

Frequency Modulation

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Angle Modulation

- Angle modulation is defined as the process in which the angle of the carrier (either frequency or phase) is varied linearly according to the message signal.
- There are two types of angle modulation:
 - Frequency Modulation
 - Phase Modulation

Equation:

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t) dt\right] \quad K_f \text{ is the frequency sensitivity of frequency modulator (Hz/ volt)}$$

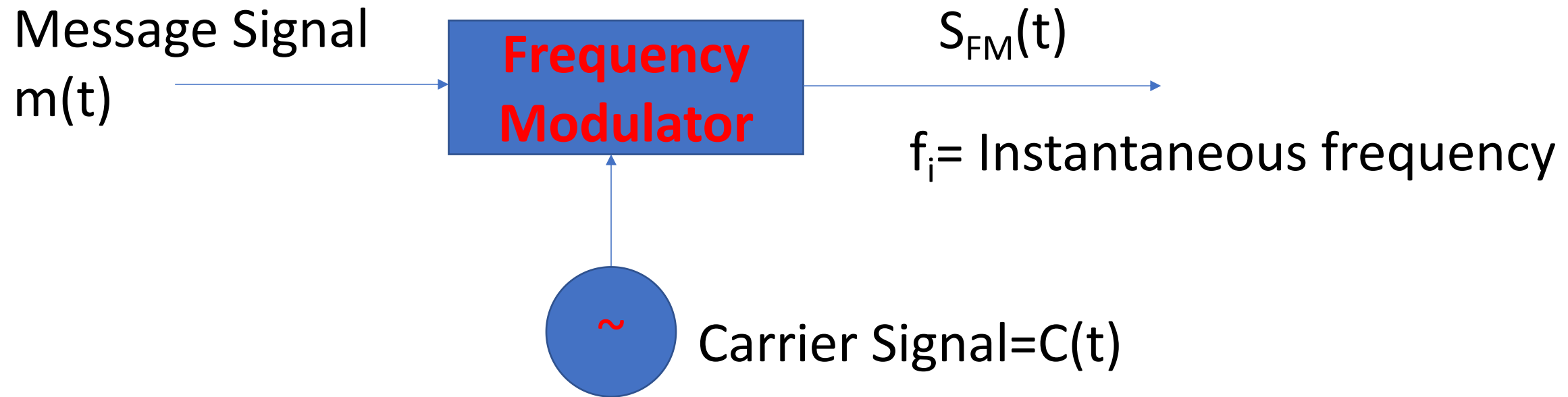
$$S_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right] \quad K_p \text{ is the phase sensitivity of the modulator (radians/volt)}$$

Frequency Modulation

- Carrier signal = $A_c \cos(2\pi f_c t + \phi)$

$$\text{Angle} = \theta(t) = 2\pi f_c t + \phi$$

$$2\pi f_c t \rightarrow \text{rad}$$



- Instantaneous frequency $f_i(t) = f_c + k_f m(t)$
- If $m(t) = 0$, then no modulation i.e. $f_i(t) = f_c$
- K_f specifies the amount of frequency change in the carrier signal for 1 volt change of the message signal.
- $K_f = 50 \text{ kHz/volt}$ (given)
- When $m(t) = 0 \text{ volt}$, $f_i = f_c$
- When $m(t) = 5 \text{ volt}$, then $f_i = f_c + 50 \text{ kHz/volt} \times 5 \text{ volt} = f_c + 250 \text{ kHz}$
- When $m(t) = -5 \text{ volt}$, then $f_i = f_c - 50 \text{ kHz/volt} \times 5 \text{ volt} = f_c - 250 \text{ kHz}$

- **Summary**

$$m(t)=0 \longrightarrow f_i=f_c$$

$$m(t)= +ve \longrightarrow f_i > f_c$$

$$m(t)= -ve \longrightarrow f_i < f_c$$

- In FM, message signal voltage variation are converted to an carrier signal frequency variation so it is also called as voltage to frequency conversion
- For $m(t)= A_m \text{Cos}2\pi f_m t$ or $A_m \text{Sin}2\pi f_m t$

- Maximum possible frequency of the resulting signal

$$f_{max} = f_c + k_f A_m$$

- Minimum possible frequency of the resulting signal

$$f_{min} = f_c - k_f A_m$$

- Maximum possible frequency deviation $\Rightarrow \Delta f = |Max[k_f m(t)]|$

$$\Delta f = |Max(k_f m(t))|$$

$$\Delta f = K_f A_m \text{ Hz}$$

$$f_{max} = f_c + \Delta f$$

$$f_{min} = f_c - \Delta f$$

Total frequency swing
of the resulting FM
signal is $f_{max} - f_{min} = 2\Delta f$

General Expression of FM signal

- Carrier signal $=C(t)=A_c \text{Cos}[2\pi f_c t + \varphi]$
- $C(t)=A_c \text{Cos}[\theta(t)]$ Where $\theta(t)=2\pi f_c t + \varphi$

$$\frac{d\theta(t)}{dt} = 2\pi f_c$$

- General expression of FM signal is

$$S_{FM}(t) = A_c \text{Cos}[\theta_i(t)]$$

- $\theta_i(t)$ = Instantaneous angle

• If $\frac{d\theta(t)}{dt} = 2\pi f_c$

$$\frac{d\theta_i(t)}{dt} = 2\pi f_i$$

$$\theta_i(t) = 2\pi \int f_i dt$$

$$\theta_i(t) = 2\pi \int [f_c + k_f m(t)] dt$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt$$

$$S_{FM}(t) = A_c \text{Cos}[\theta_i(t)]$$

$$S_{FM}(t) = A_c \text{Cos}\left[2\pi f_c t + 2\pi k_f \int_0^t m(t) dt\right]$$

Single tone FM

- Assume $m(t) = A_m \cos 2\pi f_m t$

$$\int m(t) dt = \int (A_m \cos 2\pi f_m t) dt = \frac{A_m \sin 2\pi f_m t}{2\pi f_m}$$

$$S_{FM}(t) = A_c \cos \left[2\pi f_c t + \frac{2\pi k_f A_m \sin 2\pi f_m t}{2\pi f_m} \right]$$

$$S_{FM}(t) = A_c \cos [2\pi f_c t + \beta \sin 2\pi f_m t]$$

β is the modulation index of FM

$$\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}$$