

## CHAPTER 7

### AMPLITUDE MODULATION

Transmit information-bearing (message) or baseband signal (voice-music) through a Communications Channel

Baseband = band of frequencies representing the original signal  
for music 20 Hz - 20,000 Hz, for voice 300 - 3,400 Hz

write the baseband (message) signal  $m(t) \leftrightarrow M(f)$

#### Communications Channel

Typical radio frequencies 10 KHz  $\rightarrow$  300 GHz

write  $c(t) = A_c \cos(2\pi f_c t)$

$c(t)$  = Radio Frequency Carrier Wave  
 $A_c$  = Carrier Amplitude  
 $f_c$  = Carrier Frequency

Amplitude Modulation (AM)

$\rightarrow$  Amplitude of carrier wave varies  $\pm$  a mean value in step with the baseband signal  $m(t)$

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

Mean value  $A_c$ .

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Recall a general signal

$$s(t) = a(t) \cos[2\pi f_c t + \phi(t)]$$

For AM

$$a(t) = A_c[1 + k_a m(t)]$$

$$\phi(t) = 0 \text{ or constant}$$

$k_a$  = Amplitude Sensitivity

- Note
- 1  $|k_a m(t)| < 1$  or  $[1 + k_a m(t)] > 0$
  - 2  $f_c \gg w$  = bandwidth of  $m(t)$

**AM Signal In Time and Frequency Domain**

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

$$s(t) = A_c[1 + k_a m(t)] \frac{(e^{j2\pi f_c t} + e^{-j2\pi f_c t})}{2}$$

$$\begin{aligned} s(t) &= \frac{A_c}{2} e^{j2\pi f_c t} + \frac{A_c}{2} e^{-j2\pi f_c t} \\ &\quad + \frac{A_c k_a}{2} m(t) e^{j2\pi f_c t} \\ &\quad + \frac{A_c k_a}{2} m(t) e^{-j2\pi f_c t} \end{aligned}$$

To find  $S(f)$  use:

$$m(t) \leftrightarrow M(f)$$

$$e^{j2\pi f_c t} \leftrightarrow \delta(f - f_c)$$

$$e^{-j2\pi f_c t} \leftrightarrow \delta(f + f_c)$$

$$\exp(j2\pi f_c t)m(t) \leftrightarrow M(f - f_c)$$

$$\exp(-j2\pi f_c t)m(t) \leftrightarrow M(f + f_c)$$

$$\begin{aligned} S(f) &= \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] \\ &\quad + \frac{A_c k_a}{2} [M(f - f_c) + M(f + f_c)] \end{aligned}$$

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$$\begin{aligned} s(t) &= A_c[1 + k_a m(t)] \cos 2\pi f_c t \\ &= \frac{A_c}{2}[1 + k_a m(t)][\exp(j2\pi f_c t) + \exp(-j2\pi f_c t)] \end{aligned}$$

If  $k_a m(t) > 1$ , then

→ Overmodulation

→ Envelope Distortion

see Text p. 262 - Fig. 7.1

max value of  $k_a m(t)$  = Percent Modulation

Look at AM signal in frequency domain

$$\begin{aligned} S(f) &= \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] \\ &\quad + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)] \end{aligned}$$

where  $M(f) = FT$  of  $m(t)$

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Example 1 p. 265 - Single Tone Modulation

$$m(t) = A_m \cos 2\pi f_m t \quad f_m = 1 \text{ KHz (message frequency)}$$

$$s(t) = A_c(1 + k_a m(t)) \cos 2\pi f_c t$$

$$s(t) = A_c(1 + \mu \cos 2\pi f_m t) \cos 2\pi f_c t$$

$$\mu = k_a A_m < 1$$

= Modulation Factor (Percentage)

Draw  $s(t)$  for  $\mu < 1$

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1 + \mu)}{A_c(1 - \mu)}$$

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

Rewrite  $s(t)$  using trig identity

$$\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta) \text{ with } \alpha = f_c, \quad \beta = f_m$$

$$s(t) = A_c \left[ \cos 2\pi f_c t + \frac{\mu}{2} \cos 2\pi(f_c + f_m)t \right. \\ \left. + \frac{\mu}{2} \cos 2\pi(f_c - f_m)t \right]$$

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] \\ + \frac{A_c \mu}{4} [\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] \\ + \frac{A_c \mu}{4} (\delta(f - f_c + f_m) + \delta(f + f_c - f_m))]$$

$$[s(t)]^2 = \text{power into } 1\Omega \sim |S(f)|^2$$

Carrier Power  $2 \left(\frac{A_c}{2}\right)^2 = \frac{A_c^2}{2}$   
 Positive and Negative Frequencies ↗

Upper Sideband Power  $2 \left(\frac{A_c\mu}{4}\right)^2 = \frac{A_c^2\mu^2}{8}$

Lower Sideband Power  $2 \left(\frac{A_c\mu}{4}\right)^2 = \frac{A_c^2\mu^2}{8}$

$$\frac{\text{USB + LSB Power}}{\text{Total Power}} = \frac{\frac{A_c^2\mu^2}{8} + \frac{A_c^2\mu^2}{8}}{\frac{A_c^2}{2} + \frac{A_c^2\mu^2}{8} + \frac{A_c^2\mu^2}{8}} = \frac{\mu^2}{2 + \mu^2}$$

Example

Carrier Power	% Modulation	Modulation (Audio) Power USB + LSB	Total Power	Audio Power Total
$\frac{A_c^2}{2}$	$\mu \cdot 100$	$\frac{A_c^2}{4}\mu^2$	$(2 + \mu^2)\frac{A_c^2}{4}$	$\frac{\mu^2}{2 + \mu^2}$
100	100	50	150	0.33
100	50	12.5	112.5	0.11
100	20	4	104	0.04
100	0	0	100	0

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Generation of AM Waves - P. 267

Square Law Modulator -

3 stages: Adder, Non-Linearity (NL), Bandpass Filter (BPF)

For this example, we use non-linearity  $v_2(t) = a_1v_1(t) + a_2v_1^2(t)$

$$\begin{aligned}\text{Here } v_1(t) &= A_c \cos 2\pi f_c t + m(t) \\ \rightarrow v_2(t) &= \overbrace{a_1 A_c \left[ 1 + \frac{2a_2}{a_1} m(t) \right] \cos 2\pi f_c t} \\ &\quad + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t\end{aligned}$$

The first (bracketed) term is desired AM wave at carrier  $f_c$  with

$$k_a = \frac{2a_2}{a_1}$$

Remaining terms are at baseband or  $2f_c$  and are filtered out by BPF. BPF has center frequency  $f_c$  bandwidth  $2w$ . The  $\cos^2$  term has components at baseband and at  $2f_c$  because

$$\cos^2 \alpha = \frac{1}{2}(1 + 2 \cos 2\alpha) \text{ with } \alpha = 2\pi f_c t$$

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Given  $v_2(t)$  find  $V_2(f)$

Curve

$$\begin{aligned} v_2(t) &= a_1 A_c \cos 2\pi f_c t && \text{a} \\ &+ 2a_2 A_c m(t) \cos 2\pi f_c t && \text{b} \\ &+ a_1 m(t) && \text{c} \\ &+ a_2 m^2(t) && \text{d} \\ &+ a_2 A_c^2 \underbrace{\frac{1}{2}(1 + \cos 2 \cdot 2\pi f_c t)}_{\cos^2 2\pi f_c t} && \text{e} \end{aligned}$$

$$\begin{aligned} V_2(f) &= a_1 \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] \\ &+ 2a_2 A_c [M(f - f_c) + M(f + f_c)] \\ &+ a_1 M(f) \\ &+ a_2 FT[m^2(t)] \\ &+ \frac{a_2 A_c^2}{2} [\delta(f) + \delta(f - 2f_c) + \delta(f + 2f_c)] \end{aligned}$$

### Assignment

Text: p. 384, p. 7.1, p. 7.3, p. 269 Ex. 2



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Generation of AM Waves Continued - P. 269-270

Switching Modulator

Same as square law except non linearity is now:  $\rightarrow$

$$v_2(t) = \begin{cases} v_1(t) & v_1(t) > 0 \\ 0 & v_1(t) < 0 \end{cases}$$

$$\begin{aligned} v_1(t) &= A_c \cos 2\pi f_c t + m(t) \\ &= c(t) + m(t) \end{aligned}$$

Assume  $|m(t)| \ll A_c$

$$\text{Thus } v_2(t) = \begin{cases} v_1(t) & c(t) > 0 \\ 0 & c(t) < 0 \end{cases}$$

Thus switch diode on and off at rate  $f_c$

diode on only when  $c(t) > 0$

$$v_2(t) = [A_c \cos 2\pi f_c t + m(t)]g_p(t)$$

where  $g_p(t)$  is a unipolar square wave at  $f_c$ .

Recall Fourier Series ELEC 260 p. 106 $f(t)$  with period  $T$ 

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n t}{T} + b_n \sin \frac{2\pi n t}{T}$$

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos \frac{2\pi n t}{T} dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin \frac{2\pi n t}{T} dt$$

Consider  $g_p(t)$  with period  $T_0$  and frequency  $f_c = 1/T_0$ 

$$a_0 = \frac{1}{T_0} \int_{-T_0/4}^{T_0/4} 1 \cdot dt = \frac{1}{T_0} \frac{T_0}{2} = \frac{1}{2}$$

$$\begin{aligned} a_n &= \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} \cos \frac{2\pi n t}{T_0} dt = \frac{2}{T_0} \frac{T_0}{2\pi n} \sin \frac{2\pi n t}{T_0} \Big|_{-T_0/4}^{T_0/4} \\ &= \frac{1}{\pi n} \left[ \sin \left( \frac{2\pi n T_0/4}{T_0} \right) - \sin \left( -\frac{2\pi n T_0/4}{T_0} \right) \right] \\ &= \frac{1}{\pi n} 2 \sin \left( \frac{\pi n}{2} \right) = \left\{ \begin{array}{ll} \frac{2}{\pi n} & n = 1, 5, 9 \dots \\ \frac{-2}{\pi n} & n = 3, 7, \dots \end{array} \right\} = \frac{(-1)^{n-1}}{2n-1} \cdot \frac{2}{\pi} \end{aligned}$$

$$n = 2m - 1, \quad m = 1, 2, 3, \dots$$

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Relabel  $m \rightarrow n$

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos \left[ \frac{2\pi(2n-1)f_c t}{T_o} \right] \text{ with } f_c = 1/T_o$$

write  $g_p(t)$  as Fourier cosine series (see ELEC 260)

$$\begin{aligned} g_p(t) &= \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)] \\ v_2(t) &= [A_c \cos 2\pi f_c t + m(t)]g_p(t) \\ &= [A_c \cos 2\pi f_c t + m(t)] \left[ \frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t + \dots \right] \\ &= \frac{A_c}{2} \cos 2\pi f_c t + m(t) \frac{2}{\pi} \cos 2\pi f_c t + \frac{m(t)}{2} + \dots \\ &= \underbrace{\frac{A_c}{2} \left[ 1 + \frac{4}{\pi A_c} m(t) \right] A_c \cos 2\pi f_c t}_{\text{AM wave with } k_a = \frac{4}{\pi A_c}} + \underbrace{\dots}_{\text{unwanted components}} \\ &\hspace{15em} \text{at frequencies removed from } f_c \end{aligned}$$

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Demodulation of AM Waves

Square Law Demodulator

No good unless  $m(t) \ll A_c$  i.e. % modulation very low see P. 271 - Skip this

Envelope Detector - P. 272

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## 7.2 P. 274 - Double Sideband Suppressed Carrier Modulation (DSBSC)

Carrier  $c(t) = A_c \cos 2\pi f_c t$  does not carry information  $\rightarrow$  waste of power

Recall  $\frac{\text{Power in } m(t)}{\text{Power in } c(t)} = \frac{1}{3}$  max at 100% modulation

$\rightarrow$  Suppress the Carrier

$$\begin{aligned} \text{AM: } s_{AM}(t) &= [1 + m(t)]c(t) \\ &= c(t) + m(t)c(t) \quad k_a = 1 \end{aligned}$$

$$\begin{aligned} \text{DSB: } s(t) &= m(t)c(t) \\ &= A_c \cos 2\pi f_c t m(t) \end{aligned}$$

$$S(f) = \frac{1}{2}A_c[M(f - f_c) + M(f + f_c)]$$

Phase reversal when  $m(t) < 0$ .

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### DSBSC

$$\begin{aligned} s(t) &= c(t)m(t) \\ &= A_c \cos 2\pi f_c t \cdot m(t) \\ &= \frac{A_c}{2} [\exp(-j2\pi f_c t) + \exp(j2\pi f_c t)] m(t) \end{aligned}$$

Recall shifting property

$$m(t) \leftrightarrow M(f)$$

$$\exp(j2\pi f_c t)m(t) \leftrightarrow M(f - f_c)$$

$$S(f) = \frac{A_c}{2} [M(f + f_c) + M(f - f_c)]$$

Single tone modulation

$$m(t) = A_m \cos 2\pi f_m t$$

$$M(f) = \frac{A_m}{2} [\delta(f - f_m) + \delta(f + f_m)]$$

$$M(f - f_c) = \frac{A_m}{2} [\delta(f - f_m - f_c) + \delta(f + f_m - f_c)]$$

$$M(f + f_c) = \frac{A_m}{2} [\delta(f - f_m + f_c) + \delta(f + f_m + f_c)]$$

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### 3.5 Generation of DSBSC Waves - P. 276

$$\begin{aligned}DSBs(t) &= c(t)m(t) \\ &= A_c \cos 2\pi f_c t \ m(t)\end{aligned}$$

Balanced modulator

$$\begin{aligned}s_1(t) &= A_c[1 + k_a m(t)] \cos 2\pi f_c t \\ s_2(t) &= A_c[1 - k_a m(t)] \cos 2\pi f_c t \\ s(t) &= s_1(t) - s_2(t) \\ &= 2k_a A_c \cos 2\pi f_c t \ m(t) \\ &= 2k_a c(t)m(t)\end{aligned}$$

page 7.19 P. 277 To produce DSB signal  $s(t) = m(t)c(t)$

Ring modulator (mixer) multiplies  $m(t)$  with  $c(t)$ .

For positive half-cycles of  $c(t)$

D1 and D3 conduct, D2 and D4 open

point  $a$  connected to point  $b$

point  $c$  connected to point  $d$

current through T2 primary proportional to  $m(t)$ .

For negative half-cycles of  $c(t)$

D2 and D4 conduct, D1 and D3 open

point  $a$  connected to point  $d$

point  $b$  connected to point  $c$

current through T2 primary proportional to  $-m(t)$ .

Use square wave  $c(t)$  to avoid problems with diode offsets.



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Ring Modulator

$$\begin{aligned}
 s(t) &= c(t)m(t) \\
 &= \underbrace{\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)]}_{\text{Square Wave Carrier}} \underbrace{m(t)}_{\text{mod}}
 \end{aligned}$$

Compare  $c(t)$  here with  $g_p(t)$  on page 7.12.

Here  $c(t)$  range  $-1$  to  $+1$ , no DC term

$g_p(t)$  range  $0$  to  $1$ , with DC term.

$$S(f) = C(f) \otimes M(f) = \int_{-\infty}^{\infty} C(\lambda)M(f - \lambda)d\lambda$$

$$S(f) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \overbrace{[\delta(f - f_c(2n-1)) + \delta(f + f_c(2n-1))]}^{C(f)} \otimes M(f)$$

For  $n = 1$

$$\begin{aligned}
 S(f) &= \frac{4}{2\pi} [\delta(f - f_c) + \delta(f + f_c)] \otimes M(f) \\
 &= \int_{-\infty}^{\infty} \frac{4}{2\pi} (\delta(\lambda - f_c) + \delta(\lambda + f_c)) M(f - \lambda) d\lambda \\
 &= \frac{4}{2\pi} [M(f - f_c) + M(f + f_c)]
 \end{aligned}$$

For  $n = 2 \dots$

$n = 3 \dots$

page 7.20A

### Superheterodyne Receiver

Since  $f_c$  varies from one signal to the next, the receiver is designed to convert all  $f_c$  to a fixed frequency  $f_{IF}$ . This way we can use the same filter and detector for any signal at any  $f_c$ .

RF	540-1600	KHz	AM Broadcast
LO	995-2055	KHz	
IF = LO - RF	455	KHz	

Example Consider CFAZ at 1070 KHz

Here LO = 1525 KHz      SUM = 2595  
IF = LO - RF = 455 KHz    DIF = 455

Mixer outputs sum and difference frequencies

thus another station at 1980 KHz (image frequency) will also be received since

LO = 1525 KHz

RF = 1980 KHz

RF-LO = 455 KHz

Image is filtered out by RF AMP/filter

Homodyne receiver uses zero frequency IF, i.e. LO=RF, IF=0

page 7.20X

General I-Q receiver - demodulator

applies for any signal, AM, DSB and others.

Input signal

$$\begin{aligned} s(t) &= a(t)\cos[2\pi f_{IF}t + \theta(t)] \\ &= x(t) \cos 2\pi f_{IF}t - y(t) \sin 2\pi f_{IF}t \end{aligned}$$

Output signals  $I = x(t)$  and  $Q = y(t)$ .

Exercise - show this using algebra

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## DSBSC Demodulation

### Costas Receiver

same as general I-Q receiver with extra feedback signal

Two coherent detectors	<u>Output</u>
One LO $\cos(2\pi f_c t + \phi)$	$\frac{1}{2}A_c \cos \phi \cdot m(t)$
Other LO $\sin(2\pi f_c t + \phi)$	$\frac{1}{2}A_c \sin \phi \cdot m(t)$

Here  $f_c = f = f_{IF}$ .

Phase discriminator output  $e(t)$  is a DC control signal

$$\begin{aligned} e(t) &= \left( \frac{1}{2}A_c \cos \phi \cdot m(t) \right) \cdot \left( \frac{1}{2}A_c \sin \phi \cdot m(t) \right) \\ &= \frac{A_c^2}{4} m^2(t) \cos \phi \sin \phi = \frac{A_c^2}{8} m^2(t) \sin 2\phi \end{aligned}$$

The DC control signal is used as a feedback signal to reduce  $\phi$

$$TP11 = \cos 2\pi f_c t \cos(2\pi f_c t + \phi)$$

$$TP19 = \cos 2\pi f_c t \sin(2\pi f_c t + \phi)$$

$$TP13 =$$

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### Costas receiver in more detail

Costas receiver is same as general I-Q receiver with extra feedback signal  $e(t) = I \cdot Q$  used to adjust oscillator frequency  $f$  to be close to  $f_{IF}$ . In general  $f_{IF} \neq f$ .

IF input signal  $s_{IF}(t) = m(t) \cos(2\pi f_{IF}t + \phi_{in})$ .

From table 3 page 626,

$$2 \cos \alpha \sin \beta = \sin(\alpha - \beta) + \sin(\alpha + \beta)$$

$$2 \cos \alpha \cos \beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

$$\begin{aligned} TP11 &= m(t) \cos(2\pi f_{IF}t + \phi_{in}) \cos(2\pi ft + \phi_{\ell}) \\ &= \frac{1}{2}m(t)[\cos(2\pi(f_{IF} - f)t + \phi_{in} - \phi_{\ell}) + \cos(2\pi(f_{IF} + f)t + \phi_{in} + \phi_{\ell})] \end{aligned}$$

$$\begin{aligned} TP19 &= m(t) \cos(2\pi f_{IF}t + \phi_{in}) \sin(2\pi ft + \phi_{\ell}) \\ &= \frac{1}{2}m(t)[\sin(2\pi(f_{IF} - f)t + \phi_{in} - \phi_{\ell}) + \sin(2\pi(f_{IF} + f)t + \phi_{\ell} + \phi_{in})] \end{aligned}$$

$$TP13 = \frac{1}{2}m(t) \cos[2\pi(f_{IF} - f)t + \phi_{in} - \phi_{\ell}]$$

$$TP12 = \frac{1}{2}m(t) \sin(2\pi(f_{IF} - f)t + \phi_{in} - \phi_{\ell})$$

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Costas receiver continued

$$\begin{aligned}\text{Define } \Delta f &= f_{IF} - f \\ \theta &= \phi_\ell - \phi_{in}\end{aligned}$$

$$\begin{aligned}e(t) &= (TP12) \cdot (TP13) \\ &= \frac{1}{4} m^2(t) \cos(2\pi \Delta f t - \theta) \sin(2\pi \Delta f t - \theta) \\ &= \frac{1}{8} m^2(t) \sin(4\pi \Delta f t - 2\theta)\end{aligned}$$

Using  $\sin \alpha \cos \alpha = \frac{1}{2} \sin 2\alpha$

$$e(t) = 0 \text{ if } \Delta f = 0 \text{ and } \theta = 0$$

If  $e(t) \neq 0$ , then VCO will be adjusted so that  $e(t) = 0$

After LPF 5Hz

$$\begin{aligned}e_{LP}(t) &= \frac{1}{8} \sin(4\pi \Delta f t - 2\theta) \underbrace{\langle m^2(t) \rangle}_{\text{Average Value}} \\ &= \text{DC Control Signal}\end{aligned}$$

For small  $(4\pi \Delta f t - 2\theta) = x$ ,  $\sin x \sim x$ .

Thus  $e_{LP}(t) \simeq 4\pi \Delta f t - 2\theta$

### P. 283 - Quadrature Carrier Multiplexing

- Combine two independent DSBSC modulated waves on the same frequency  $f_c$  with two modulations  $m_1(t)$   $m_2(t)$
- To achieve this we use two carrier waves at  $f_c$  but  $90^\circ$  out of phase

$$s(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$$

To receive the two independent modulations  $m_1(t)$   $m_2(t)$  use the general I-Q receiver. The I-Q receiver contains two separate DSBSC demodulators with local oscillators (carriers)  $90^\circ$  out of phase,  $\cos 2\pi f_c t$  and  $\sin 2\pi f_c t$ .

For this to work, we need to have the local oscillators to have frequency  $f_c$  exactly the same as the frequency of  $s(t)$ .

Note that  $s(t)$  is in the general format with  $x(t) = m_1(t)$  and  $y(t) = -m_2(t)$ .

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### Quadrature Carrier Multiplexing - continued

We will show that there is no mixing of the two modulations, even though the carrier frequency is the same

$$\begin{aligned}v(t) &= s(t) \cos 2\pi f_c t \\ &= A_c m_1(t) \cos 2\pi f_c t \cos 2\pi f_c t \\ &\quad + A_c m_2(t) \sin 2\pi f_c t \cos 2\pi f_c t \\ \cos^2 \alpha &= \frac{1}{2}(1 + \cos 2\alpha) \\ \cos \alpha \sin \alpha &= \frac{1}{2} \sin 2\alpha \\ v(t) &= \frac{1}{2} A_c m_1(t) [1 + \cos 4\pi f_c t] \\ &\quad + \frac{1}{2} A_c m_2(t) \sin 4\pi f_c t\end{aligned}$$

After LPF

$$v_o(t) = \frac{1}{2} A_c m_1(t)$$



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### **P. 284 - Single Sideband Modulation**

In DSBSC, the two sidebands carry redundant information thus we can eliminate one sideband to get SSB-SC usually referred to as SSB.

SSB with carrier is called SSB-AM

We begin with frequency domain description of SSB

1. Single tone modulation  $m(t) = \cos 2\pi f_m t$
2. General modulation  $m(t)$

Looking only at positive frequencies

$m(t)$

AM

DSB-SC

SSB

page 7.31

### **3.9 Generation of SSB Waves**

1 - Filtering (Frequency Discrimination)

2 - Phase Discrimination (general I-Q transmitter)

#### 1 Filter Method

Produce Modulator + Filter

Need sharp filter to pass USB and reject LSB

e.g 8-pole crystal filter at 10.7 MHz not practical

Upconvert to desired carrier frequency

Assignment #4 P. 388 P. 7.4 # 16, 19a, 20a

page 7.31A

2 General I-Q transmitter

$$s(t) = x(t) \cos 2\pi f_c t - y(t) \sin 2\pi f_c t$$

page 7.32

text P. 288

### Time Domain Description of SSB

$$s(t) = \frac{A_c}{2} [m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t]$$

– For upper sideband

+ For lower sideband

$\hat{m}(t)$  = Hilbert transform of  $m(t)$  see notes p. 3.3-3.4

–90° Phase shift for all positive frequencies

+90° Phase shift for all negative frequencies

$H(f)$  special kind of “filter”

Recall  $H(f) = -j \operatorname{sgn} f$  p.3.3-3.4

$$h(t) = 1/\pi t$$

$$\hat{m}(t) = m(t) \otimes h(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(\tau)}{t - \tau} d\tau$$

$$\begin{aligned} \hat{M}(f) &= M(f)H(f) \\ &= -j \operatorname{sgn} f M(f) \end{aligned}$$

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Single tone modulation for SSB

Example 3 - P. 291

$$\text{If } m(t) = A_m \cos 2\pi f_m t$$

$$\hat{m}(t) = A_m \sin 2\pi f_m t$$

$$\begin{aligned} s(t) &= \frac{A_c}{2} [m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t] \\ &= \frac{A_c A_m}{2} [\cos 2\pi f_m t \cos 2\pi f_c t \mp \sin 2\pi f_m t \sin 2\pi f_c t] \end{aligned}$$

Recall  $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$ , thus

$$s(t) = \frac{A_c A_m}{2} \cos[2\pi(f_c \pm f_m)t]$$

$s(t)$  is a single tone at frequency  $f_c \pm f_m$ . See also text figure 7.3 1 p. 266

SSB - Time  $\leftrightarrow$  Frequency Domain

$$s(t) = \frac{A_c}{2} [m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t]$$

$$\hat{M}(f) = M(f)H(f) = -j \operatorname{sgn} f M(f) \quad \text{p. 7.32}$$

$$\begin{aligned} S(f) &= \frac{A_c}{2} [M(f) \otimes \frac{1}{2} \{\delta(f - f_c) + \delta(f + f_c)\}] \\ &\quad \pm -j \operatorname{sgn} f M(f) \otimes \frac{1}{2j} \{\delta(f - f_c) - \delta(f + f_c)\}] \\ &= \frac{A_c}{4} [M(f) \otimes \delta(f - f_c) \\ &\quad + M(f) \otimes \delta(f + f_c) \\ &\quad \pm \operatorname{sgn} f M(f) \otimes \delta(f - f_c) \\ &\quad \mp \operatorname{sgn} f M(f) \otimes \delta(f + f_c)] \\ &= \frac{A_c}{4} [M(f - f_c) + M(f + f_c) \\ &\quad \pm \operatorname{sgn}(f - f_c) M(f - f_c) \mp \operatorname{sgn}(f + f_c) M(f + f_c)] \\ &= \frac{A_c}{2} M(f - f_c), \quad f - f_c > 0, f > f_c \end{aligned}$$

similarly

$$S(f) = \frac{A_c}{2} M(f + f_c), \quad f + f_c > 0, f < -f_c$$

**P. 293 Demodulation of SSB**

$$c_{LO}(t) = \cos 2\pi f_c t$$

$$s(t) = \frac{A_c}{2} [m(t) \cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t]$$

$$v(t) = s(t) \cos 2\pi f_c t$$

$$= \frac{A_c}{2} m(t) \cos 2\pi f_c t \cos 2\pi f_c t$$

$$+ \frac{A_c}{2} \hat{m}(t) \cos 2\pi f_c t \sin 2\pi f_c t$$

$$\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha)$$

$$\cos \alpha \sin \alpha = \frac{1}{2} \sin 2\alpha$$

$$v(t) = \underbrace{\frac{A_c}{4} m(t)}_{\text{Message Signal}} + \frac{A_c}{4} m(t) \cos 4\pi f_c t + \underbrace{\frac{A_c}{4} \hat{m}(t) \sin 4\pi f_c t}_{\text{Unwanted Terms}}$$

This assumes no phase or frequency error in local oscillator  $C_{LO}(t)$

**Demodulation of SSB with Frequency Error**

$$\begin{aligned}
C_{LO}(t) &= \cos 2\pi(f_c + \Delta f)t \\
s(t) &= \frac{A_c}{2}[m(t) \cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t] \\
v(t) &= s(t) \cos 2\pi(f_c + \Delta f)t \\
&= \frac{A_c}{2}m(t) \cos 2\pi(f_c + \Delta f)t \cos 2\pi f_c t \\
&\quad + \frac{A_c}{2}\hat{m}(t) \cos 2\pi(f_c + \Delta f)t \sin 2\pi f_c t \\
\cos \alpha \cos \beta &= \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\
\cos \alpha \sin \beta &= \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)] \\
\alpha &= 2\pi(f_c + \Delta f)t \quad \beta = 2\pi f_c t \quad \alpha - \beta = 2\pi \Delta f t \\
v(t) &= \frac{A_c}{2}m(t)\frac{1}{2} \cos 2\pi \Delta f t \\
&\quad + \frac{A_c}{2}\frac{\hat{m}(t)}{2} \sin 2\pi \Delta f t + \text{Double Frequency Terms}
\end{aligned}$$

After LPF,  $v_0(t)$  is an SSB signal on a low frequency carrier  $\Delta f$ .



Carrier Plus SSB Wave

$$\begin{aligned}
s(t) &= \underbrace{A_c \cos 2\pi f_c t}_{\text{Carrier}} + \underbrace{m(t) \cos 2\pi f_c t - \hat{m}(t) \sin 2\pi f_c t}_{\text{USB}} \\
&= (A_c + m(t)) \cos 2\pi f_c t - \hat{m}(t) \sin 2\pi f_c t \\
&= a(t) \cos(2\pi f_c t + \phi)
\end{aligned}$$

$$\text{where } a(t) = \sqrt{[A_c + m(t)]^2 + [\hat{m}(t)]^2}$$

$$a(t) = \text{output of envelope detector}$$

$$= \sqrt{A_c^2 + 2A_c m(t) + m^2(t) + \hat{m}^2(t)}$$

$$\text{If } A_c \gg |m(t)| \text{ and } A_c \gg |\hat{m}(t)|$$

$$\begin{aligned}
a(t) &= \sqrt{A_c^2 + 2A_c m(t)} && \text{recall } (1+x)^{1/2} \simeq 1 + \frac{1}{2}x \\
&= A_c \sqrt{1 + \frac{2}{A_c} m(t)} \\
&= A_c \left[ 1 + \frac{1}{A_c} m(t) \right] = A_c + m(t)
\end{aligned}$$

Thus  $a(t) = m(t) + \text{DC bias term}$  as long as  $A_c \gg |m(t)|$   
 $A_c \gg |\hat{m}(t)|$

To tune in #2 change LO to 10.559 MHz

$$\text{SIG} - \text{LO} = 10.559 - 10.104 = 455 \text{ Khz}$$

Tunable filter to eliminate image frequency

could also choose LO = 9.645 MHz for Station #1.

Question: Can we demodulate LSB with this receiver?

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In practice it is not cost effective to filter at frequencies much greater than 10.7 MHz. This is because the filter skirts cannot be made steep enough .

Typical Crystal Filter Spec.

Typical Voice Audio signal 300 Hz - 2600 Hz

If 2300 Hz passband centered on audio

then -60 dB attenuation will be at 400 Hz in opposite sideband.

Summary of modulation types, assuming the message  $|m(t)| < 1$

AM

$$s(t) = [1 + k_a m(t)] \cos(2\pi f_c t + \phi)$$

DSB

$$s(t) = m(t) \cos(2\pi f_c t + \phi)$$

USB

$$s(t) = m(t) \cos(2\pi f_c t + \phi) - \hat{m}(t) \sin(2\pi f_c t + \phi)$$

FM

$$\begin{aligned} s(t) &= \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\alpha) d\alpha] \\ &= \cos[2\pi f_c t + \phi(t)] = \cos[\theta_i(t)] \end{aligned}$$

PM

$$s(t) = \cos[2\pi f_c t + k_p m(t)]$$

For  $m(t) = A_m \cos 2\pi f_m t$  with  $A_m = 1$

$$\text{AM } m(t) = [1 + k_a \cos 2\pi f_m t] \cos(2\pi f_c t + \phi)$$

$$\text{DSB } s(t) = \cos 2\pi f_m t \cos(2\pi f_c t + \phi)$$

$$\begin{aligned} \text{USB } s(t) &= \cos 2\pi f_m t \cos(2\pi f_c t + \phi) \\ &\quad - \sin 2\pi f_m t \sin(2\pi f_c t + \phi) \end{aligned}$$

$$\text{FM } s(t) = \cos\left(2\pi f_c t + \frac{k_f}{f_m} \sin 2\pi f_m t\right)$$

$$\text{PM } s(t) = \cos[2\pi f_c t + k_p \cos 2\pi f_m t]$$