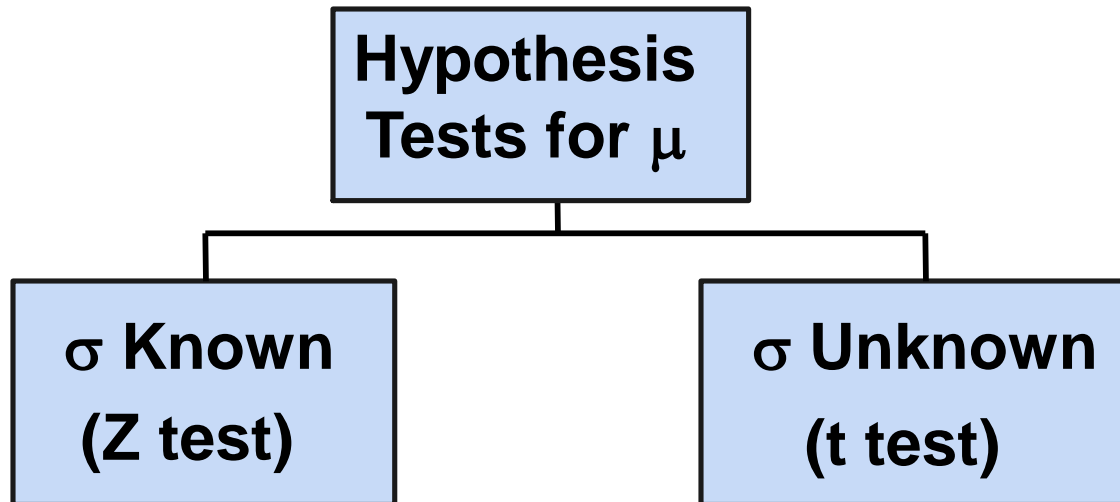
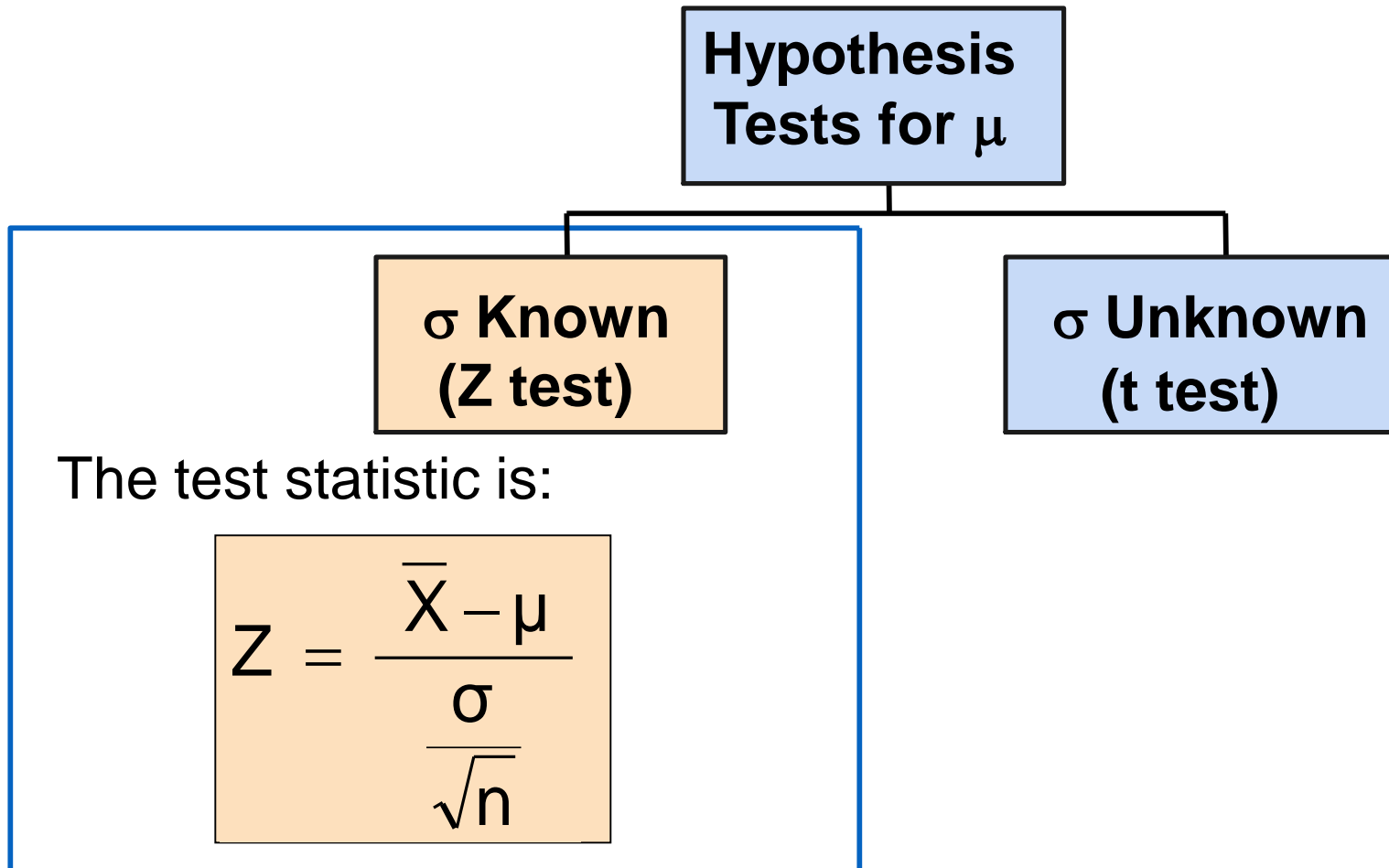


Hypothesis Tests for the Mean



Z Test of Hypothesis for the Mean (σ Known)

- Convert sample statistic () to a \bar{X} test statistic



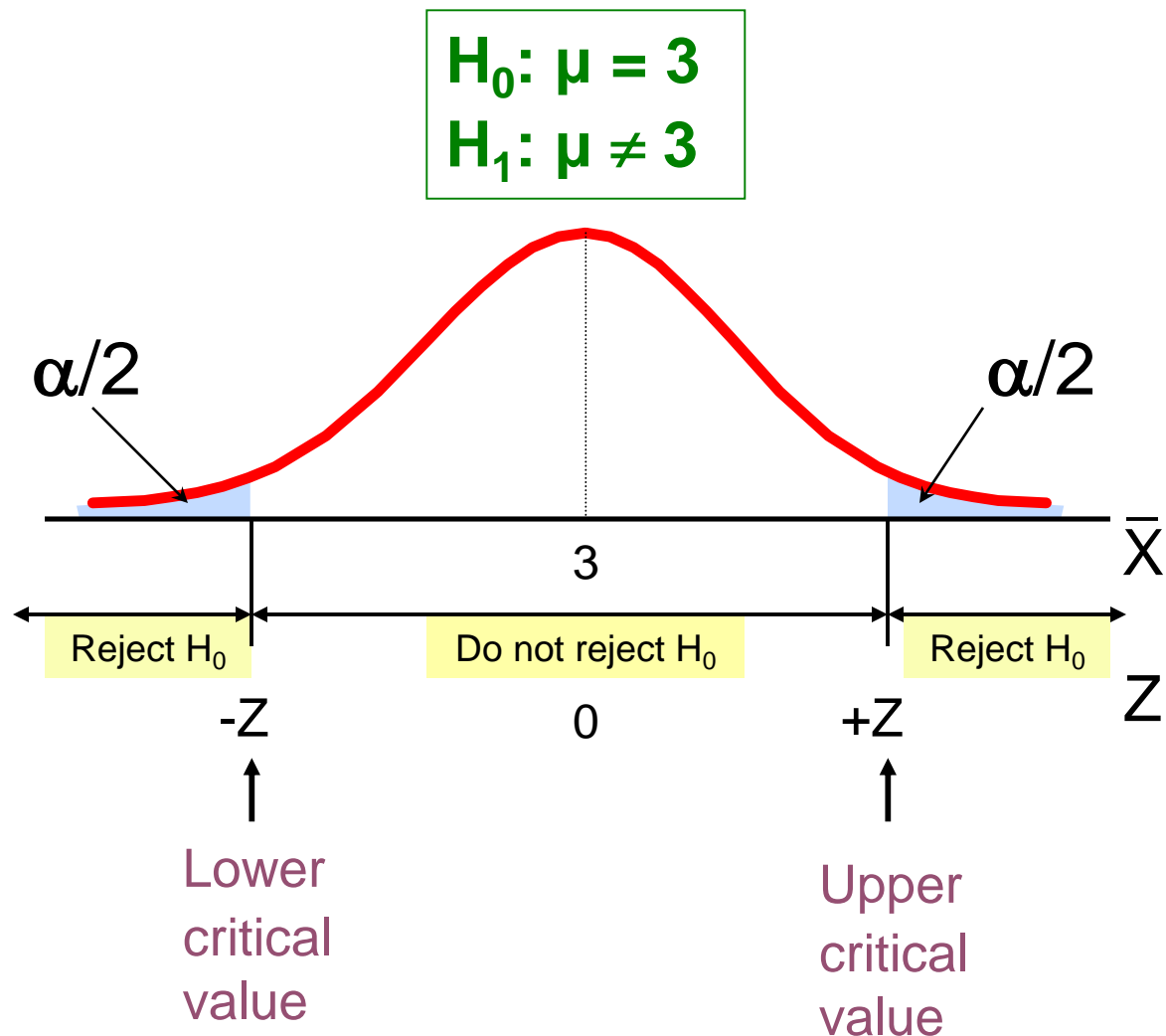
Critical Value Approach to Testing

- For a two-tail test for the mean, σ known:
- Convert sample statistic (\bar{x}) to test statistic (Z statistic)
- Determine the critical Z values for a specified level of significance α from a table or computer
- **Decision Rule:** If the test statistic falls in the rejection region, reject H_0 ; otherwise do not reject H_0



Two-Tail Tests

- There are two cutoff values (critical values), defining the regions of rejection



6 Steps in Hypothesis Testing

1. State the null hypothesis, H_0 and the alternative hypothesis, H_1
2. Choose the level of significance, α , and the sample size, n
3. Determine the appropriate test statistic and sampling distribution
4. Determine the critical values that divide the rejection and nonrejection regions

6 Steps in Hypothesis Testing

(continued)

5. Collect data and compute the value of the test statistic
6. Make the statistical decision and state the managerial conclusion. If the test statistic falls into the nonrejection region, do not reject the null hypothesis H_0 . If the test statistic falls into the rejection region, reject the null hypothesis. Express the managerial conclusion in the context of the problem

Hypothesis Testing Example

Test the claim that the true mean # of TV sets in US homes is equal to 3.
(Assume $\sigma = 0.8$)

1. State the appropriate null and alternative hypotheses
 - $H_0: \mu = 3$ $H_1: \mu \neq 3$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
 - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test



Hypothesis Testing Example

(continued)

3. Determine the appropriate technique
 - σ is known so this is a Z test.
4. Determine the critical values
 - For $\alpha = 0.05$ the critical Z values are ± 1.96
5. Collect the data and compute the test statistic
 - Suppose the sample results are
 $n = 100$, $\bar{X} = 2.84$ ($\sigma = 0.8$ is assumed known)

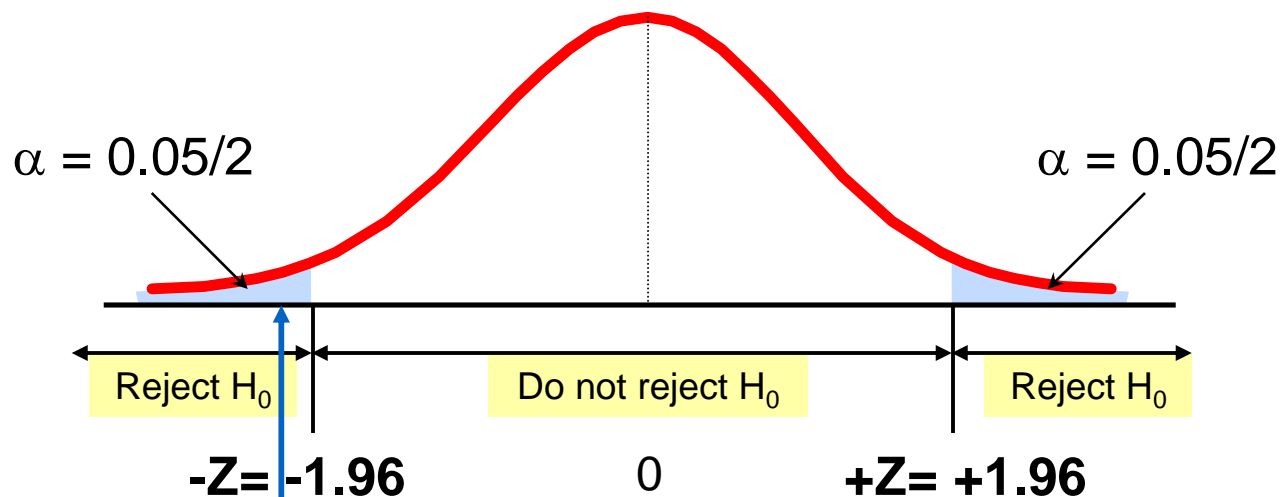
So the test statistic is:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$



Hypothesis Testing Example *(continued)*

- 6. Is the test statistic in the rejection region?



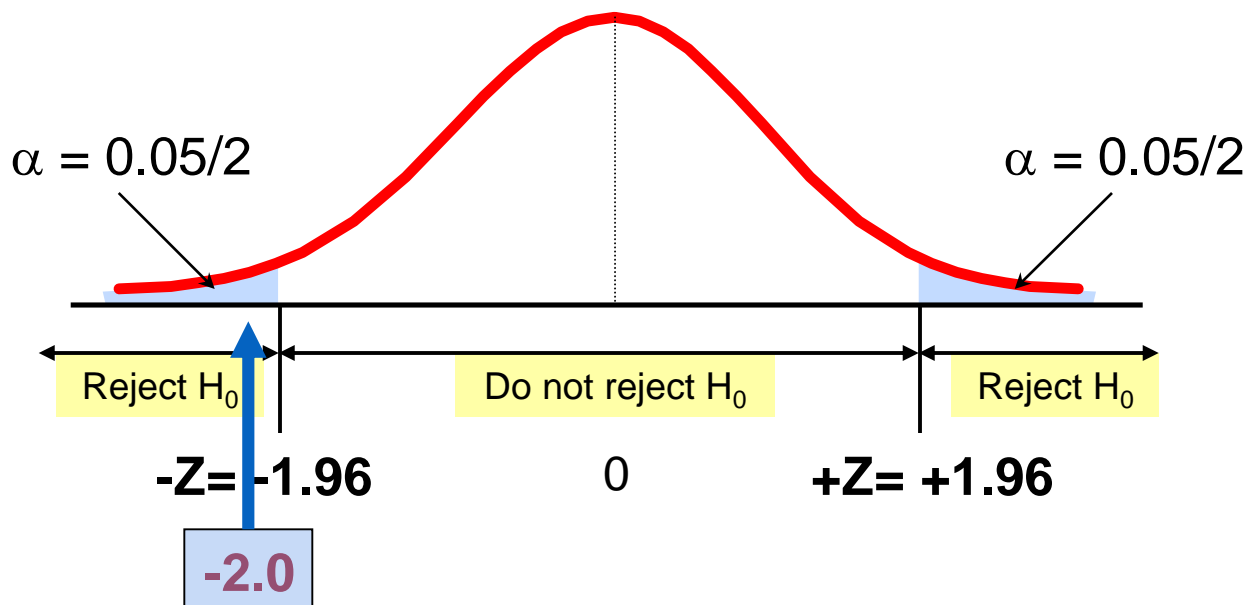
Reject H_0 if
 $Z < -1.96$ or
 $Z > 1.96$;
otherwise
do not
reject H_0

Here, $Z = -2.0 < -1.96$, so the
test statistic is in the rejection
region



Hypothesis Testing Example *(continued)*

6(continued). Reach a decision and interpret the result



Since $Z = -2.0 < -1.96$, we reject the null hypothesis and conclude that there is sufficient evidence that the mean number of TVs in US homes is not equal to 3



p-Value Approach to Testing

- p-value: Probability of obtaining a test statistic more extreme (\leq or \geq) than the observed sample value **given H_0 is true**
 - Also called observed level of significance
 - Smallest value of α for which H_0 can be rejected

p-Value Approach to Testing

(continued)

- Convert Sample Statistic (e.g., \bar{x}) to ~~\bar{x}~~ Test Statistic (e.g., Z statistic)
- Obtain the **p-value** from a table or computer
- Compare the **p-value** with α

If $p\text{-value} \geq \alpha$, do not reject H_0



p-Value Example

- Example:** How likely is it to see a sample mean of 2.84 (or something further from the mean, in either direction) if the true mean is $\mu = 3.0$?

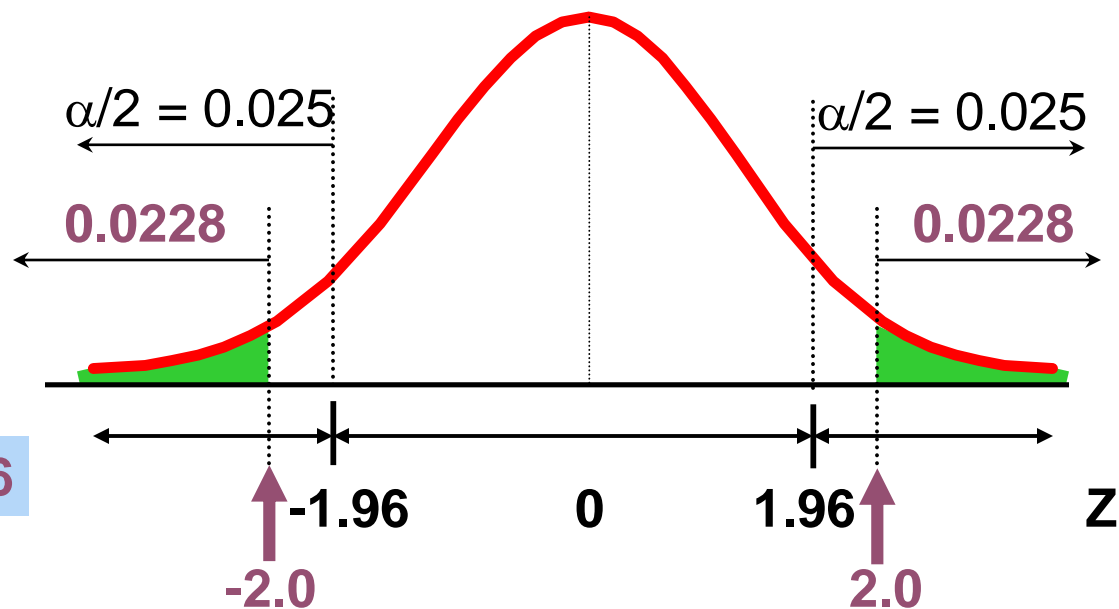
$\bar{X} = 2.84$ is translated to a Z score of $Z = -2.0$ ($\sigma = 0.8$ is assumed known), $n=100$

$$P(Z < -2.0) = 0.0228$$

$$P(Z > 2.0) = 0.0228$$

p-value

$$= 0.0228 + 0.0228 = 0.0456$$





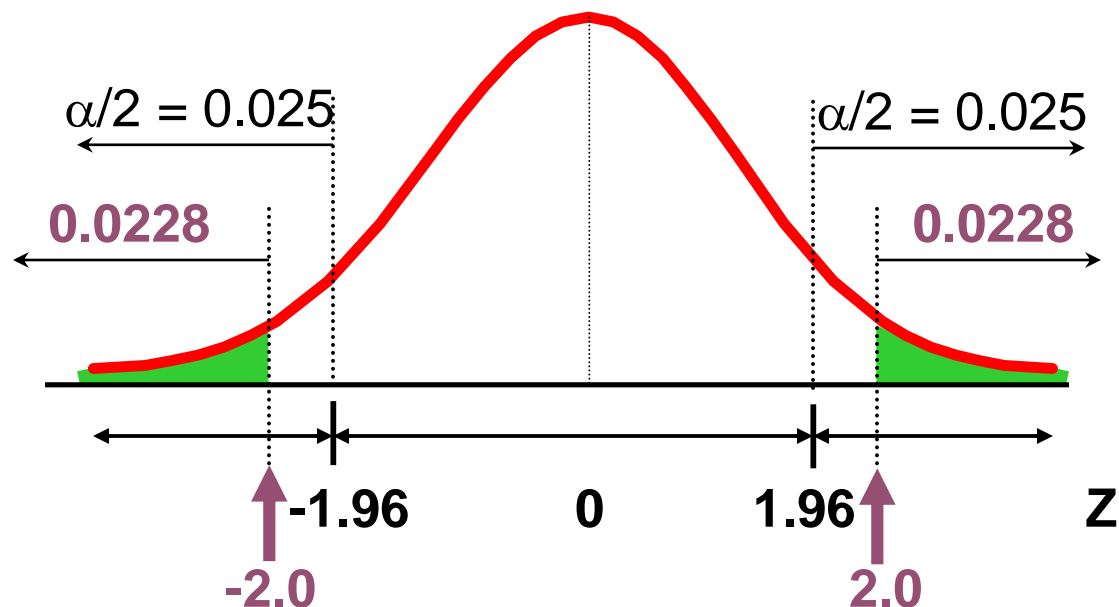
p-Value Example

(continued)

- Compare the p-value with α
 - If p-value $< \alpha$, reject H_0
 - If p-value $\geq \alpha$, do not reject H_0

Here: p-value = 0.0456
 $\alpha = 0.05$

Since $0.0456 < 0.05$,
we reject the null
hypothesis



Connection to Confidence Intervals

- For $\bar{X} = 2.84$, $\sigma = 0.8$ and $n = 100$, the 95% confidence interval is:

$$2.84 - (1.96) \frac{0.8}{\sqrt{100}} \quad \text{to} \quad 2.84 + (1.96) \frac{0.8}{\sqrt{100}}$$

$$2.6832 \leq \mu \leq 2.9968$$

- Since this interval does not contain the hypothesized mean (**3.0**), we reject the null hypothesis at $\alpha = 0.05$



One-Tail Tests

- In many cases, the alternative hypothesis focuses on a particular direction

$$H_0: \mu \geq 3$$

$$H_1: \mu < 3$$



This is a **lower**-tail test since the alternative hypothesis is focused on the lower tail below the mean of 3

$$H_0: \mu \leq 3$$

$$H_1: \mu > 3$$

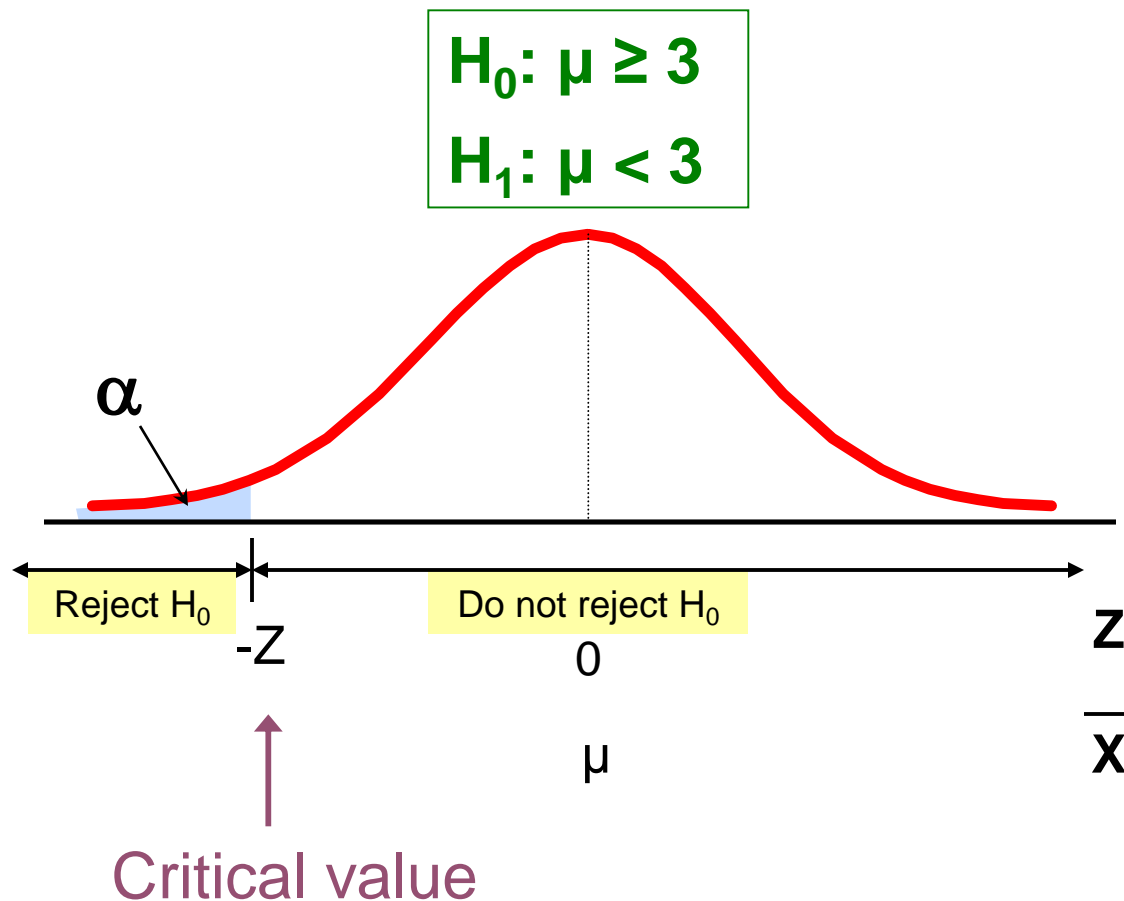


This is an **upper**-tail test since the alternative hypothesis is focused on the upper tail above the mean of 3



Lower-Tail Tests

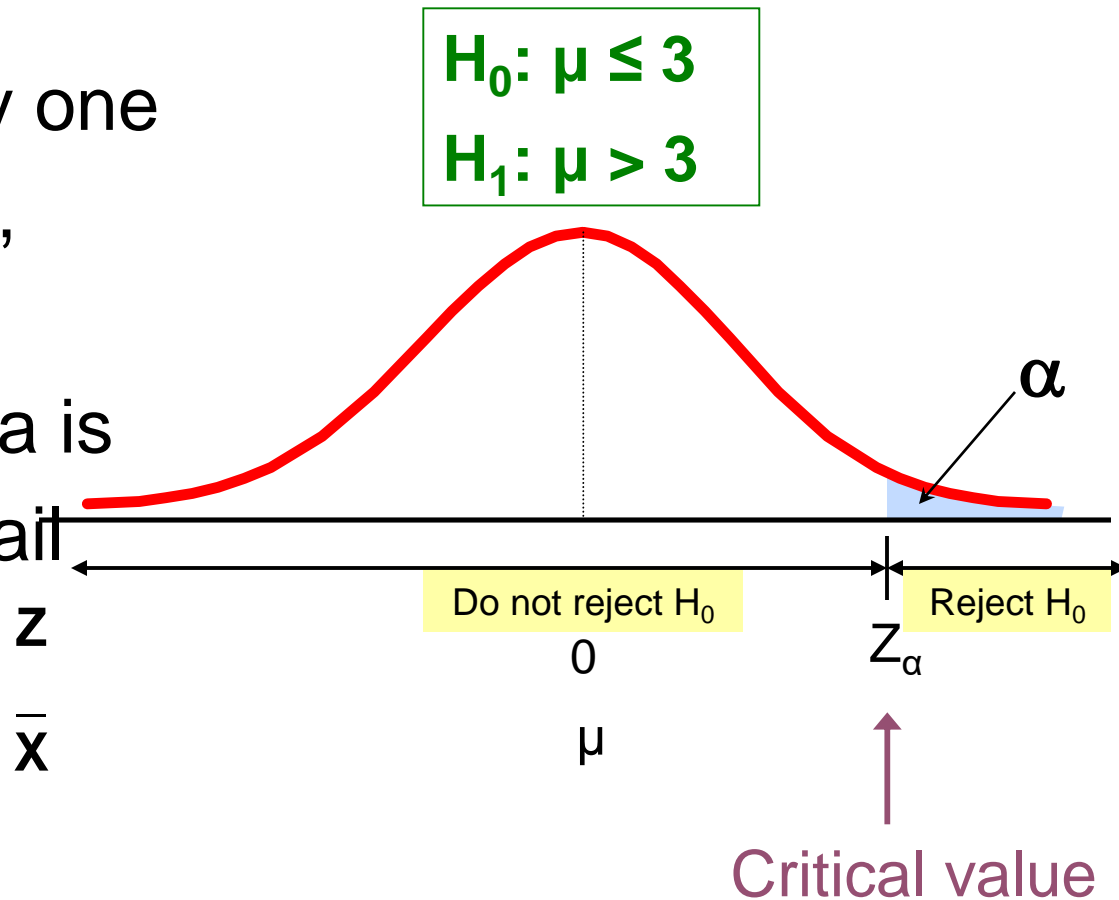
- There is only one critical value, since the rejection area is in only one tail





Upper-Tail Tests

- There is only one critical value, since the rejection area is in only one tail



Example: Upper-Tail Z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bills have increased, and now average over \$52 per month. The company wishes to test this claim. (Assume $\sigma = 10$ is known)

Form hypothesis test:

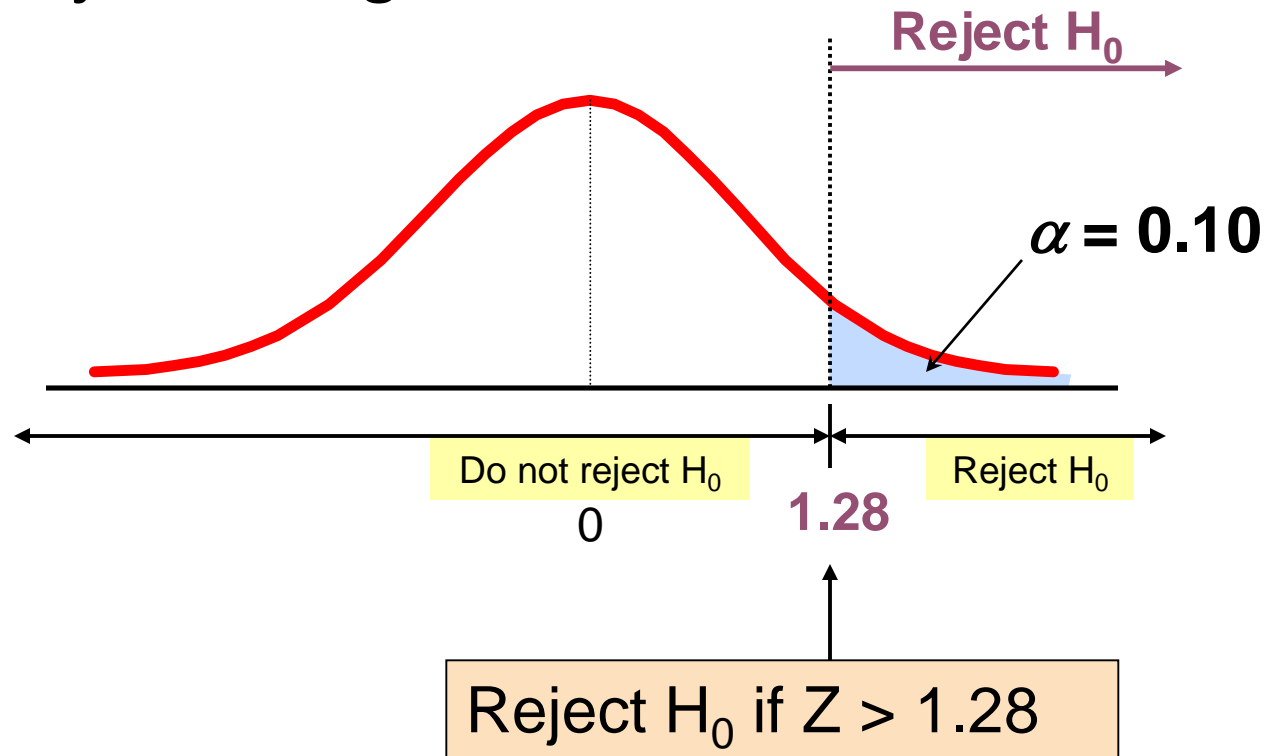
$H_0: \mu \leq 52$ the average is not over \$52 per month

$H_1: \mu > 52$ the average **is** greater than \$52 per month
(i.e., sufficient evidence exists to support the manager's claim)

Example: Find Rejection Region

(continued)

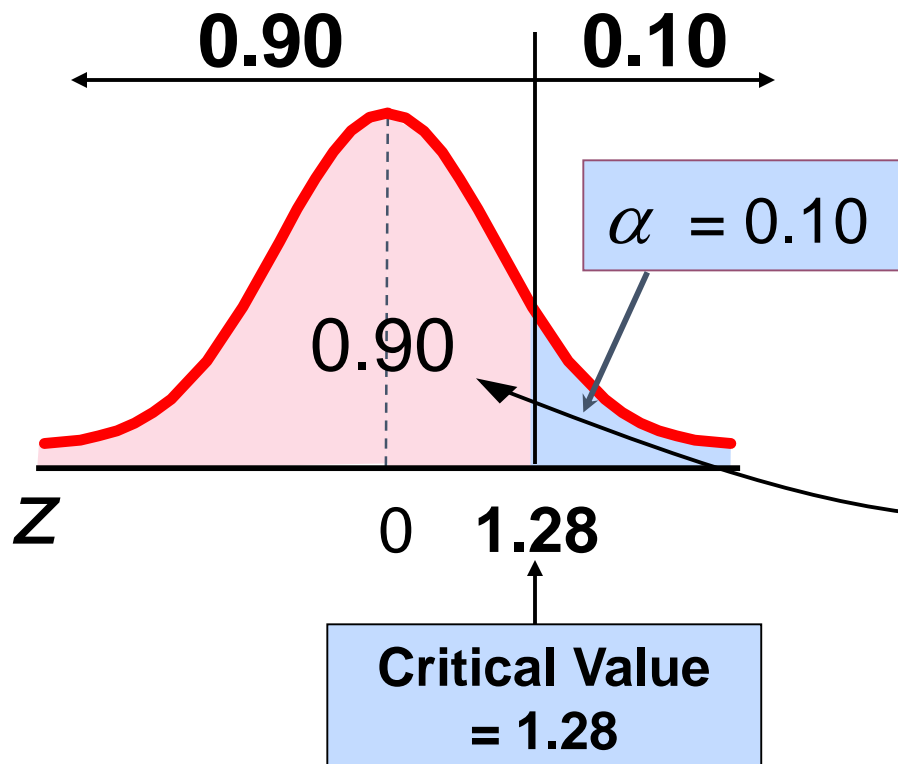
- Suppose that $\alpha = 0.10$ is chosen for this test
Find the rejection region:





Review: One-Tail Critical Value

What is Z given $\alpha = 0.10$?



Standardized Normal
Distribution Table (Portion)

Z	.07	.08	.09
1.1	.8790	.8810	.8830
1.2	.8980	.8997	.9015
1.3	.9147	.9162	.9177



Example: Test Statistic

(continued)

Obtain sample and compute the test statistic

Suppose a sample is taken with the following results: $n = 64$,
 $\bar{X} = 53.1$ ($\sigma = 10$ was assumed known)

- Then the test statistic is:

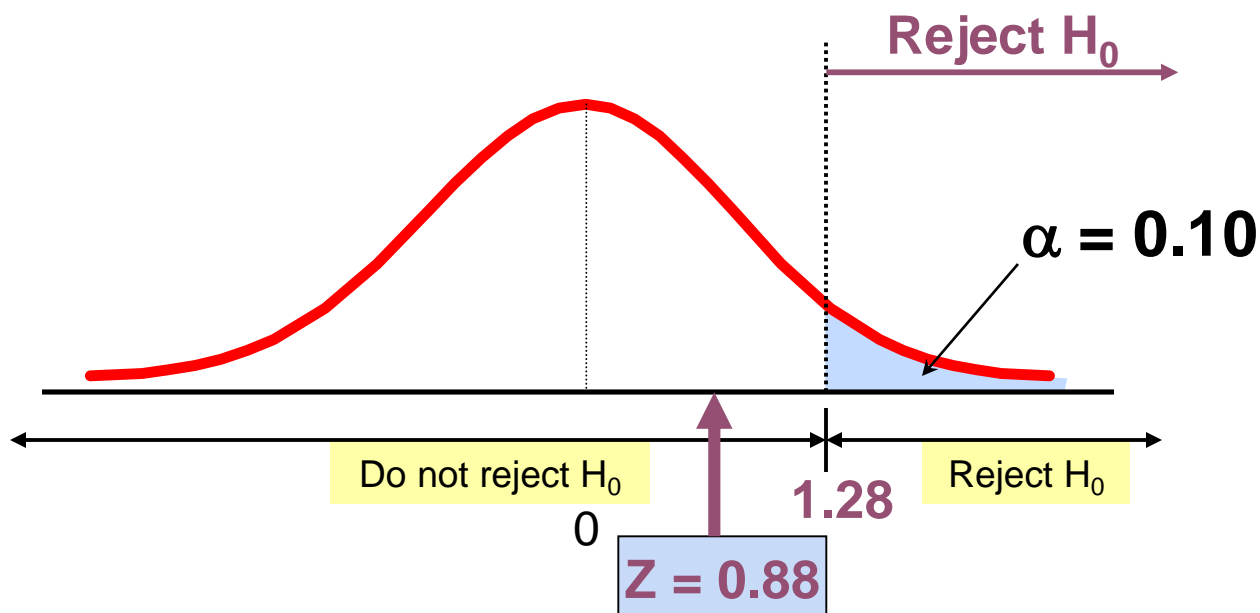
$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$



Example: Decision

(continued)

Reach a decision and interpret the result:



Do not reject H_0 since $Z = 0.88 \leq 1.28$

i.e.: there is not sufficient evidence that the mean bill is over \$52

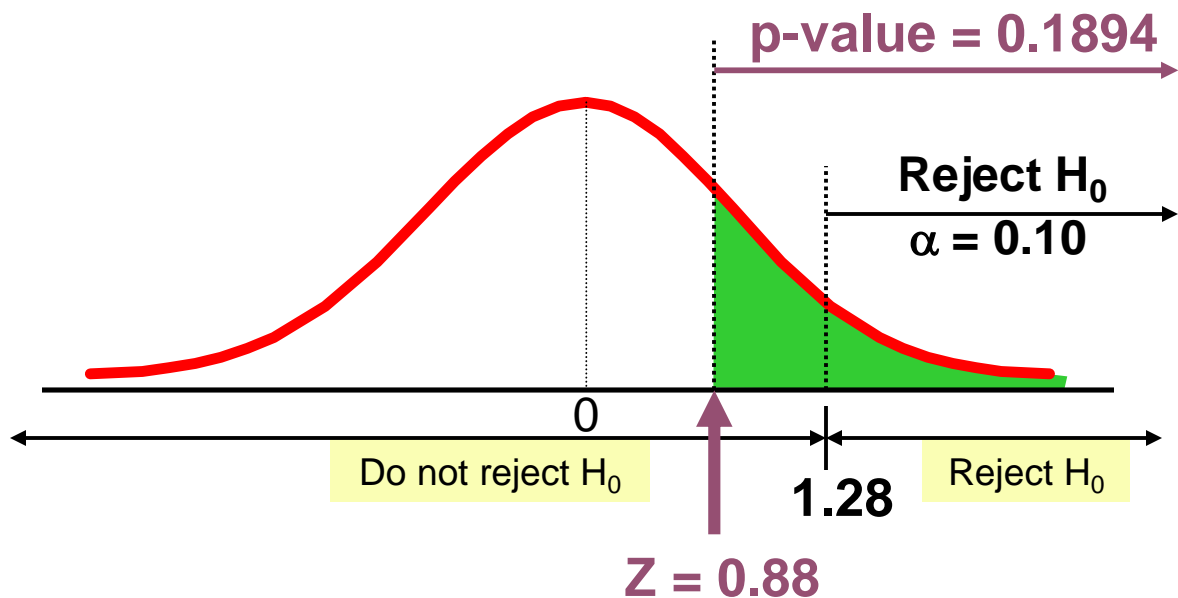


p -Value Solution

(continued)

Calculate the p -value and compare to α

(assuming that $\mu = 52.0$)



$$P(\bar{X} \geq 53.1)$$

$$= P\left(Z \geq \frac{53.1 - 52.0}{10/\sqrt{64}}\right)$$

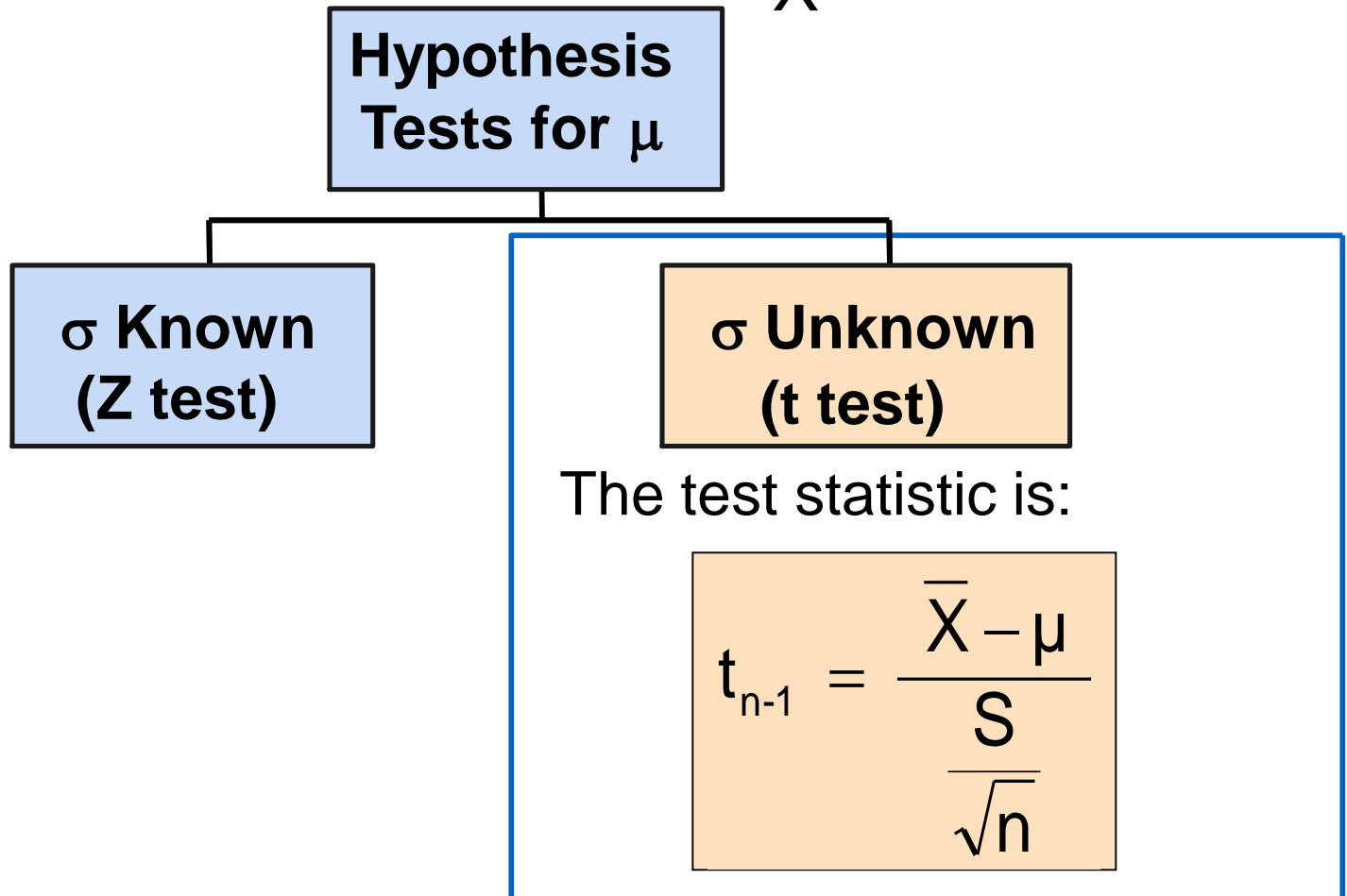
$$= P(Z \geq 0.88) = 1 - 0.8106$$

$$= 0.1894$$

Do not reject H_0 since $p\text{-value} = 0.1894 > \alpha = 0.10$

t Test of Hypothesis for the Mean (σ Unknown)

- Convert sample statistic () to a t test statistic





Example: Two-Tail Test (σ Unknown)

The average cost of a hotel room in Parlakhemundi is said to be Rs.168 per night. A random sample of 25 hotels resulted in $\bar{X} = \text{Rs.}172.50$ and $S = \text{Rs.}15.40$. Test at the $\alpha = 0.05$ level.

(Assume the population distribution is normal)



$$H_0: \mu = 168$$

$$H_1: \mu \neq 168$$

Example Solution: Two-Tail Test

$H_0: \mu = 168$
 $H_1: \mu \neq 168$

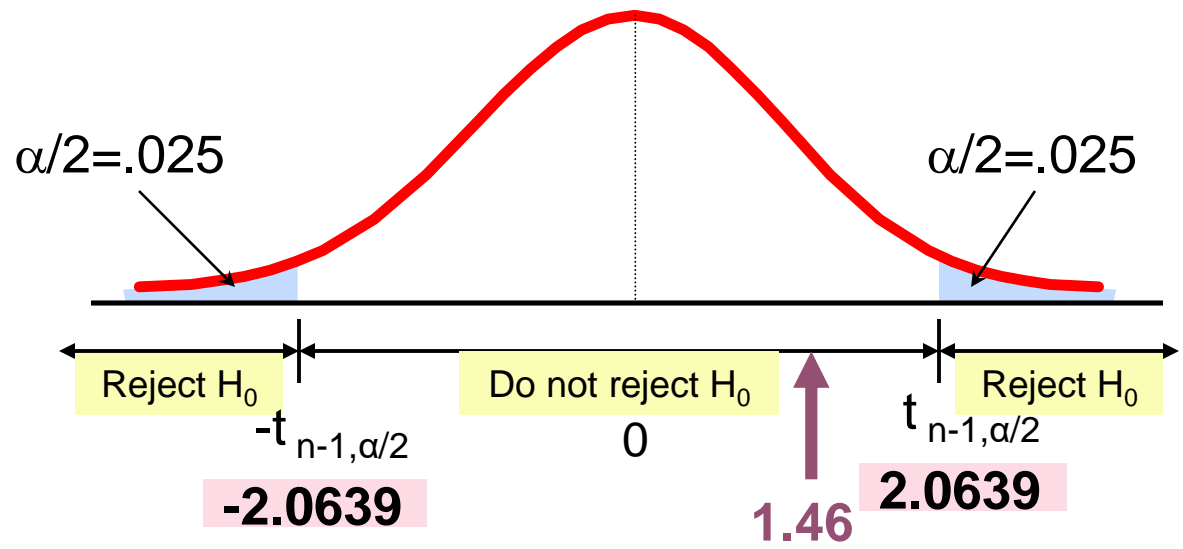
• $\alpha = 0.05$

• $n = 25$

• [Redacted]

• **Critical Value:**

[Redacted]



$$t_{n-1} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0 : not sufficient evidence that true mean cost is different than Rs.168



Connection to Confidence Intervals

- For $\bar{X} = 172.5$, $S = 15.40$ and $n = 25$, the 95% confidence interval is:

$$172.5 - (2.0639)15.4/\sqrt{25} \text{ to } 172.5 + (2.0639)15.4/\sqrt{25}$$

$$166.14 \leq \mu \leq 178.86$$

- Since this interval contains the Hypothesized mean (**168**), we do not reject the null hypothesis at $\alpha = 0.05$

Hypothesis Tests for Proportions

- Involves categorical variables
- Two possible outcomes
 - “Success” (possesses a certain characteristic)
 - “Failure” (does not possess that characteristic)
- Fraction or proportion of the population in the “success” category is denoted by p



Proportions

(continued)

- Sample proportion in the success category is denoted by p_s

- $$p_s = \frac{X}{n} = \frac{\text{number of successes in sample}}{\text{sample size}}$$

- When $np \geq 10$ and $n(1-p) \geq 10$, the distribution of p_s can be approximated by a normal distribution with mean and standard deviation

-

$$\mu_{p_s} = p$$

$$\sigma_{p_s} = \sqrt{\frac{p(1-p)}{n}}$$

Potential Pitfalls

- Use randomly collected data to reduce selection biases
- Choose the level of significance, α , and the type of test (one-tail or two-tail) before data collection
- Do not employ “data snooping” to choose between one-tail and two-tail test, or to determine the level of significance
- Do not practice “data cleansing” to hide observations that do not support a stated hypothesis
- Report all pertinent findings