

# Lecture - 5

## 2.4.1 The Radiated Power

The time average complex Poynting vector in the far field of the center-fed dipole is

$$S = \frac{1}{2} E \times H^* = \frac{1}{2} [E_\theta a_\theta \times H_\phi^* a_\phi] = \frac{1}{2} \left[ E_\theta \frac{E_\theta^*}{\eta} a_r \right] = \frac{|E_\theta|^2}{2\eta} a_r$$

$$= \frac{\eta |I_0|^2}{8\pi^2 r^2} \left[ \frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin \theta} \right]^2 \hat{a}_r$$

The radiation intensity function for the center-fed dipole is given by

$$U(\theta, \phi) = r^2 S$$

$$U(\theta, \phi) = \frac{\eta |I_0|^2}{8\pi^2} \left[ \frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin \theta} \right]^2$$

We may plot that normalized radiation intensity function  $[U(\theta) = B_0 F(\theta)]$  to determine the effect of the antenna length on its radiation pattern.

$$F(\theta) = \left[ \frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin \theta} \right]^2$$

The total real power radiated by the center-fed dipole is

$$P_{rad} = \int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin \theta d\theta d\phi$$

$$= \frac{\eta |I_0|^2}{4\pi} \int_0^\pi \frac{\left[ \cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right) \right]^2}{\sin \theta} d\theta$$

The  $\theta$ -dependent integral in the radiated power expression cannot be integrated analytically. However, the integral may be manipulated, using several transformations of variables, into a form containing some commonly encountered special functions (integrals) known as the sine integral and cosine integral.

$$S_i(x) = \int_0^x \frac{\sin x}{x} dx \quad (\text{sin integral})$$

$$C_i(x) = -\int_x^\infty \frac{\cos x}{x} dx \quad (\text{cosine integral})$$

$$C_{in}(x) = \int_0^x \left( \frac{1 - \cos x}{x} \right) dx \quad (\text{related integral})$$

$$= C + \ln(x) - C_i(x)$$

$$C = 0.5775 \quad (\text{Euler's constant})$$

The radiated power of the center-fed dipole becomes

$$P_{rad} = \frac{\eta |I_0|^2}{4\pi} \left\{ C + \ln(kl) - C_i(kl) + \frac{1}{2} \sin(kl) [S_i(2kl) - 2S_i(kl)] \right. \\ \left. + \frac{1}{2} \cos(kl) [C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)] \right\}$$

## 2.4.2 Radiation resistance

The radiated power is related to the radiation resistance of the antenna by

$$P_{rad} = \frac{1}{2} |I_0|^2 R_r$$

which gives the radiation resistance of a Center-fed dipole

$$R_r = \frac{\eta}{2\pi} \left\{ C + \ln(kl) - C_i(kl) + \frac{1}{2} \sin(kl) [S_i(2kl) - 2S_i(kl)] \right. \\ \left. + \frac{1}{2} \cos(kl) [C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)] \right\}$$

## 2.4.3 Directivity and Effective aperture

The directivity function of the center-fed dipole is given by

$$D(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{P_{rad}}$$

$$U(\theta, \phi) = \frac{\eta |I_0|^2}{8\pi^2} \left[ \frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin \theta} \right]^2 = \frac{\eta |I_0|^2}{8\pi^2} F(\theta, \phi)$$

The radiated power is expressed as

$$P_{rad} = \frac{\eta |I_0|^2}{4\pi^2} f(kl)$$

$$f(kl) = C + \ln(kl) - C_i(kl) + \frac{1}{2} \sin(kl) [S_i(2kl) - 2S_i(kl)] \\ + \frac{1}{2} \cos(kl) [C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]$$

Directivity of the dipole is

$$D(\theta, \phi) = 4\pi \frac{\frac{1}{2\pi} F(\theta, \phi)}{f(kl)} = 2 \frac{F(\theta, \phi)}{f(kl)}$$

The maximum directivity is

$$D_0 = 2 \frac{[F(\theta, \phi)]_{\max}}{f(kl)}$$

### Effective aperture

$$A_e = \frac{\lambda^2}{4\pi} D_0$$

$$A_e = \frac{\lambda^2}{2\pi} \frac{[F(\theta, \phi)]_{\max}}{f(kl)}$$