\eta}{2} \int_{-\infty}^{\infty} \frac{df}{1 + (f/f_c)^2}$$

$$\text{noting that } \int_{-\infty}^{\infty} dx/(1 + x^2) = \pi, \qquad N_o = \frac{\pi}{2} \eta f_c$$

Ideal Low Pass Filter: $H(f) = \begin{cases} 1 & |f| \leq B \\ 0 & \text{elsewhere} \end{cases}$

$$G_{n_o}(f) = \begin{cases} \frac{\eta}{2} & -B \leq f \leq B \\ 0 & \text{elsewhere} \end{cases} \qquad N_o = \eta B$$

Noise and Band Pass Filter

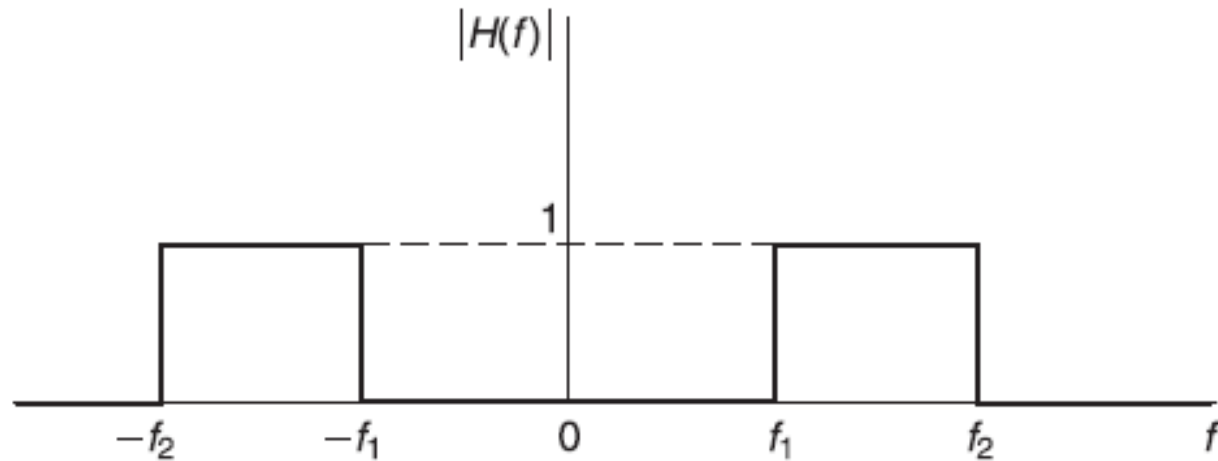


Fig. 7.8 A rectangular bandpass filter.

$$N_o = 2\frac{\eta}{2} (f_2 - f_1) = \eta (f_2 - f_1)$$

Noise and Differentiator

A differentiator follows $H(f) = j2\pi\tau f$

If white noise with $G_{n_i}(f) = \eta/2$

$$G_{n_o}(f) = |H(f)|^2 G_{n_i}(f) = 4\pi^2 \tau^2 f^2 \frac{\eta}{2}$$

If the differentiator is followed by a rectangular low-pass filter having a bandwidth B ,

noise power at the output of the low-pass filter

$$N_o = \int_{-B}^B 4\pi^2 \tau^2 f^2 \frac{\eta}{2} df = \frac{4\pi^2}{3} \eta \tau^2 B^3$$

Noise and Integrator-1

An integrator follows

$$H(f) = \frac{1}{j\omega\tau} - \frac{e^{-j\omega T}}{j\omega\tau} = \frac{1 - e^{-j\omega T}}{j\omega\tau}$$

with $\omega = 2\pi f$,

$$|H(f)|^2 = \left(\frac{T}{\tau}\right)^2 \left(\frac{\sin \pi T f}{\pi T f}\right)^2$$

$$N_o = \int_{-\infty}^{\infty} \frac{\eta}{2} |H(f)|^2 df = \frac{\eta}{2} \left(\frac{T}{\tau}\right)^2 \int_{-\infty}^{\infty} \left(\frac{\sin \pi T f}{\pi T f}\right)^2 df = \frac{\eta T}{2\tau^2}$$

$$n_o(T) = \frac{1}{\tau} \int_0^T n_i(t) dt$$

$$m_{n_o} = E[n_o(T)] = E\left\{\frac{1}{\tau} \int_0^T n_i(t) dt\right\} = \int_{-\infty}^{\infty} dn_i f(n_i) \left\{\frac{1}{\tau} \int_0^T n_i(t) dt\right\}$$

$$= \frac{1}{\tau} \int_0^T dt \left\{ \int_{-\infty}^{\infty} n_i f(n_i) dn_i \right\}$$

$$m_{n_i} = E(n_i) = \int_{-\infty}^{\infty} n_i f(n_i) dn_i$$

Noise and Integrator-2

$$N_o = \sigma_{n_o}^2 = E\{[n_o(T)]^2\} = E\left\{\frac{1}{\tau^2} \int_0^T n_i(t) dt \cdot \int_0^T n_i(\lambda) d\lambda\right\}$$

If we define $\alpha \equiv n_i(t)$ and $\beta \equiv n_i(\lambda)$

$$N_o = \frac{1}{\tau^2} \int_0^T dt \int_0^T d\lambda \int_{-\infty}^{\infty} \alpha \beta f(\alpha, \beta) d\alpha d\beta$$

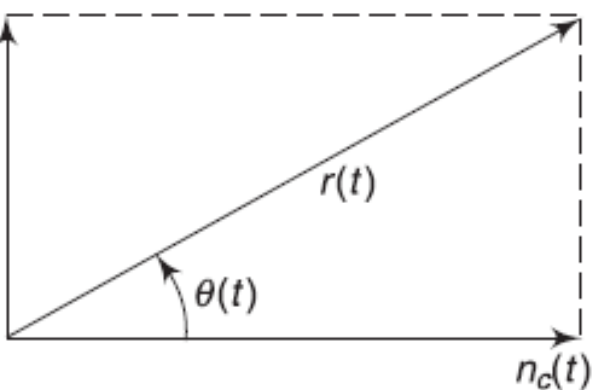
$$R_{n_i}(t - \lambda) = E[n_i(t)n_i(\lambda)] = E(\alpha\beta) = N_o$$

$$E[n_i(t)n_i(\lambda)] = R_{n_i}(t - \lambda) = \mathcal{F}\{G(f)\} = \int_{-\infty}^{\infty} \eta/2 e^{-j2\pi f(t - \lambda)} df = (\eta/2) \delta(t - \lambda)$$

$$\begin{aligned} N_o &= \frac{1}{\tau^2} \int_0^T dt \int_0^T d\lambda (\eta/2) \delta(t - \lambda) \\ &= \frac{1}{\tau^2} \int_0^T dt \cdot \eta/2 = \frac{\eta T}{2\tau^2} \end{aligned}$$

Quadrature Components of Noise

It is sometimes more advantageous to represent Narrowband noise centred around f_0 as

$$n(t) = n_c(t) \cos 2\pi f_0 t - n_s(t) \sin 2\pi f_0 t$$


Now,

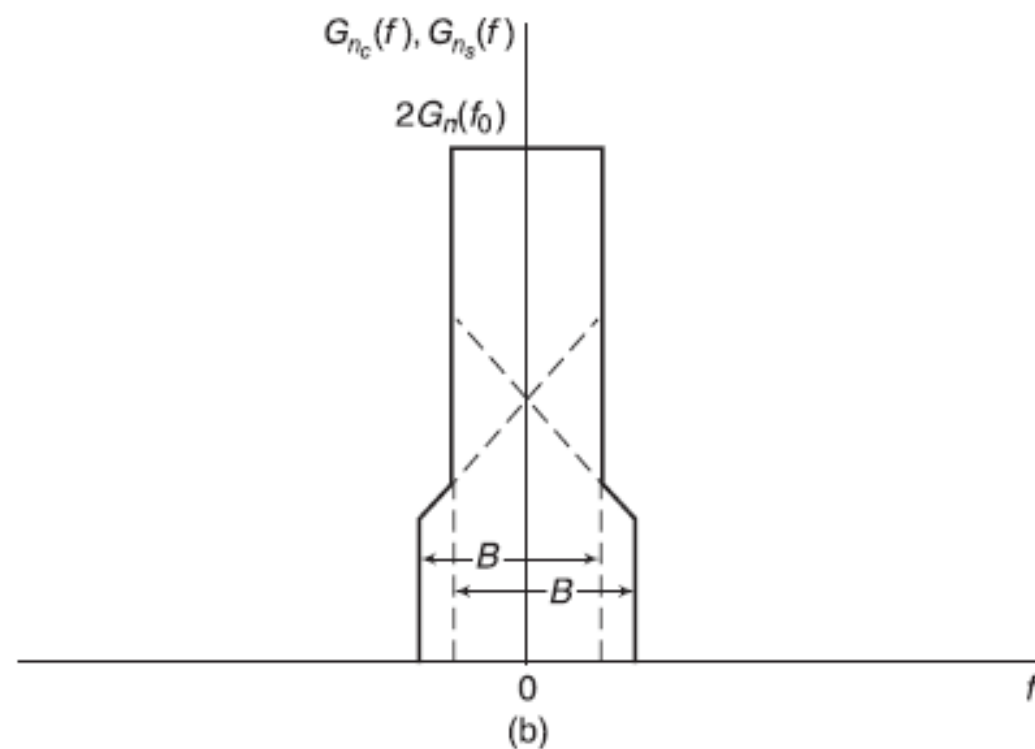
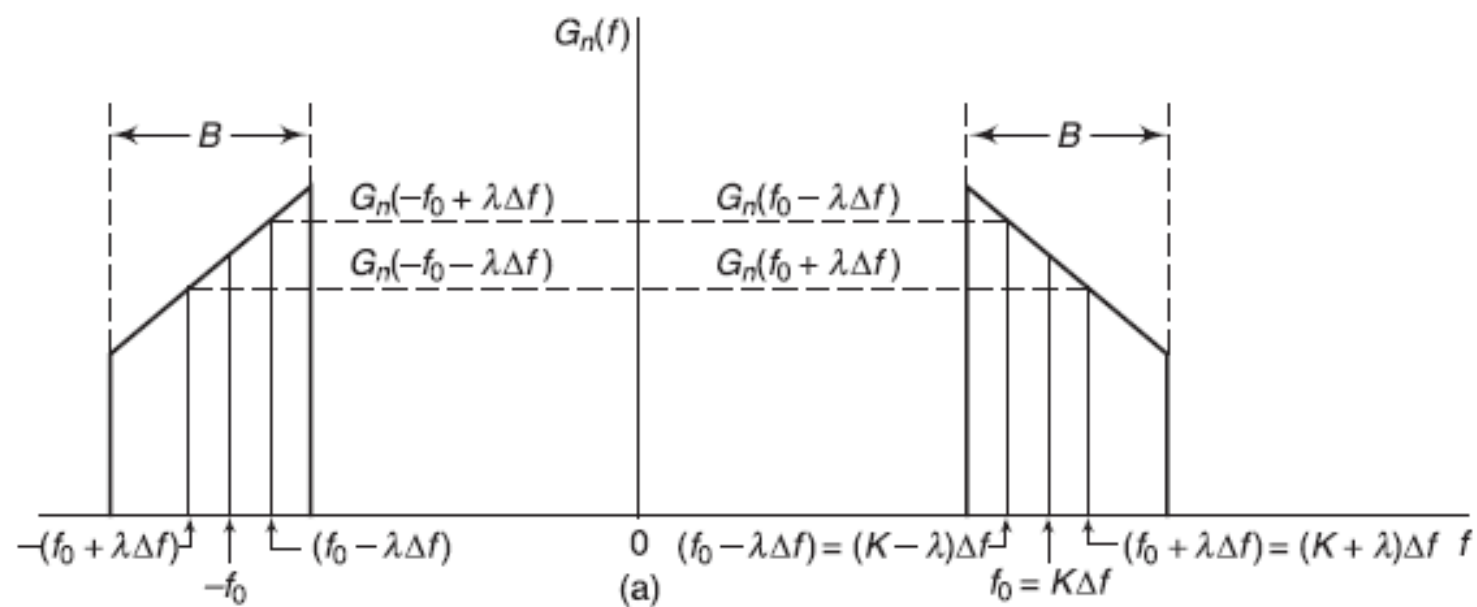
$$n(t) = \lim_{\Delta f \rightarrow 0} \sum_{k=1}^{\infty} (a_k \cos 2\pi k \Delta f t + b_k \sin 2\pi k \Delta f t)$$

$$n(t) = \lim_{\Delta f \rightarrow 0} \sum_{k=1}^{\infty} \{a_k \cos 2\pi [f_0 + (k - K) \Delta f] t + b_k \sin 2\pi [f_0 + (k - K) \Delta f] t\}$$

$$n_c(t) = \lim_{\Delta f \rightarrow 0} \sum_{k=1}^{\infty} [a_k \cos 2\pi (k - K) \Delta f t + b_k \sin 2\pi (k - K) \Delta f t]$$

$$n_s(t) = \lim_{\Delta f \rightarrow 0} \sum_{k=1}^{\infty} [a_k \sin 2\pi (k - K) \Delta f t - b_k \cos 2\pi (k - K) \Delta f t]$$

$$r(t) = [n_c^2(t) + n_s^2(t)]^{1/2} \quad \theta(t) = \tan^{-1} [n_s(t)/n_c(t)]$$



(a) Power spectrum of bandlimited noise.
 (b) Power spectral density of n_c and n_s .



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