

Lecture-2

Example-1:

Obtain the pattern of two isotropic point sources with identical amplitude and phase currents, and spaced one-half wavelength apart.

Ans:

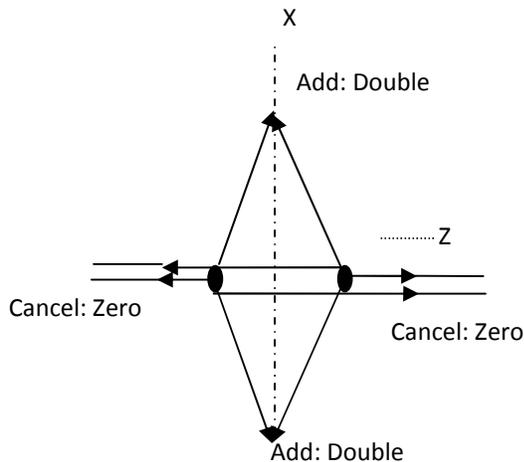


Fig 6.3 Two element array

The two isotropic antennas are placed on the z-axis with equal distance apart from the origin. The pattern of this array can be approximated by simple observations. The points in the far field along the perpendicular bisector of the line joining the point sources, the path lengths from each point sources are equal. As the point sources are of identical amplitude and phase, the waves arrive in phase and equal in amplitude in the far field along the x-axis and y-axis. So the total field is double that of the single source. Along the z-axis, the scenario is different. If we move along the +z axis, the wave coming from the left source must travel one half wavelength before reaching the source on the right. This amounts to a 180° phase lag. The waves then continue traveling to the right +z axis and maintain the same phase relationship. Thus in the far field, the waves from the two sources traveling in +z directions are 180° out of phase, so the total field is zero. The same reasoning can be used to see the effects in the -z direction. The total pattern has a relative value of 2 in the x- direction, 0 in z-direction and a smooth variation in between.

In a mathematical form the AF is

$$AF = E_0 e^{+j(kd \cos \theta + S)/2} + E_0 e^{-j(kd \cos \theta + S)/2}$$

$$(AF)_n = \cos \left[\frac{kd \cos \theta + S}{2} \right]$$

Distance between the two elements $d = \lambda/2$

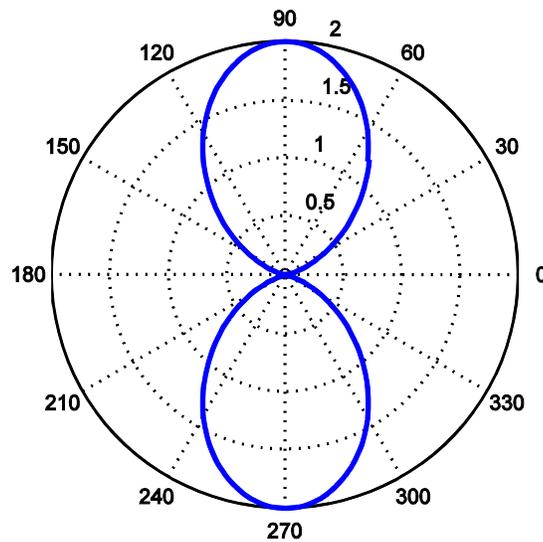
$S = 0$, as both the elements have the identical phase.

So $kd\cos\theta = (2/\lambda) \times (\lambda/2) \cos\theta = \cos\theta$

$$AF = \cos \left[\frac{\cos\theta}{2} \right]$$

So AF is maximum for $\theta = \lambda/2$ and 0 for $\theta = 0$.

	0	$\lambda/2$	$\lambda/3$
AF	0	1	1/2



Polar Plot for two Isotropic source placed at one-half wavelength apart and having identical amplitude and phase

Example-2:

Obtain the pattern of two isotropic point sources with identical amplitude and opposite phase currents, and spaced one-half wavelength apart.

Ans:

$$AF = E_0 e^{+j(kd\cos\theta)/2} - E_0 e^{-j(kd\cos\theta)/2}$$

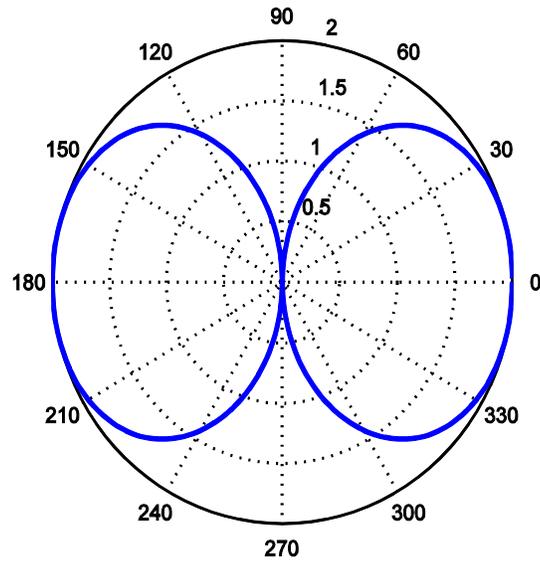
(-ve sign due to the opposite phase)

$$AF = 2jE_0 \sin \left[\frac{kd\cos\theta}{2} \right]$$

The normalized array factor will be

$$(AF)_n = \sin \left[\frac{\cos\theta}{2} \right] \quad (\text{For } d = \lambda/2)$$

	0	/2	/3
AF	1	0	1/ 2



Polar plot of array factor of two isotropic sources placed at one-half wavelength apart and having same amplitude and opposite phase

Example-3:

Obtain the pattern of two isotropic point sources with identical amplitude and in phase quadrature currents, and spaced (i) one-half wavelength, (ii) quarter wavelength apart .

Ans:

(i) Isotropic point sources at one-half wavelength apart

$$AF = E_0 e^{+j(kd \cos \theta + \pi)/2} + E_0 e^{-j(kd \cos \theta + \pi)/2}$$

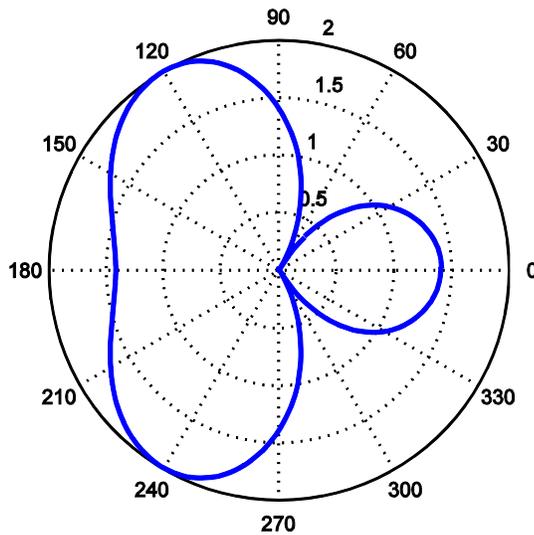
Here the value of π is $\pi/2$ and $d = \lambda/2$

$$\text{So } AF = E_0 e^{+j\left(\frac{f}{2} \cos \theta + \frac{\pi}{4}\right)} + E_0 e^{-j\left(\frac{f}{2} \cos \theta + \frac{\pi}{4}\right)}$$

$$AF = 2E_0 \cos \left[\frac{f}{2} \cos \theta + \frac{\pi}{4} \right]$$

$$(AF)_n = \cos \left[\frac{f}{2} \cos \theta + \frac{\pi}{4} \right]$$

	0	$\pi/3$	$\pi/2$	$2\pi/3$	
AF	$1/2$	0	$1/2$	1	$1/2$



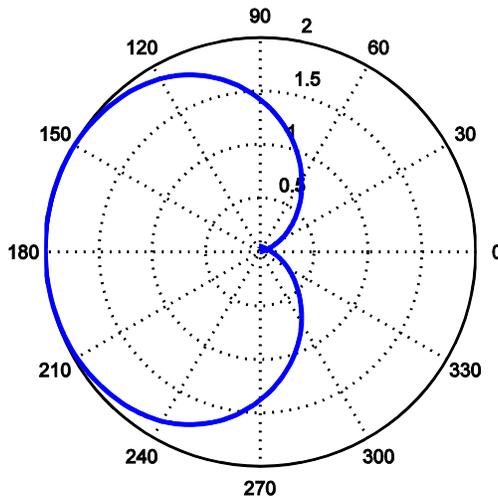
Polar plot of two isotropic sources in phase quadrature

(ii) Isotropic point sources at quarter wavelength apart

Similarly the array factor can be obtained by putting $d = \lambda/4$.

$$(AF)_n = \cos\left[\frac{f}{4} \cos \theta + \frac{f}{4}\right]$$

	0	$\pi/2$	$2\pi/3$	$5\pi/6$	
AF	0	$1/2$	0.9239	0.9945	1



Polar plot of two isotropic sources in phase quadrature and quarter-wavelength apart

Example-4 (Principle of Pattern Multiplication)

An array consists of two horizontal dipoles located along z-axis at one-half wavelength apart. If the excitation magnitudes and phases are same, then obtain the shape of the radiation pattern.

Ans:

The total field radiated by the array can be expressed as

$$E_t = \hat{a}_r j\gamma \frac{kI_o l e^{-jkr}}{4\pi r} |\cos \theta| \left[e^{+j(kd \cos \theta + s)/2} + e^{-j(kd \cos \theta + s)/2} \right]$$

$$E(\text{total}) = E_{\text{Single Element}} \times AF$$

The field due to the single element at the origin is

$$\vec{E} = \frac{j\gamma kI_o l e^{-jkr}}{4\pi r} |\cos \theta| \hat{a}_r$$

$\vec{E}_n = |\cos \theta| \hat{a}_r$ and it does not depend on s.

The AF of the dipole array can be obtained similar to Example-1. So the total field of the array is

$$E_t = \cos \theta \times \cos \left[\frac{f \cos \theta}{2} \right]$$

