Lecture-5

5.8. Cavity model for the rectangular patch

The transmission line model is very limited in its description of the real processes taking place when a patch is excited and the performance of the patch is also affected by higher-order modes.

The cavity model is a more general model of the patch which imposes open-end conditions at the side edges of the patch. It represents the patch as a dielectric - loaded cavity with:

- Electrical walls (above and below), and

- Magnetic Walls (around the perimeter of the patch).

At the magnetic wall

$\hat{n} \times H = 0$ (The H- field is purely normal)

$\hat{n} \cdot E = 0$ (The **E**- field is purely tangential)

It is analogous to the open end termination in the theory of transmission lines.

If we treat the microstrip antenna only as a cavity, we cannot represent radiation because an ideal loss-free cavity does not radiate and its input impedance is purely reactive. To account for the radiation, a loss mechanism is introduced. This is done by introducing an effective loss tangent, $¢_{eff}$.

The wave generated and propagating beneath the patch undergo considerable reflection at the edges of the patch. Only a very small fraction of them is being radiated. Thus, the antenna is quite inefficient. The cavity model assumes that the **E** field is purely tangential to the slots formed between the ground plane and the patch edges (magnetic walls). Moreover, it considers only TM^x modes, i.e., modes with no H_x component. These assumptions are, basically, very much true.

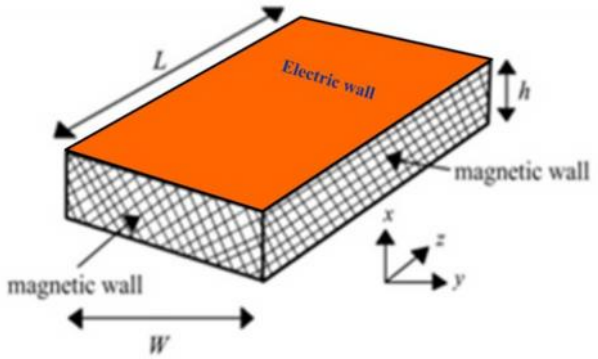

Fig.5.15 Rectangular microstrip patch geometry

The TM^x modes are fully described by a single scalar function A_x - the *x*- component of the magnetic vector potential:

$$\mathrm{A} = A_x \hat{\mathrm{x}} \tag{5.17}$$

In a homogeneous source-free medium, A_x satisfies the wave equation:

$$\nabla^2 A_x + k^2 A_x = 0 \tag{5.18}$$

For regular shapes (like the rectangular cavity), it is advantageous to use the separation of variables:

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} + k^2 A_x = 0 \tag{5.19}$$

$$A_x = X(x)Y(y)Z(z) \tag{5.20}$$

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} XY\frac{\partial^2 Z}{\partial z^2} = -k^2 XYZ$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -k^2 \tag{5.21}$$

$$\frac{d^2 X}{dx^2} + k_x^2 X = 0,\ \frac{d^2 Y}{dy^2} + k_y^2 Y = 0, \frac{d^2 Z}{dz^2} + k_z^2 Z = 0 \tag{5.22}$$

The eigen value equation is

$$k_x^2 + k_y^2 + k_z^2 = k^2 \tag{5.23}$$

The solutions of (5.22) are harmonic functions:

$$\begin{aligned} X(x) &= \sum_n A_n^c \cos\left(k_{xn}x\right) + A_n^s \sin\left(k_{xn}x\right), \\ Y(y) &= \sum_n B_n^c \cos\left(k_{yn}y\right) + B_n^s \sin\left(k_{yn}y\right), \\ Z(z) &= \sum_n C_n^c \cos\left(k_{zn}z\right) + C_n^s \sin\left(k_{zn}z\right). \end{aligned} \tag{5.24}$$

When the functions in (5.24) are substituted in (5.20), that give the general solution of (5.18). The particular solution of (5.18) depends on the boundary conditions.

In our case, there are electric walls at x = 0 and x = h. There, the tangential E-field components must vanish, i.e., $E_y = E_z = 0\big|_{x=0,h}$

$$E_x = \frac{1}{j\mu\varepsilon}\left(\frac{\partial^2 A_x}{\partial x^2} + k^2 A_x\right), E_y = \frac{1}{j\mu\varepsilon}\left(\frac{\partial^2 A_x}{\partial x \partial y}\right), E_z = \frac{1}{j\mu\varepsilon}\left(\frac{\partial^2 A_x}{\partial x \partial z}\right) \tag{5.25}$$

We set A_x at the top and bottom walls as

$$\frac{\partial A_x}{\partial x}\Big|_{x=0,h} = 0 \tag{5.26}$$

For all the side walls, we set a vanishing normal derivative for A_x :

$$\frac{\partial A_x}{\partial x}\Big|_{z=0,L} = 0, \quad \frac{\partial A_x}{\partial y}\Big|_{y=0,W} = 0 \tag{5.27}$$

This ensures vanishing H_x and H_y at $z = 0$ and $z = L$, as well as vanishing H_x and H_z at $y = 0$ and $y = W$ (magnetic walls).

$$H_x = 0,\ H_y = \frac{1}{\mu}\left(\frac{\partial A_x}{\partial z}\right),\ H_z = \frac{1}{\mu}\left(\frac{\partial A_x}{\partial y}\right) \tag{5.28}$$

It is now obvious that the solution must appear in terms of the functions

$$\begin{aligned} X(x) &= \sum_n A_n^c \cos\left(k_{xn} x\right), \quad k_{xn} = n\frac{\pi}{h}, \\ Y(y) &= \sum_n B_n^c \cos\left(k_{yn} y\right), \quad k_{yn} = n\frac{\pi}{W}, \\ Z(z) &= \sum_n C_n^c \cos\left(k_{zn} z\right), \quad k_{zn} = n\frac{\pi}{L} \end{aligned} \tag{5.29}$$

The spectrum of the eigen modes in the cavity is discrete. The frequencies of those modes (the resonant frequencies) can be calculated from (5.23) as

$$\left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{W}\right)^2 + \left(\frac{p\pi}{L}\right)^2 = \left(\mu_r^{(mnp)}\right)^2 \varepsilon \tag{5.30}$$

$$f_r^{(mnp)} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{W}\right)^2 + \left(\frac{p\pi}{L}\right)^2} \tag{5.31}$$

The mode with the lowest resonant frequency is the ***dominant mode***. Since usually $L < W$, the lowest-frequency mode is the TM^x_{001}, for which

$$f_r^{(001)} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\frac{\pi}{L} = \frac{c}{2L\sqrt{\varepsilon_r}} \tag{5.32}$$

The field distribution of some low-order modes is given in the following figure.

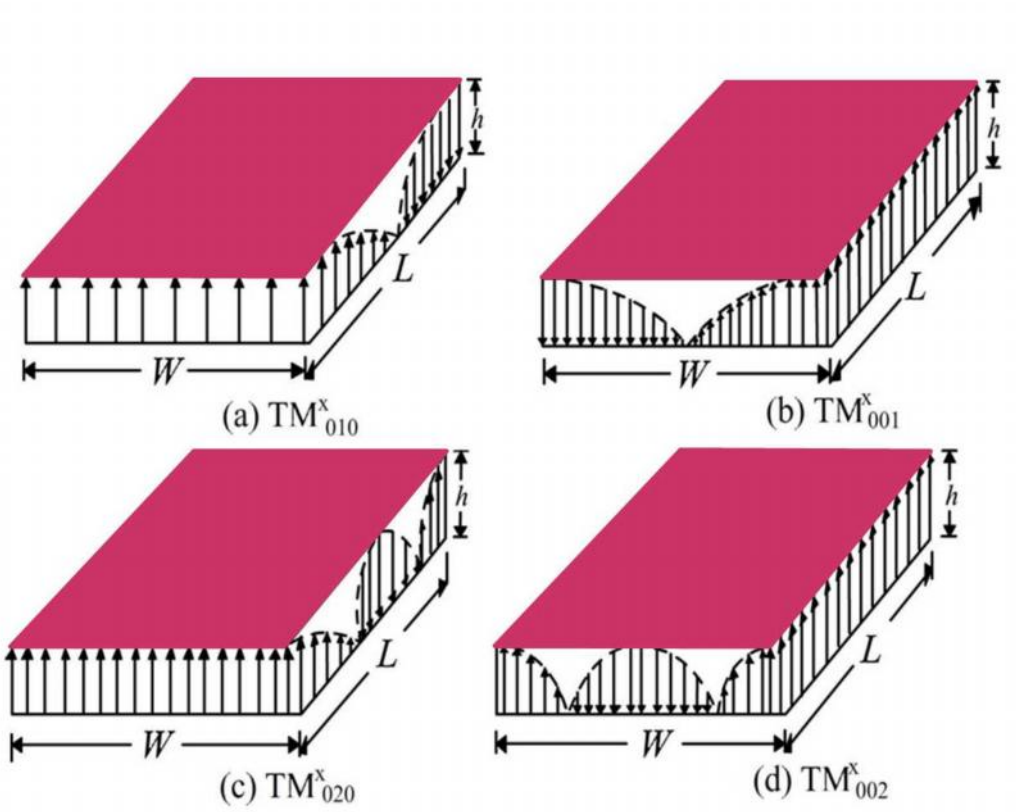

Fig. 5.16 Field configurations (modes) for rectangular microstrip patch

The general solution for $A_x^{(mnp)}$ [see (5.20) and (5.24)] is

$$A_x^{(mnp)} = \left[A_x^c \cos\left(m\frac{\pi}{h}x\right)\right]\left[B_x^c \cos\left(n\frac{\pi}{W}y\right)\right]\left[C_x^c \cos\left(p\frac{\pi}{L}z\right)\right] \tag{5.33}$$

or

$$A_x^{(mnp)} = A_{mnp} \cdot \cos\left(m\frac{®}{h}x\right) \cdot \cos\left(n\frac{®}{L}y\right) \cdot \cos\left(p\frac{®}{W}z\right) \tag{5.34}$$

The respective field solution for the (m, n, p) mode is

$$E_x = -j\frac{\left(k^2 - k_x^2\right)}{\mu«£} A_{mnp} \cdot \cos\left(k_x x\right) \cdot \cos\left(k_y y\right) \cdot \cos\left(k_z z\right) \tag{5.35}$$

$$E_y = -j\frac{k_x k_y}{\mu«£} A_{mnp} \cdot \sin\left(k_x x\right) \cdot \sin\left(k_y y\right) \cdot \cos\left(k_z z\right) \tag{5.36}$$

$$E_x = -j\frac{k_x k_z}{\mu«£} A_{mnp} \cdot \sin\left(k_x x\right) \cdot \cos\left(k_y y\right) \cdot \sin\left(k_z z\right) \tag{5.37}$$

$$H_x = 0 \tag{5.38}$$

$$H_y = -\frac{k_z}{«} A_{mnp} \cdot \cos\left(k_x x\right) \cdot \cos\left(k_y y\right) \cdot \sin\left(k_z z\right) \tag{5.39}$$

$$H_z = \frac{k_y}{«} A_{mnp} \cdot \cos\left(k_x x\right) \cdot \sin\left(k_y y\right) \cdot \cos\left(k_z z\right) \tag{5.40}$$

For the dominant TM_{001}^{x} mode,

$$E_x = \left[-j\left(k^2 - ®^2 / h^2\right) / \left(\mu«£\right)\right] A_{001} \cos\left(® z / L\right)$$

$$E_y = E_z = 0$$

$$H_y = -\left(® / «L\right) A_{001} \sin\left(® z / L\right)$$

$$H_x = H_z = 0.$$