



Probability is:

- A quantitative measure of **uncertainty**
- A measure of the **strength of belief** in the occurrence of an uncertain event
- A measure of the degree of **chance or likelihood of occurrence** of an uncertain event
- Measured by a number between 0 and 1 (or between 0% and 100%)



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- An *experiment* is a process that is performed to understand and observe possible outcomes.
- An *event* is an outcome of an experiment.
- Set of all outcomes of an experiment is called the *sample space*.



Example

- In a manufacturing unit three parts from the assembly are selected. You observe whether they are defective or not.
- Let us designate a defective as D, and a non-defective as G.
- Now sample space $S = \{GGG, GGD, GDG, DGG, GDD, DGD, DDG, DDD\}$; all possible outcomes.
- An event of getting at least two defectives is
 $E = \{GDD, DGD, DDG, DDD\}$



- Probability of an event A is defined as the ratio between two numbers m and n.

$$P(A) = \frac{m}{n}.$$

Where m= no. of ways that are favorable to the occurrence of A and

n= the total no. of outcomes of the experiment.

- Always $P(A) \geq 0$ and ≤ 1 .



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TYPES OF PROBABILITY

- Classical
- Relative
- Subjective



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- Classical Probability – Here the probability can be assessed without actually having to perform any experiment.
- Example: Drawing an ace out of a pack of cards.



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- Relative Probability – Here probability cannot be assessed without performing the experiment or using the past data.
- Example: What is the probability that you will get a score of at least 75% in the course Quantitative Techniques?

Types of Probability (Continued)

- **Subjective Probability**

- ✓ based on personal beliefs, experiences, prejudices, intuition - personal judgment
- ✓ different for all observers (subjective)
- ✓ examples: elections, new product introduction, snowfall



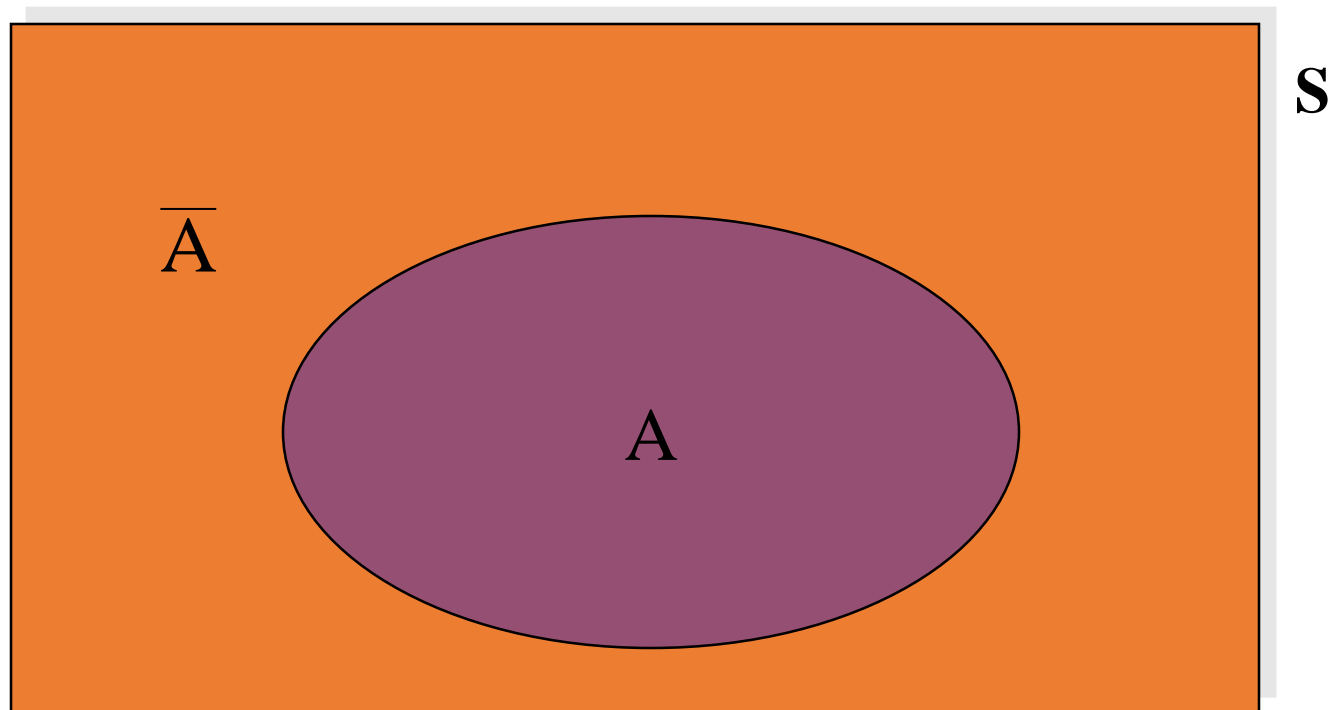
Basic Definitions

- **Set - a collection of elements or objects of interest**
 - ✓ Empty set (denoted by \emptyset)
 - a set containing no elements
 - ✓ Universal set (denoted by S)
 - a set containing all possible elements
 - ✓ Complement (**Not**). The complement of A is
 - a set containing all elements of S not in A

$$(\bar{A})$$



Complement of a Set



Venn Diagram illustrating the Complement of an event

Basic Definitions (Continued)

✓ Intersection (And)

$$(A \cap B)$$

- a set containing all elements in both A and B

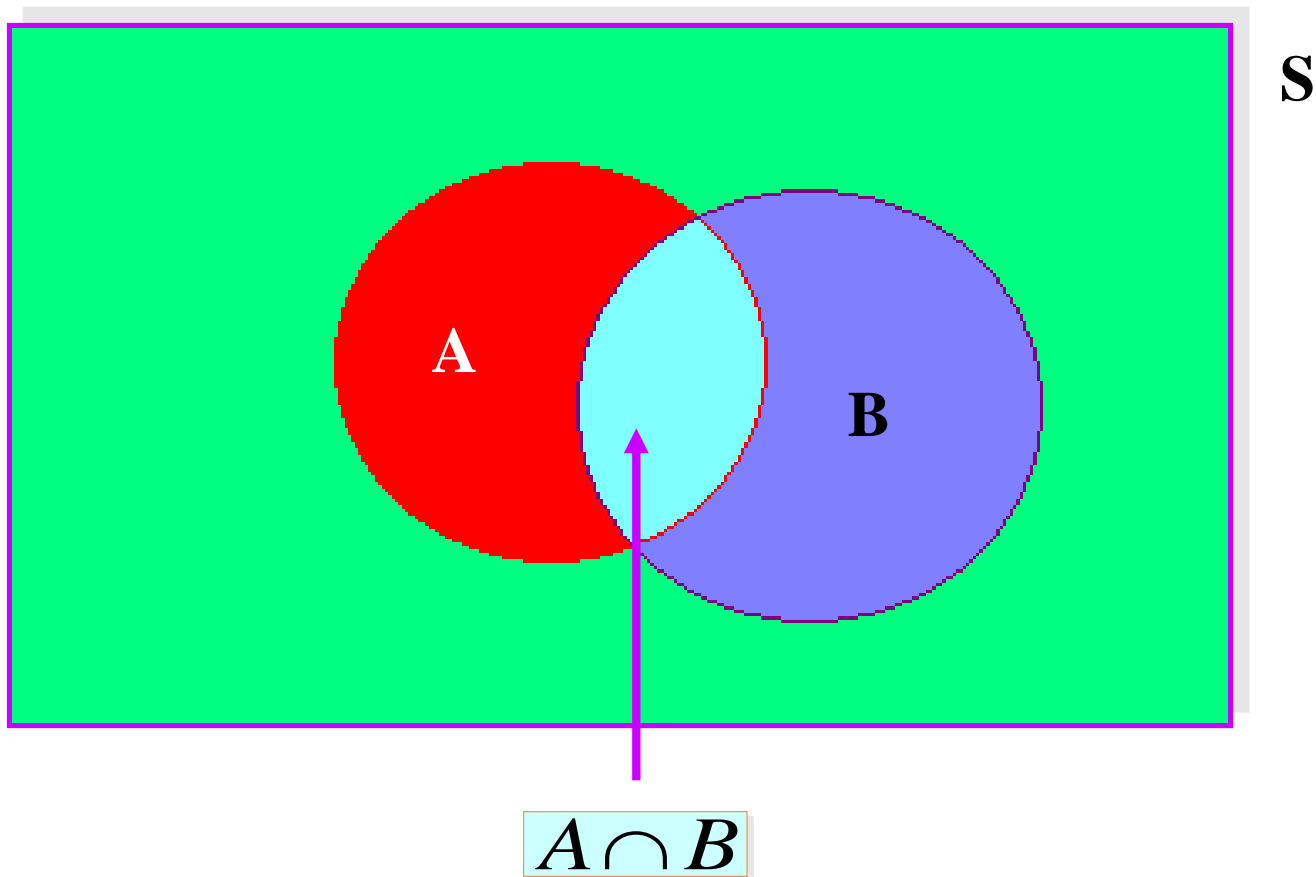
✓ Union (Or)

$$(A \cup B)$$

- a set containing all elements in A or B or both



Sets: A Intersecting with B

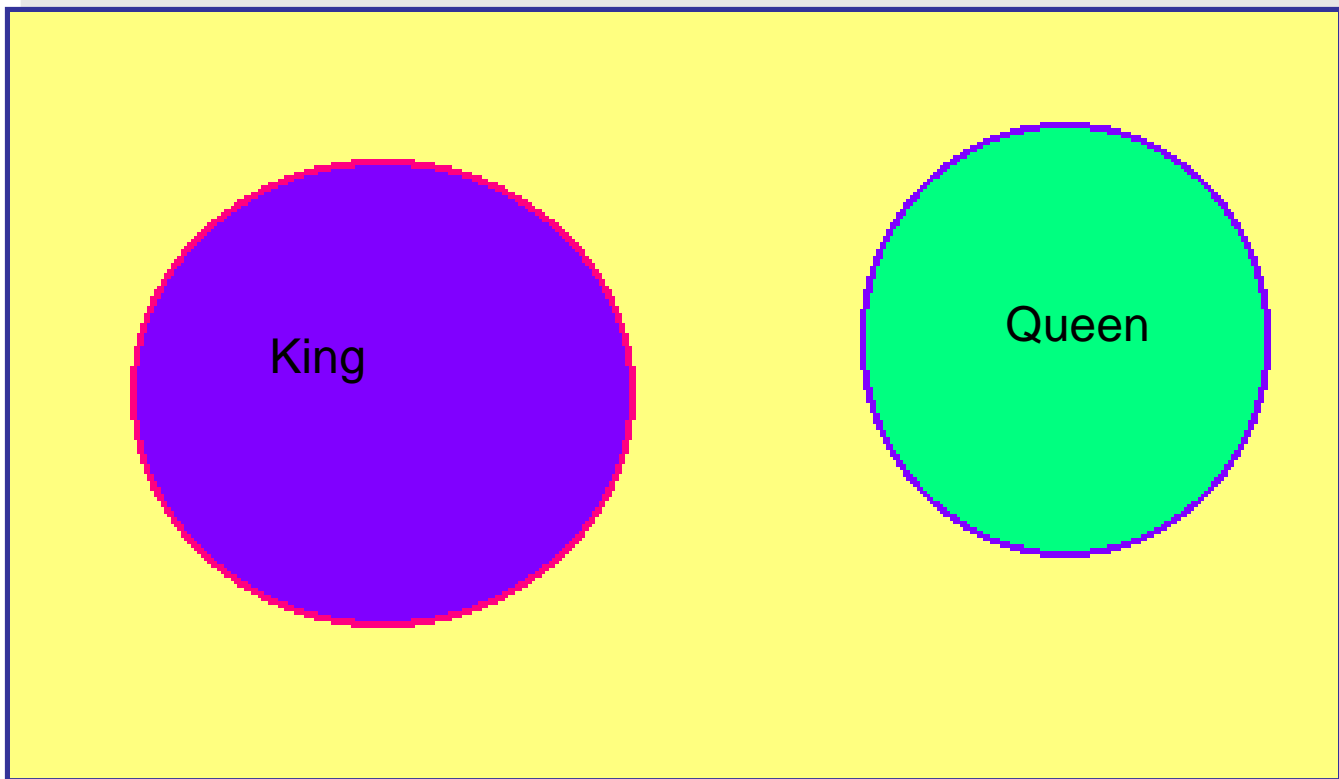


Basic Definitions (Continued)

- **Mutually exclusive or disjoint sets**
 - sets having no elements in common, having no intersection, whose intersection is the empty set.
 - Eg. King and Queen in cards.
- **Partition**
 - a collection of mutually exclusive sets which together include all possible elements, whose union is the universal set.

Mutually Exclusive or Disjoint Sets

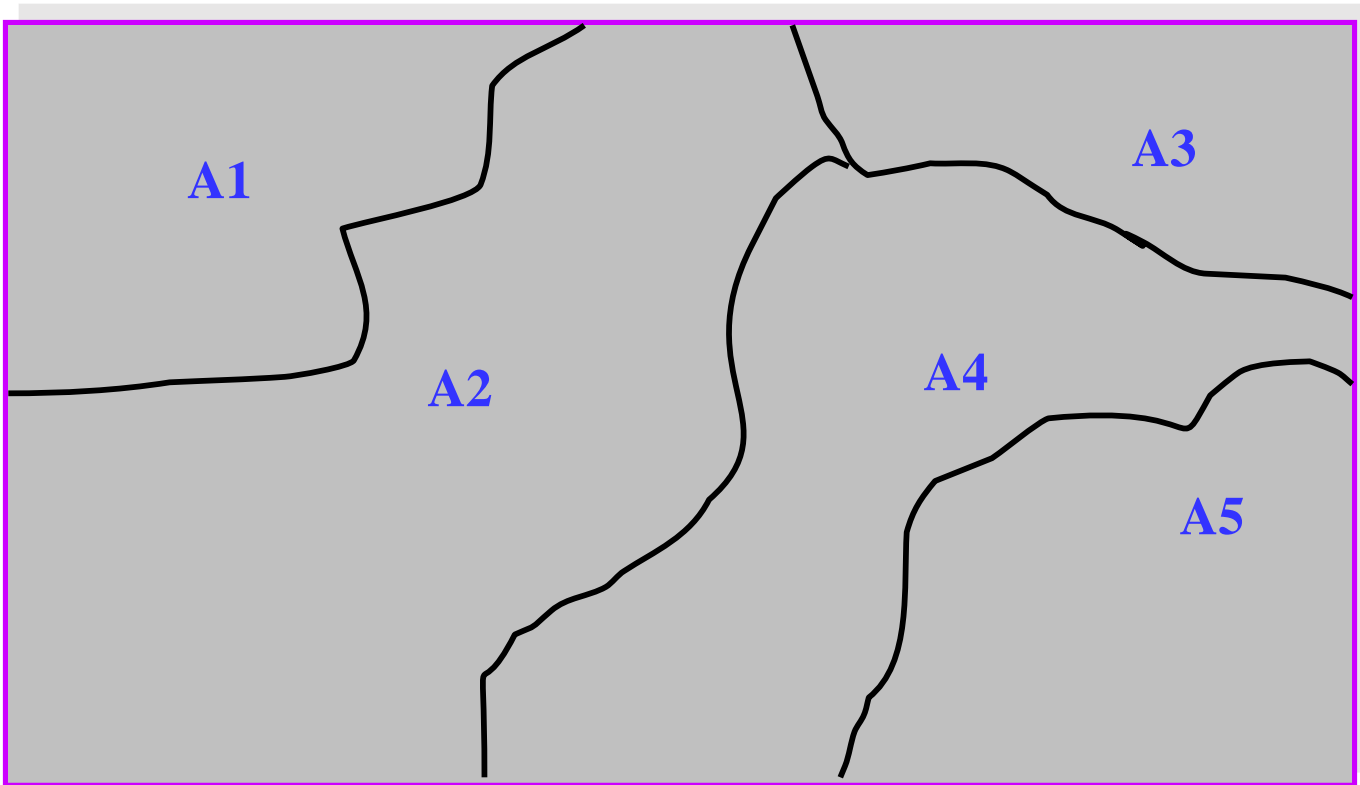
Sets have nothing in common



S



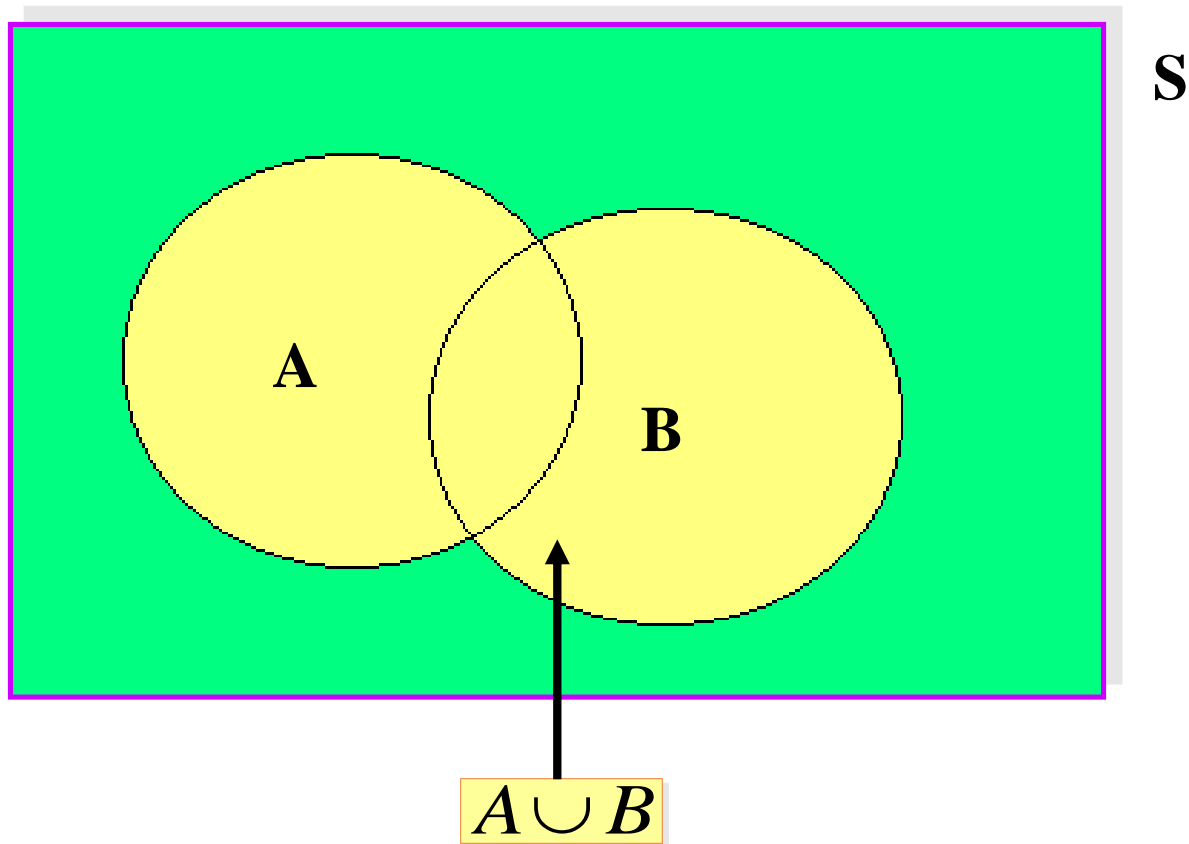
Sets: Partition



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Sets: A Union B





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- Independent Events-

Two events A and B are said to be independent if the occurrence of A is in no way influenced by the occurrence of B. Likewise, the occurrence of B is no way influenced by the occurrence of A.

Eg. When you toss a coin twice, getting a head in the second trial is not influenced by what happened in the first trial.



Experiment

- Process that leads to one of several possible outcomes *, e.g.:
 - ✓ Coin toss
 - Heads, Tails
 - ✓ Throw die
 - 1, 2, 3, 4, 5, 6
 - ✓ Pick a card
 - AH, KH, QH, ...
 - ✓ Introduce a new product
- Each trial of an experiment has a single observed outcome.
- The precise outcome of a random experiment is unknown before a trial.

* Also called a basic outcome, elementary event, or simple event



Events : Definition

- **Sample Space or Event Set**

- ✓ Set of all possible outcomes (universal set) for a given experiment

- E.g.: Roll a regular six-sided die

- $S = \{1,2,3,4,5,6\}$

- **Event**

- ✓ Collection of outcomes having a common characteristic

- E.g.: Even number

- $A = \{2,4,6\}$

- Event A occurs if an outcome in the set A occurs

- **Probability of an event**

- ✓ Sum of the probabilities of the outcomes of which it consists

- $P(A) = P(2) + P(4) + P(6)$



Pick a Card: Sample Space

	H hearts	D diamonds	C clubs	S spades
	← Event 'Ace'			
		K	K	K
		Q	Q	Q
		J	J	J
		10	10	10
		9	9	9
		8	8	8
		7	7	7
		6	6	6
		5	5	5
		4	4	4
		3	3	3
		2	2	2
	↑ Event 'Heart'			

Union of Events 'Heart' and 'Ace'

$$P(\text{Heart} \cup \text{Ace}) = \frac{n(\text{Heart} \cup \text{Ace})}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

Event 'Ace'

$$P(\text{Ace}) = \frac{n(\text{Ace})}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

Event 'Heart'

$$P(\text{Heart}) = \frac{n(\text{Heart})}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

The intersection of the events 'Heart' and 'Ace' comprises the single point circled twice: the ace of hearts

$$P(\text{Heart} \cap \text{Ace}) = \frac{n(\text{Heart} \cap \text{Ace})}{n(S)} = \frac{1}{52}$$

Basic Rules for Probability

- **Range of Values for $P(A)$:** $0 \leq P(A) \leq 1$

- **Complements** - Probability of *not* A

$$P(\bar{A}) = 1 - P(A)$$

- **Intersection** - Probability of both A *and* B

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

- ✓ **Mutually exclusive events** (A and C) :

$$P(A \cap C) = 0$$

Basic Rules for Probability (Continued)

- **Union** - Probability of A *or* B or **both** (rule of unions)

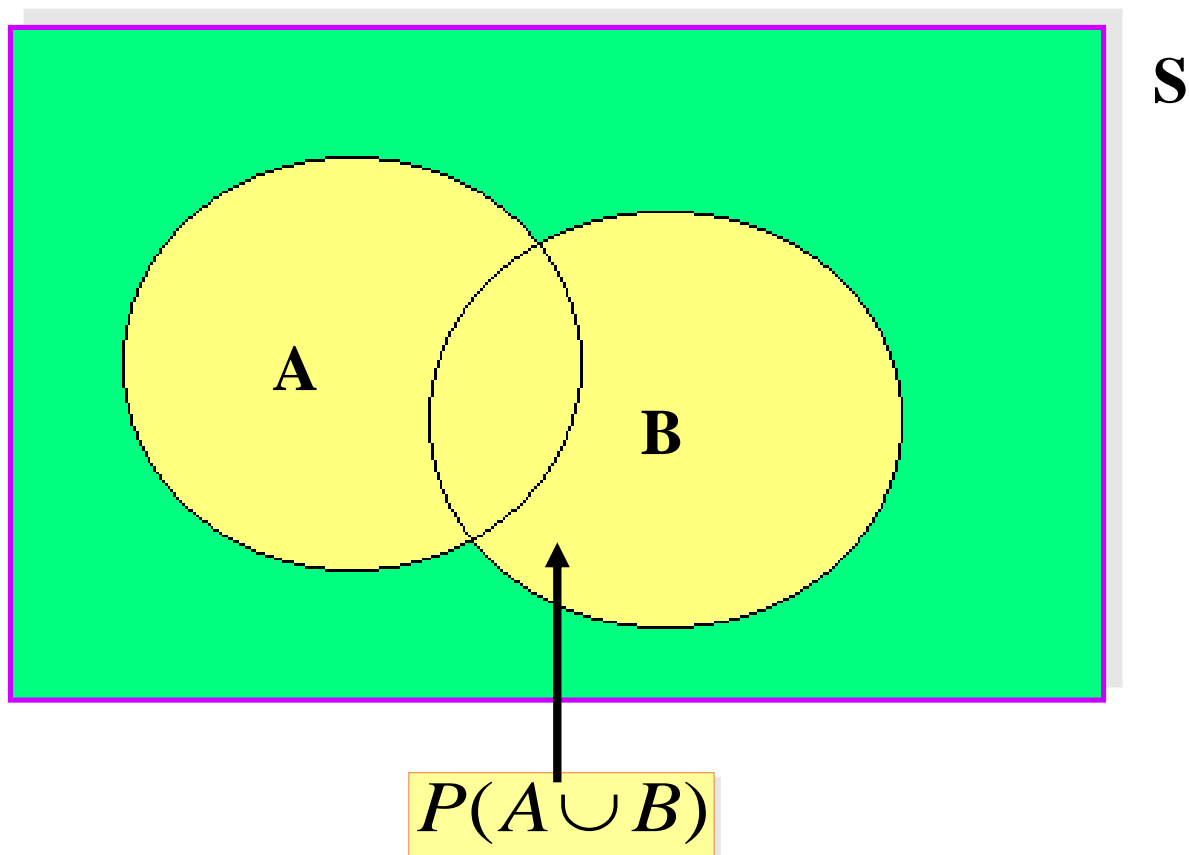
$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = P(A) + P(B) - P(A \cap B)$$

✓ Mutually exclusive events: If A and B are mutually exclusive, then

$$P(A \cap B) = 0 \text{ so } P(A \cup B) = P(A) + P(B)$$



Sets: $P(A \cup B)$



Multiplication Rule

- Independent Events

For two independent events A and B, the probability of simultaneous occurrence of A and B, also known as probability of intersection equals the product of probability of A and probability of B.

$$P(A \cap B) = P(A) \cdot P(B)$$

- Example: The probability that you will get a A grade in QT is 0.7 and that you will get a A grade in Marketing is 0.6. What is the probability that you will get a A grade in both these subjects?

Multiplication Rule

- If events are not independent

Two events A and B which are not independent – the intersection of the two events equals the product of the probability of A and the probability of B given that A has happened.

$$P(A \cap B) = P(A) \cdot P(B/A).$$

- It can also be written as

$$P(A \cap B) = P(B) \cdot P(A/B).$$

- $P(A/B)$ or $P(B/A)$ can also be called Conditional Probability.



Conditional Probability

- **Conditional Probability** - Probability of A *given* B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ where } P(B) \neq 0$$

✓ Independent events:

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

Conditional Probability (continued)

Rules of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{so} \quad P(A \cap B) = P(A|B)P(B) \\ = P(B|A)P(A)$$

If events A and D are **statistically independent**:

$$P(A|D) = P(A)$$

$$\text{so} \quad P(A \cap D) = P(A)P(D)$$

$$P(D|A) = P(D)$$



Example

- From a pack of cards, 2 cards are drawn without replacement. What is the probability that both cards drawn are heart?

Let A = getting a heart in first draw

B= getting a heart in second draw

$$P(A) = 13/52$$

$$P(B/A) = 12/51$$

Therefore

$$P(A \cap B) = P(A) \cdot P(B/A) = (13/52) (12/51) = 1/17.$$

- There is a 30% chance that you will catch the 9.00AM bus. If you catch the bus you will be on time. If you miss the bus, there is a 60% chance that you will be late. What is the probability that you will be late?



- There is a 30% chance that you will catch the 9.00AM bus. If you catch the bus you will be on time. If you miss the bus, there is a 60% chance that you will be late. What is the probability that you will be late?
- Hint let $P(A)$ = Miss the bus
 $P(B)$ = Late
- $P(A \cap B) = P(A) \cdot P(B/A)$



- A survey was conducted for 200 families.
- What is the probability that a randomly selected family is a buyer of car?
- What is the probability that a randomly selected family is both a buyer of a car and belongs to income 4 lakhs and above?
- A family selected at random is found to be belonging to income of 4 lakhs and above. What is the probability that this family is buyer of car?

Family	Income below 4 Lakhs/year.	Income above 4 Lakhs/year	total
Buyer of car	38	42	80
Non-Buyer of car	82	38	120
Total	120	80	200



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- Marginal probability – simple event $P(A)$ or $P(B)$
- Joint Probability – simultaneous occurrence of events $P(A \cap B)$
- Conditional probability – $P(A/B)$ or $P(B/A)$.

Contingency Table - Example

Counts

	AT& T	IBM	Total
Telecommunication	40	10	50
Computers	20	30	50
Total	60	40	100

Probabilities

	AT& T	IBM	Total
Telecommunication	.40	.10	.50
Computers	.20	.30	.50
Total	.60	.40	1.00

Probability that a project is undertaken by IBM *given* it is a telecommunications project:

$$\begin{aligned}
 P(IBM | T) &= \frac{P(IBM \cap T)}{P(T)} \\
 &= \frac{0.10}{0.50} = 0.2
 \end{aligned}$$

Probability Tree (Decision Tree)

- There are three Machines designated as A,B,C are producing same item. The output of A,B,C are 40%, 35%, 25% respectively. A produces 5% defectives. B produces 10% defectives. C produces 12% defectives.
- If an item is selected from the total output, what is the probability that it is defective?
- If an item is selected what is the probability that it is good?
- An item is selected and found to be defective. What is the chance that it is produced from A?

Independence of Events

Conditions for the statistical independence of events A and B:

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

and

$$P(A \cap B) = P(A)P(B)$$

$$\begin{aligned} P(Ace|Heart) &= \frac{P(Ace \cap Heart)}{P(Heart)} \\ &= \frac{1}{\frac{52}{13}} = \frac{1}{13} = P(Ace) \\ &\quad \frac{52}{52} \end{aligned}$$

$$\begin{aligned} P(Heart|Ace) &= \frac{P(Heart \cap Ace)}{P(Ace)} \\ &= \frac{1}{\frac{52}{4}} = \frac{1}{4} = P(Heart) \\ &\quad \frac{52}{52} \end{aligned}$$

$$P(Ace \cap Heart) = \frac{4}{52} * \frac{13}{52} = \frac{1}{52} = P(Ace)P(Heart)$$

Independence of Events – Example

Events *Television* (T) and *Billboard* (B) are assumed to be **independent**.

$$a) P(T \cap B) = P(T)P(B)$$

$$= 0.04 * 0.06 = 0.0024$$

$$b) P(T \cup B) = P(T) + P(B) - P(T \cap B)$$

$$= 0.04 + 0.06 - 0.0024 = 0.0976$$



2-7 The Law of Total Probability and Bayes' Theorem

The law of total probability:

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$

In terms of conditional probabilities:

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap \bar{B}) \\ &= P(A|B)P(B) + P(A|\bar{B})P(\bar{B}) \end{aligned}$$

More generally (where B_i make up a partition):

$$\begin{aligned} P(A) &= \sum P(A \cap B_i) \\ &= \sum P(A|B_i)P(B_i) \end{aligned}$$

The Law of Total Probability- Example

Event U: Stock market will go up in the next year

Event W: Economy will do well in the next year

$$P(U|W) = .75$$

$$P(U|\bar{W}) = .30$$

$$P(W) = .80 \Rightarrow P(\bar{W}) = 1 - .8 = .2$$

$$\begin{aligned} P(U) &= P(U \cap W) + P(U \cap \bar{W}) \\ &= P(U|W)P(W) + P(U|\bar{W})P(\bar{W}) \\ &= (.75)(.80) + (.30)(.20) \\ &= .60 + .06 = .66 \end{aligned}$$



Bayes' Theorem

- **Bayes'** theorem enables you, knowing just a little more than the probability of A given B, to find the probability of B given A.
- Based on the definition of conditional probability and the law of total probability.

$$\begin{aligned}P(B|A) &= \frac{P(A \cap B)}{P(A)} \\ &= \frac{P(A \cap B)}{P(A \cap B) + P(A \cap \bar{B})} \\ &= \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})}\end{aligned}$$

Applying the law of total probability to the denominator

Applying the definition of conditional probability throughout



Bayes' Theorem - Example

- A medical test for a rare disease (affecting 0.1% of the population) is imperfect:
 - ✓ When administered to an ill person, the test will indicate so with probability 0.92
 - ✓ When administered to a person who is not ill, the test will erroneously give a positive result (false positive) with probability 0.04 .



Example (continued)

$$P(I) = 0.001$$

$$P(\bar{I}) = 0.999$$

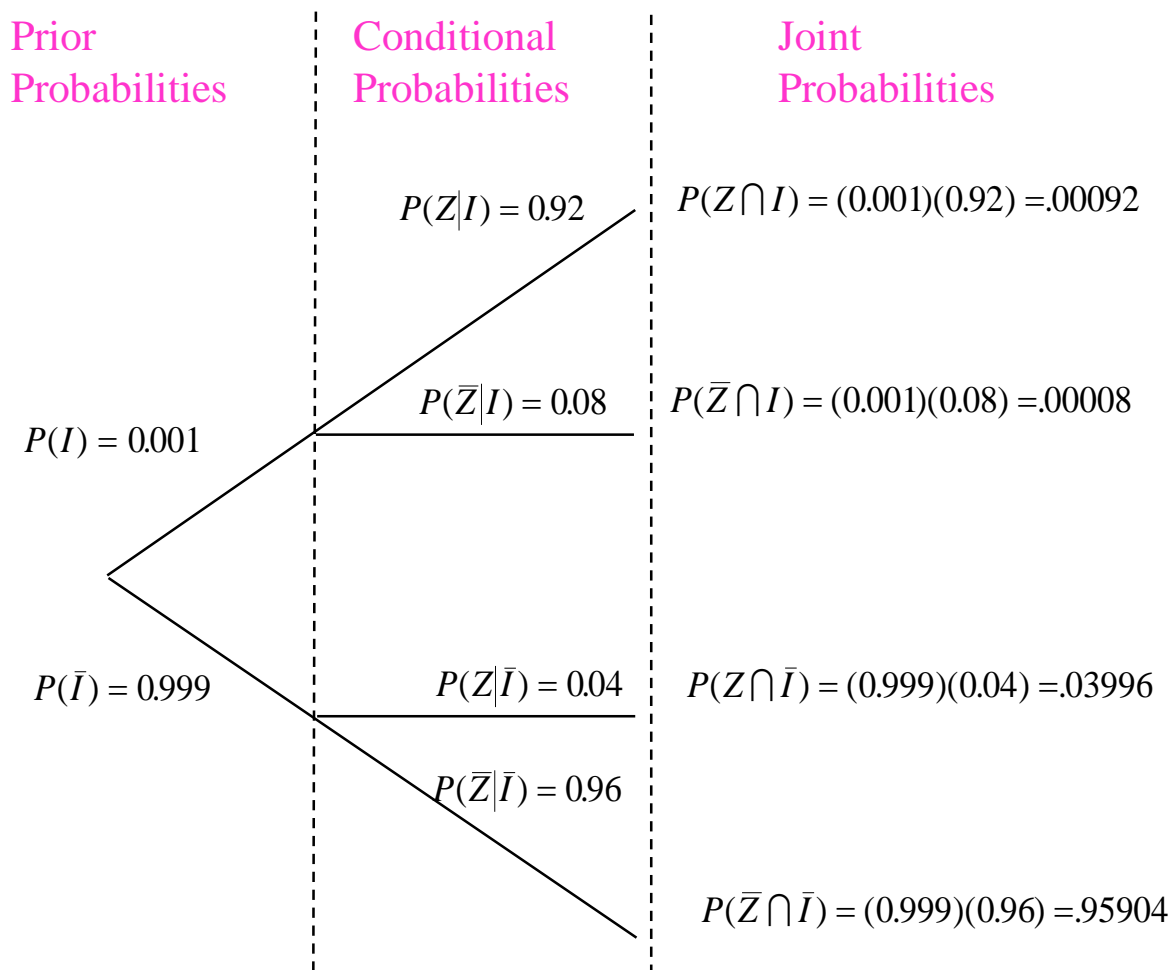
$$P(Z|I) = 0.92$$

$$P(Z|\bar{I}) = 0.04$$

$$\begin{aligned} P(I|Z) &= \frac{P(I \cap Z)}{P(Z)} \\ &= \frac{P(I \cap Z)}{P(I \cap Z) + P(I \cap \bar{Z})} \\ &= \frac{P(Z|I)P(I)}{P(Z|I)P(I) + P(Z|\bar{I})P(\bar{I})} \\ &= \frac{(.92)(0.001)}{(.92)(0.001) + (0.04)(.999)} \\ &= \frac{0.00092}{0.00092 + 0.03996} = \frac{0.00092}{.04088} \\ &= .0225 \end{aligned}$$



Example (Tree Diagram)





Bayes' Theorem Extended

- Given a partition of events B_1, B_2, \dots, B_n :

$$\begin{aligned} P(B_1|A) &= \frac{P(A \cap B_1)}{P(A)} \\ &= \frac{P(A \cap B_1)}{\sum P(A \cap B_i)} \\ &= \frac{P(A|B_1)P(B_1)}{\sum P(A|B_i)P(B_i)} \end{aligned}$$

Applying the law of total probability to the denominator

Applying the definition of conditional probability throughout

Bayes' Theorem Extended - Example

- An economist believes that during periods of high economic growth, the U.S. dollar appreciates with probability 0.70; in periods of moderate economic growth, the dollar appreciates with probability 0.40; and during periods of low economic growth, the dollar appreciates with probability 0.20.
- During any period of time, the probability of high economic growth is 0.30, the probability of moderate economic growth is 0.50, and the probability of low economic growth is 0.20.
- Suppose the dollar has been appreciating during the present period. What is the probability we are experiencing a period of high economic growth?

Partition:

H - High growth $P(H) = 0.30$

M - Moderate growth $P(M) = 0.50$

L - Low growth $P(L) = 0.20$

Event A – Appreciation

$$P(A|H) = 0.70$$

$$P(A|M) = 0.40$$

$$P(A|L) = 0.20$$

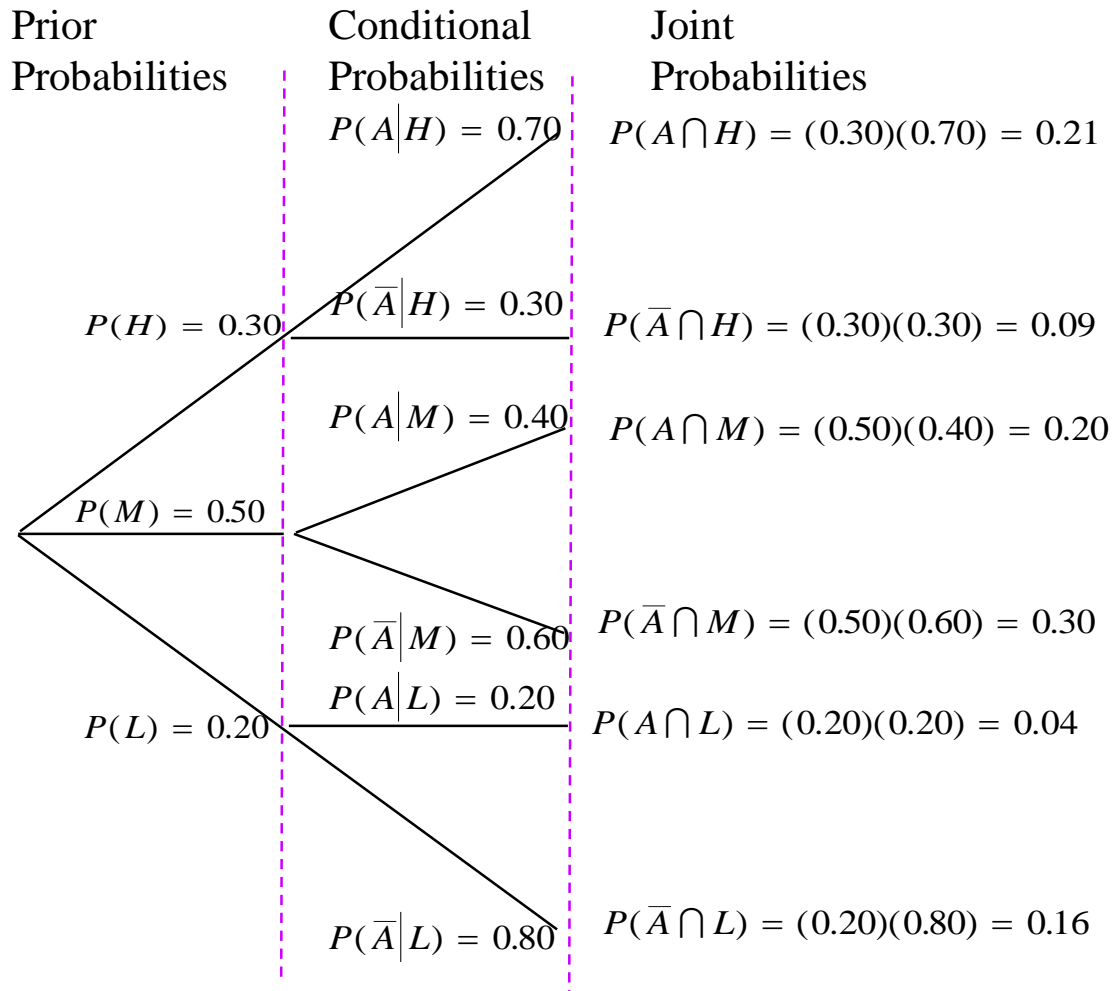


Example (continued)

$$\begin{aligned}P(H|A) &= \frac{P(H \cap A)}{P(A)} \\&= \frac{P(H \cap A)}{P(H \cap A) + P(M \cap A) + P(L \cap A)} \\&= \frac{P(A|H)P(H)}{P(A|H)P(H) + P(A|M)P(M) + P(A|L)P(L)} \\&= \frac{(0.70)(0.30)}{(0.70)(0.30) + (0.40)(0.50) + (0.20)(0.20)} \\&= \frac{0.21}{0.21 + 0.20 + 0.04} = \frac{0.21}{0.45} \\&= 0.467\end{aligned}$$



Example (Tree Diagram)



The Joint Probability Table

- A **joint probability** table is similar to a contingency table , except that it has probabilities in place of frequencies.
- The row totals and column totals are called **marginal probabilities**.