

Lecture-7

6.3.8 Dolph–Chebyshev array:

Dolph proposed (in 1946) a method to design arrays with any desired sidelobe levels and any HPBW. This method is based on the approximation of the pattern of the array by a Chebyshev polynomial of order m , high enough to meet the requirement for the side-lobe levels.

Chebyshev Polynomials: The Chebyshev polynomial of order m is defined by

$$T_m(z) = \begin{cases} (-1)^m \cosh(m \cdot \cos^{-1}(|z|)), & z \leq -1 \\ \cos(m \cdot \cos^{-1}(z)), & -1 \leq z \leq 1, \\ \cosh(m \cdot \cos^{-1}(z)), & z \geq 1 \end{cases} \quad (6.53)$$

A Chebyshev polynomial $T_m(z)$ of any order m can be derived from a recursion formula, provided $T_{m-1}(z)$ and $T_{m-2}(z)$ are known, in the following way:

$$T_m(z) = 2zT_{m-1}(z) - T_{m-2}(z) \quad (6.54)$$

From the above equation we can express the Chebyshev polynomials as:

$$\begin{aligned} m = 0, T_0(z) &= 1 \\ m = 1, T_1(z) &= z \\ m = 2, T_2(z) &= 2z^2 - 1 \\ m = 3, T_3(z) &= 4z^3 - 3z \\ m = 4, T_4(z) &= 8z^4 - 8z^2 + 1 \\ m = 5, T_5(z) &= 16z^5 - 20z^3 + 5z \end{aligned} \quad (6.55)$$

The array factor expressed in (6.46) and (6.47) is the summation of cosine terms and the largest harmonics of the cosine terms is one less than the total number of elements in the array. Each cosine term having an argument which is the interger multiple of the fundamental frequency can be expressed as a series of cosine functions with the fundamental frequency as the argument. i.e.

$$\begin{aligned} m = 0 \quad \cos(mu) &= 1 \\ m = 1 \quad \cos(mu) &= \cos u \\ m = 2 \quad \cos(mu) &= \cos 2u = 2\cos^2 u - 1 \\ m = 3 \quad \cos(mu) &= \cos 3u = 4\cos^3 u - 3\cos u \end{aligned} \quad (6.56)$$

If the $-1 \leq z \leq 1$, the Chebyshev polynomials are related to the cosine functions. So by comparing equation (6.55) and (6.56) we see that the Chebyshev argument z is related to cosine argument u by

$$z = \cos u \Rightarrow u = \cos^{-1} z$$

e.g. $\cos(2u)$ can be expressed as

$$\cos(2 \cos^{-1} z) = 2[\cos(\cos^{-1} z)]^2 - 1 = 2z^2 - 1 = T_2(z)$$

Similarly:

$$m = 0, \cos(mu) = T_0(z) = 1$$

$$m = 1, \cos(mu) = \cos(u) = T_1(z) = z$$

$$m = 2, \cos(mu) = \cos(2u) = T_2(z) = 2z^2 - 1$$

$$m = 3, \cos(mu) = \cos(3u) = T_3(z) = 4z^3 - 3z$$

$$m = 4, \cos(mu) = \cos(4u) = T_4(z) = 8z^4 - 8z^2 + 1$$

$$m = 5, \cos(mu) = \cos(5u) = T_5(z) = 16z^5 - 20z^3 + 5z$$

(6.57)

Properties of the Chebyshev polynomials:

- 1) All polynomials of any order m pass through the point $(1, 1)$.
- 2) Within the range $-1 \leq z \leq 1$, the polynomials have values within $[-1, 1]$.
- 3) All nulls occur within $-1 \leq z \leq 1$.
- 4) The maxima and minima in the $z \in [-1, 1]$ range have values $+1$ and -1 , respectively.
- 5) The higher the order of the polynomial, the steeper the slope for $|z| > 1$.

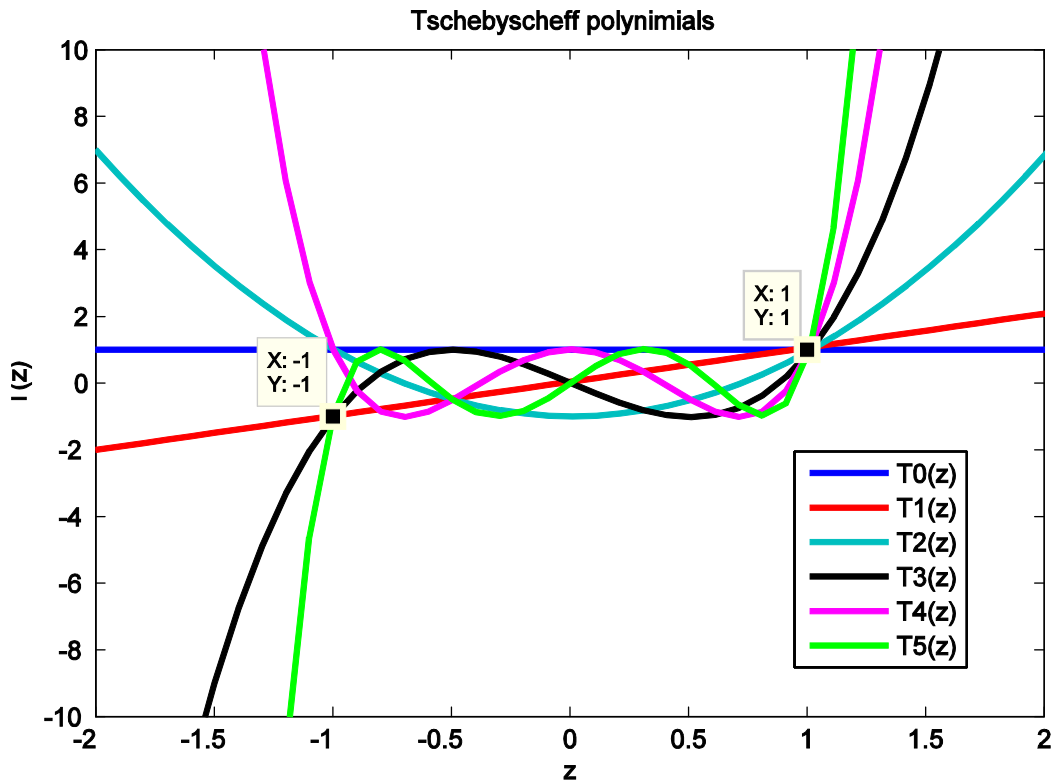


Fig. 6.18 Plot of Chebyshev polynomials