

Assignment-2

1. Show that $\int_0^{\infty} \frac{\cos\left(\frac{\pi w}{2}\right) \cos(xw)}{(1-w)^2} dw = \begin{cases} \frac{\pi}{2} \cos x & , |x| < \frac{\pi}{2} \\ 0 & , |x| > \frac{\pi}{2} \end{cases}$
2. Find the Fourier cosine integral representation of the function $f(x) = \begin{cases} x & , 0 < x < a \\ 0 & , x > a \end{cases}$
3. Find the Fourier sine integral representation of the function
 - a. $f(x) = \begin{cases} e^x & , 0 < x < 1 \\ 0 & , x > 1 \end{cases}$
 - b. $f(x) = \begin{cases} \sin x & , 0 \leq x \leq \pi \\ 0 & , x > \pi \end{cases} \sin x$
 - c. $f(x) = \begin{cases} \pi - x & , 0 < x < \pi \\ 0 & , x > \pi \end{cases}$
4. Prove that $\int_0^{\infty} \frac{w^3 \cos xw}{w^4+4} dw = \frac{\pi}{2} e^{-x} , x > 0$
5. Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 0 & , x < 0 \\ \pi e^{-x} & , x > 0 \end{cases} \quad \text{Hence evaluate } \int_0^{\infty} \frac{\cos xw + w \sin xw}{1+w^2} dw.$$

6. Find the Fourier integral representation of the function

$$f(x) = \begin{cases} \cos x & , |x| < \frac{\pi}{2} \\ 0 & , \text{otherwise} \end{cases}$$