

Lecture - 4

2.4 Finite-length infinitesimally thin dipole

A good approximation of the current distribution along the dipole's length is the sinusoidal one:

$$I_e(z') = \begin{cases} I_0 \sin \left[k \left(\frac{l}{2} - z' \right) \right], & 0 \leq z' \leq l/2 \\ I_0 \sin \left[k \left(\frac{l}{2} + z' \right) \right], & -l/2 \leq z' \leq 0 \end{cases} \quad (2.19)$$

This distribution assumes that the antenna is center-fed and current vanishes at the end points $\left(z' = \pm \frac{l}{2} \right)$.

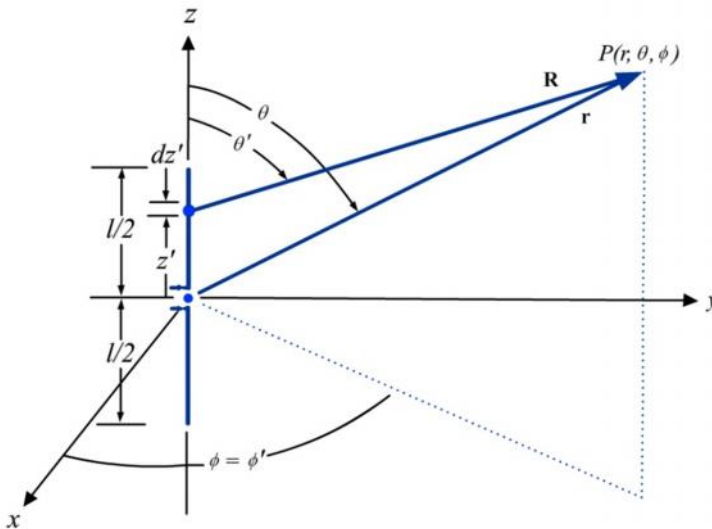


Fig. 2.3: Finite length dipole

It can be shown that the vector potential integral

$$A = \hat{z} \frac{\tilde{I}}{4f} \int_{-l/2}^{l/2} I(z') \frac{e^{-jkR}}{R} dz' \quad (2.20)$$

has an analytical (closed form) solution. Here, however, we follow a standard approach used to calculate the far field for an arbitrary wire antenna. It is based on the solution for the field of the infinitesimal dipole. The finite-length is subdivided into an infinite number of infinitesimal dipoles of length dz' . For an infinitesimal dipole of length dz' positioned along z -axis at z' , the electric and the magnetic field components at far-field is given as:

$$\begin{aligned}
dE_n &= j\eta k I_e(z') \frac{e^{-jkR}}{4\pi R} \sin \theta \cdot dz' \\
dH_\phi &= jk I_e(z') \frac{e^{-jkR}}{4\pi R} \sin \theta \cdot dz' \\
dE_r = dE_\phi = dH_r = dH_\theta &= 0
\end{aligned} \tag{2.21}$$

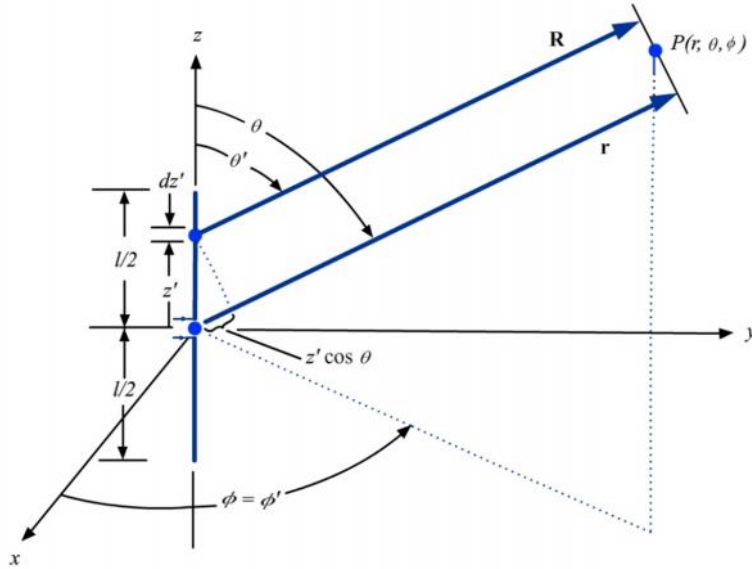


Fig. 2.4 Geometrical arrangement for the far field approximation

where $R = [x^2 + y^2 + (z - z')^2]^{1/2}$ and $I_e(z')$ denotes the value of the current element at z' . Using the far-zone approximations,

$$\begin{cases} \frac{1}{R} \approx \frac{1}{r}, & \text{for the amplitude factor} \\ R \approx r - z' \cos \theta, & \text{for the phase factor} \end{cases} \tag{2.22}$$

The following approximation of the elementary far field is obtained:

$$dE_n = j\eta k I_e \frac{e^{-jkr}}{4\pi r} e^{jkz' \cos \theta} \cdot \sin \theta \cdot dz' \tag{2.23}$$

Using the superposition principle, the total far field is obtained as:

$$E_n = \int_{-l/2}^{l/2} dE_n = j\eta k \frac{e^{-jkr}}{4\pi r} \cdot \sin \theta \cdot \int_{-l/2}^{l/2} I_e(z') e^{jkz' \cos \theta} dz' \tag{2.24}$$

The first factor

$$EF(\theta) = jYk \frac{e^{-jkr}}{r} \sin \theta \quad (2.25)$$

is called the **element factor**. The element factor in this case is the far field produced by an infinitesimal dipole of unit current element. The element factor is the same for any current element, provided the angle θ is always associated with the current axis.

The second factor

$$AF(\theta) = \int_{-l/2}^{l/2} I_e(z') e^{jkz' \cos \theta} dz' \quad (2.26)$$

is the **space factor (or pattern factor, array factor)**. This factor is dependent on the amplitude and phase distribution of the current at the antenna (the source distribution in space).

Total field = (Element Factor) × (Space Factor)

For the specific current distribution described by (2.19), the pattern factor is

$$AF(\theta) = I_0 \left\{ \int_{-l/2}^0 \sin \left[k \left(\frac{l}{2} + z' \right) \right] e^{jkz' \cos \theta} dz' + \int_0^{l/2} \sin \left[k \left(\frac{l}{2} - z' \right) \right] e^{jkz' \cos \theta} dz' \right\} \quad (2.26)$$

The above integrals were solved with the help of the following integral:

$$\int \sin(a + b \cdot x) e^{c \cdot x} dx = \frac{e^{cx}}{b^2 + c^2} [c \sin(a + bx) - b \cos(a + bx)] \quad (2.27)$$

where $a = kl/2$

$$b = \pm k$$

$$c = \pm jk \cos \theta$$

The far field of the finite-length dipole is obtained as

$$E_\theta = EF(\theta) \cdot AF(\theta) = jYI_0 \frac{e^{-jkr}}{2fr} \cdot \frac{\left[\cos \left(\frac{kl}{2} \cos \theta \right) - \cos \left(\frac{kl}{2} \right) \right]}{\sin \theta} \quad (2.28)$$

The amplitude pattern:

$$\bar{E}(\theta, \phi) = \frac{\cos \left(\frac{kl}{2} \cos \theta \right) - \cos \left(\frac{kl}{2} \right)}{\sin \theta} \quad (2.29)$$

$$H_w \frac{E_r}{y} j \frac{I_0 e^{-jkr}}{2f r} \left[\frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin \theta} \right] \quad (2.30)$$

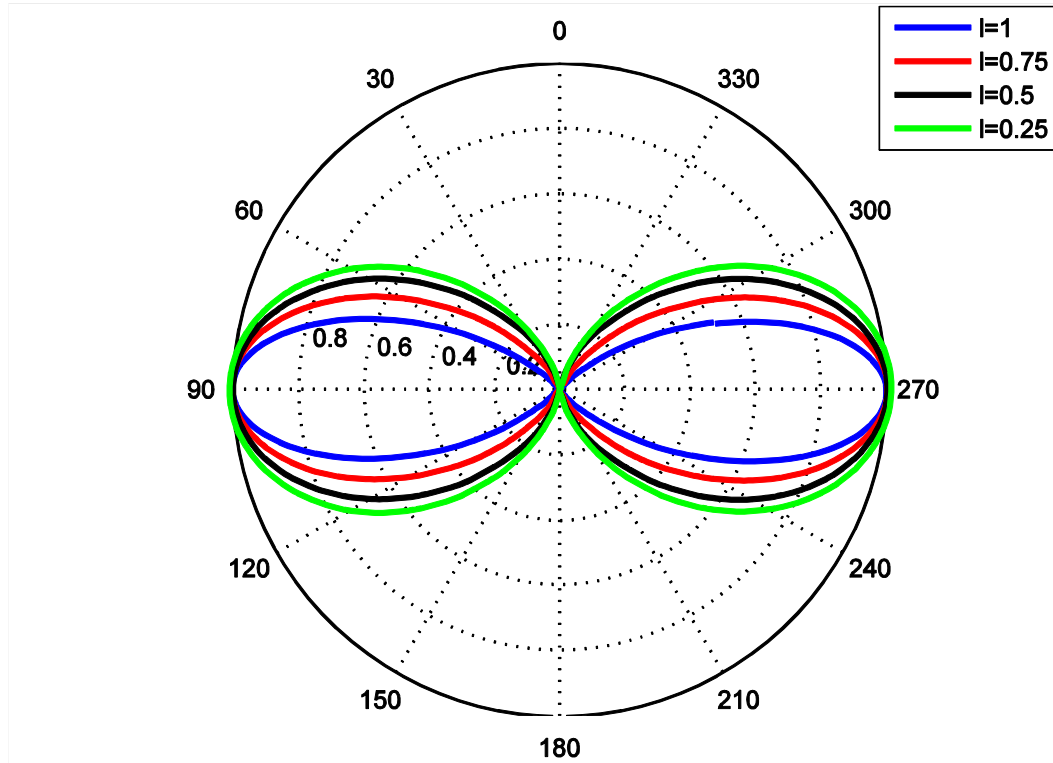


Fig. 2.4 Amplitude pattern for different length dipole antenna

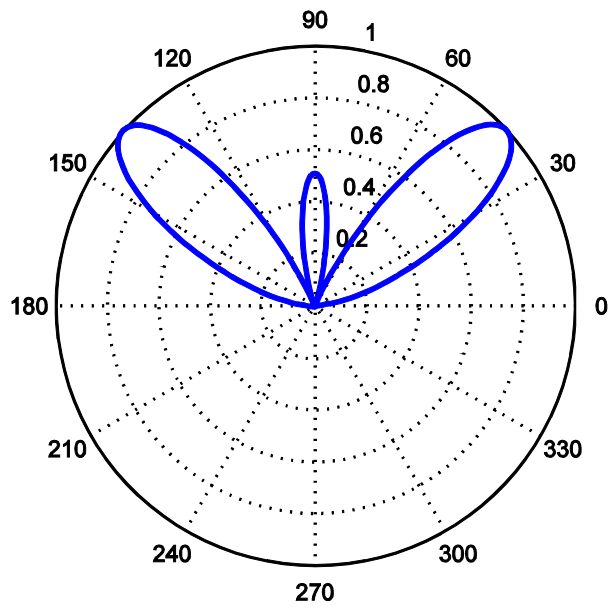


Fig. 2.5 Pattern for a dipole of length 1.5