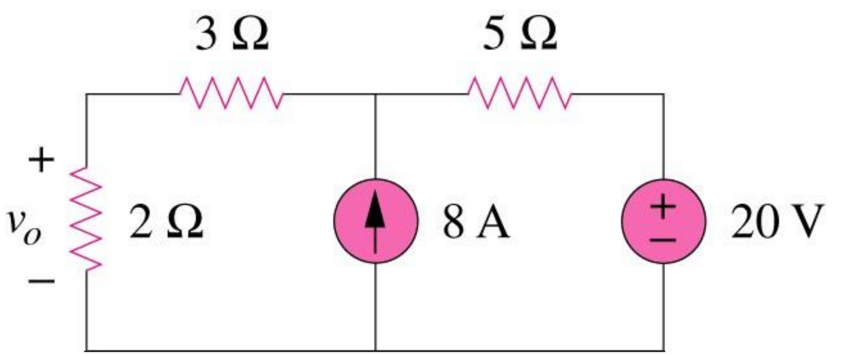
Chapter 5: Circuit Theorems

1. Motivation
2. Source Transformation
3. Superposition (2.1 Linearity Property)
4. Thevenin's Theorem
5. Norton's Theorem
6. Maximum Power Transfer
7. Summary

5.1 Motivation

If you are given the following circuit, are there any other alternative(s) to determine the voltage across 2Ω resistor?



In Chapter 4, a circuit is analyzed without tampering with its original configuration.

What are they? And how? Can we work it out by inspection?

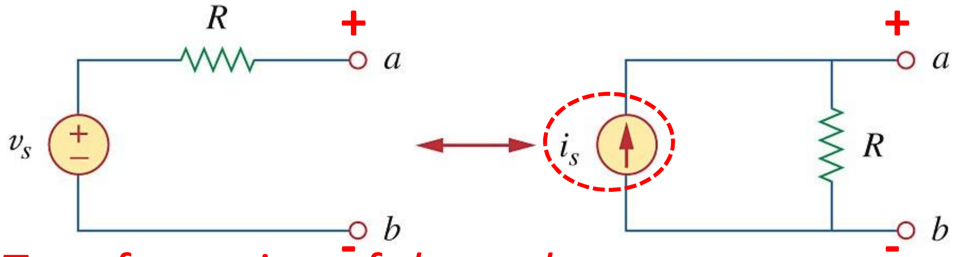
In Chapter 5, some theorems have been developed to simplify circuit analysis such as **Thevenin's and Norton's theorems**. The theorems are applicable to ***linear circuits***.

Discussion: ***Source Transformation, Linearity,*** & ***superposition***.

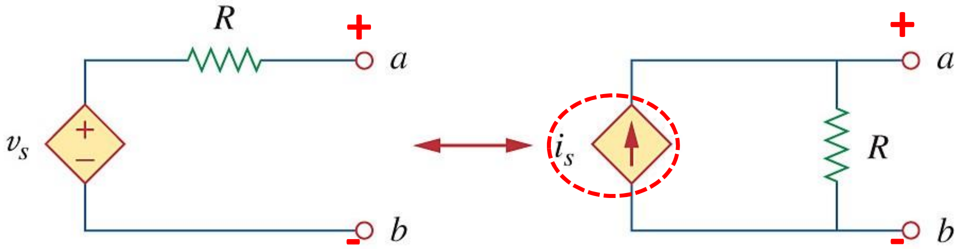
5.2 Source Transformation (1)

- Like series-parallel combination and wye-delta transformation, source transformation is another tool for simplifying circuits.
- An **equivalent circuit** is one whose v-i characteristics are identical with the original circuit.
- A **source transformation** is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, and vice versa.

• Transformation of independent sources



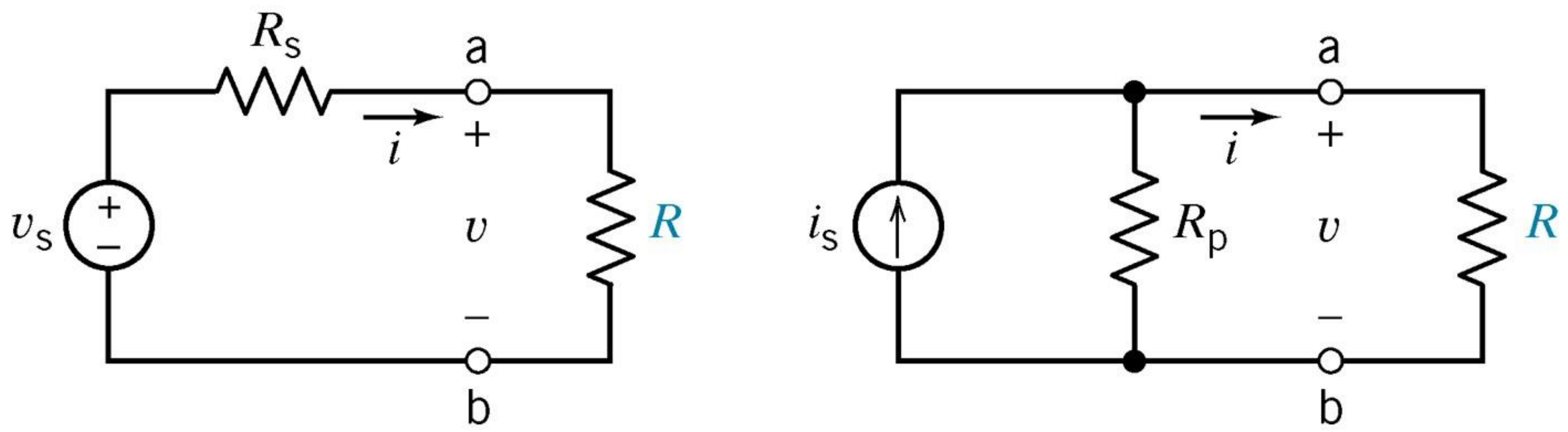
• Transformation of *dependent* sources



✓ The arrow of the current source is directed toward the positive terminal of the voltage source.

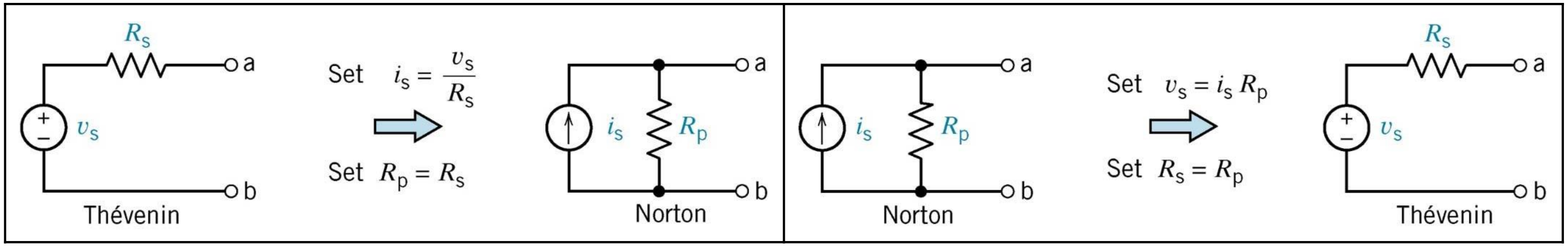
✓ The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source.

5.2 Source Transformation (1)



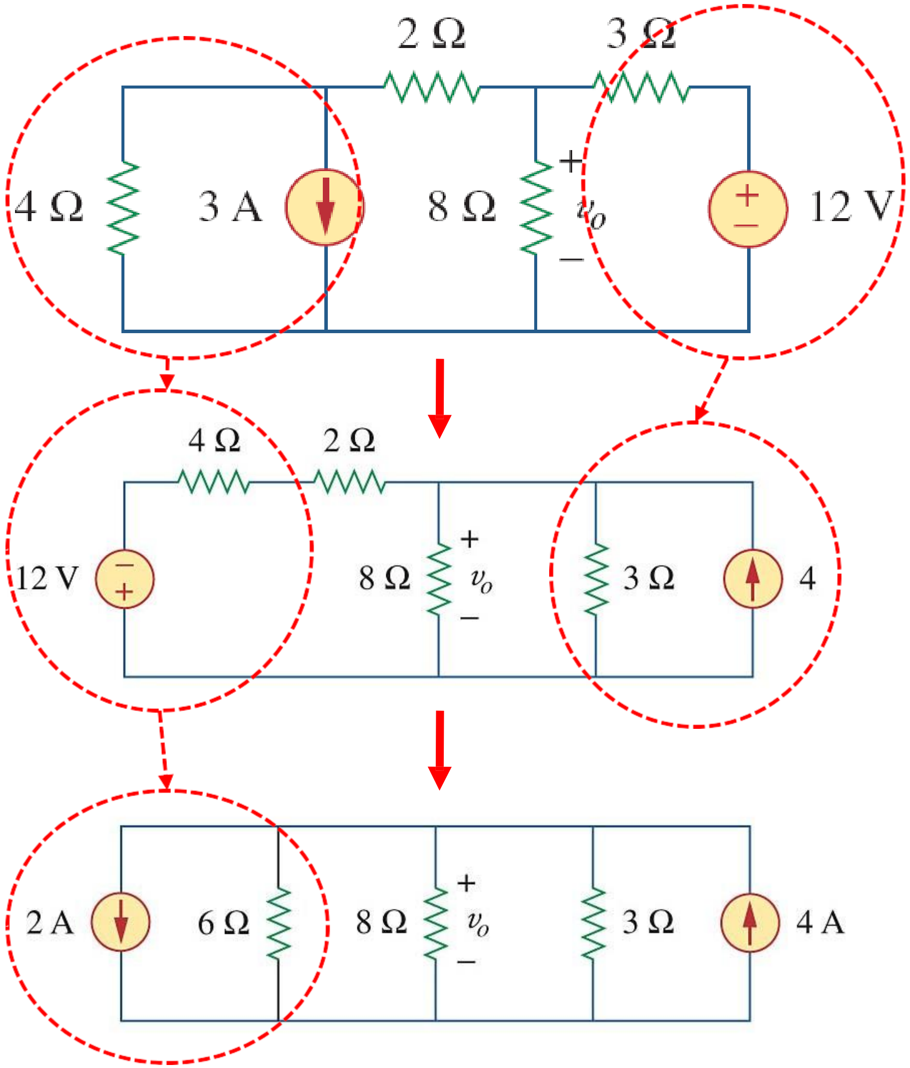
A voltage source v_s connected in series with a resistor R_s and a current source i_s is connected in parallel with a resistor R_p are equivalent circuits provided that

$$\boxed{R_p = R_s \ \& \ v_s = R_s i_s}$$



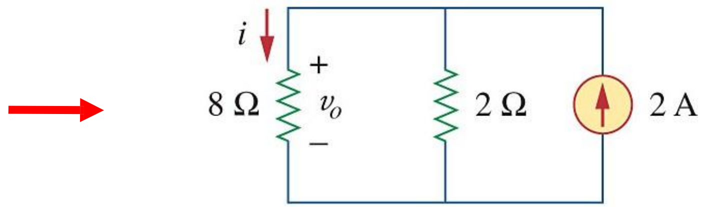
5.2 Source Transformation (3)

Example: Find v_o in the circuit using source transformation.



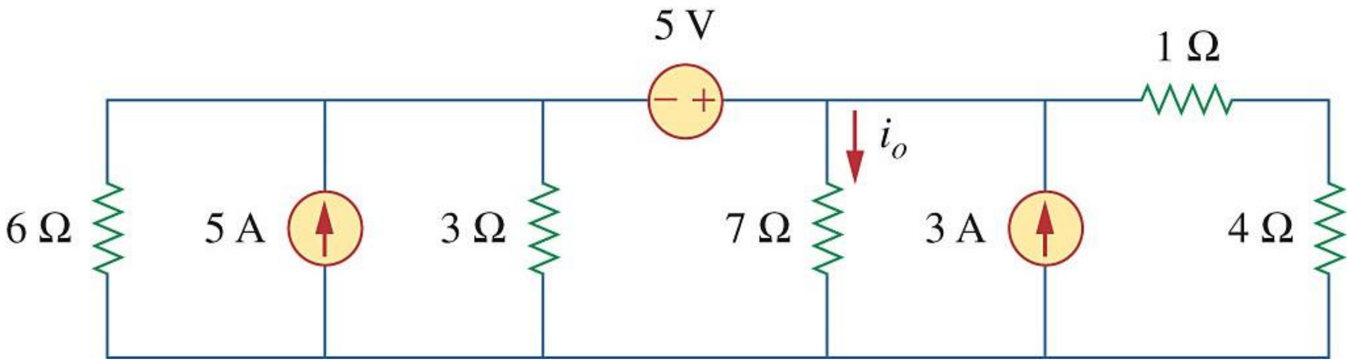
$$i = \frac{2}{2+8}(2) = 0.4 \text{ A}$$

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

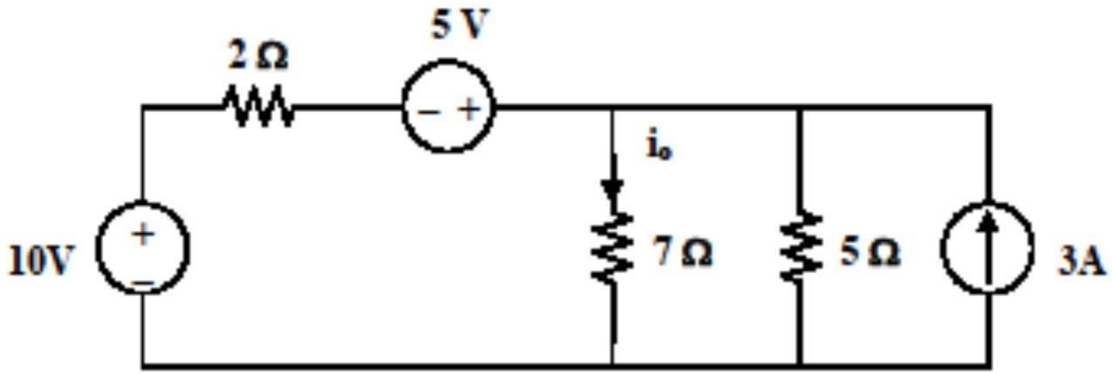


5.2 Source Transformation (4)

Example: Find i_o in the circuit using source transformation.



Combining the 6-Ω and 3-Ω resistors in parallel gives 2Ω. Adding the 1-Ω and 4-Ω resistors in series gives $1 + 4 = 5\Omega$. Transforming the left current source in parallel with the 2-Ω resistor gives the equivalent circuit.



*Refer to in-class illustration, textbook, answer $i_0 = 1.78\text{ A}$

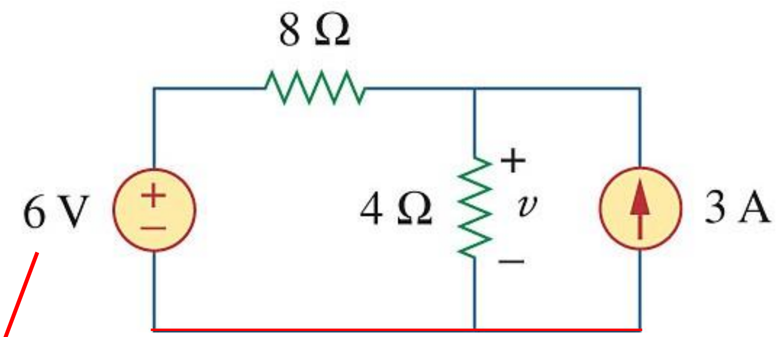
5.3 Superposition Theorem (1)

- **Superposition** states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to *EACH independent source acting alone*.
- The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.
- ***Steps to Apply Superposition Principle***:
  1. Turn off all indep. sources except one source. Find the output (v or i) due to that active source using techniques in Chapters 2 & 3.
  2. Repeat Step 1 for each of the other indep. sources.
  3. Find Total contribution by adding all contributions from indep. sources.

Note: In Step 1, this implies that we replace every **voltage source by $\mathbf{0\ V}$** (or a **short circuit**), and every **current source by $\mathbf{0\ A}$** (or an **open circuit**). Dependent sources are left intact because they are controlled by others.

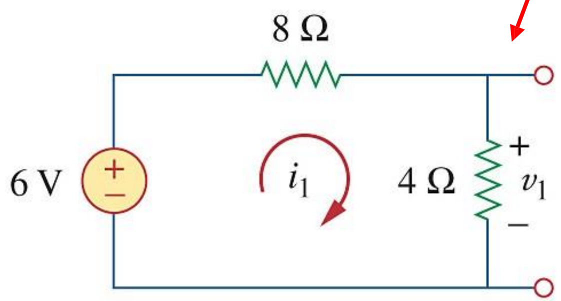
5.3 Superposition Theorem (2)

Example: Use the superposition theorem to find *v* in the circuit.



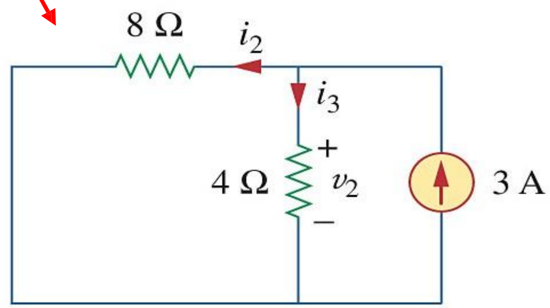
3A is discarded by open circuit

6V is discarded by short circuit



$$\boxed{v = v_1 + v_2}$$

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$



$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5 \text{ A}$$

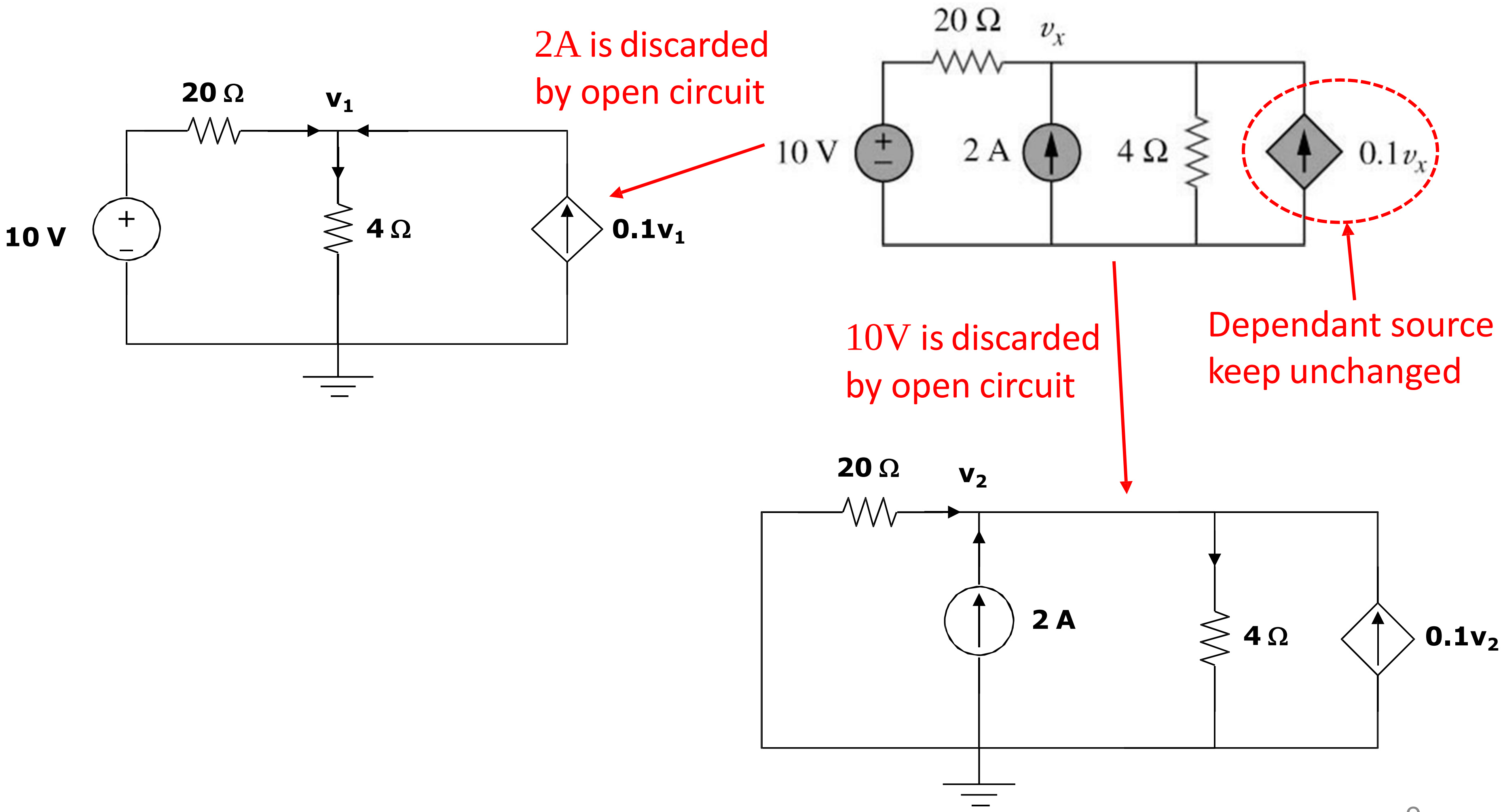
$$v_1 = 4i_1 = 2 \text{ V}$$

$$i_3 = \frac{8}{4 + 8}(3) = 2 \text{ A}$$

$$v_2 = 4i_3 = 8 \text{ V}$$

5.3 Superposition Theorem (3)

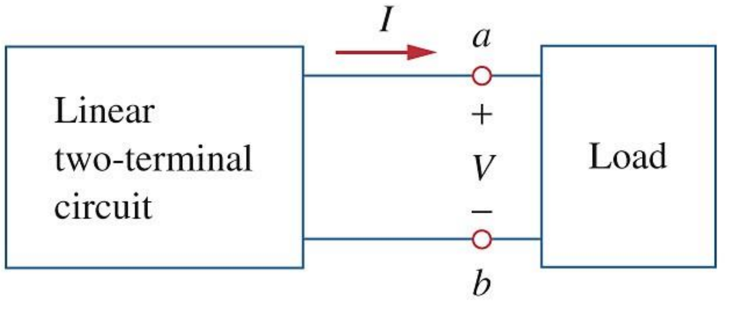
Example: Use superposition to find v_x in the circuit.



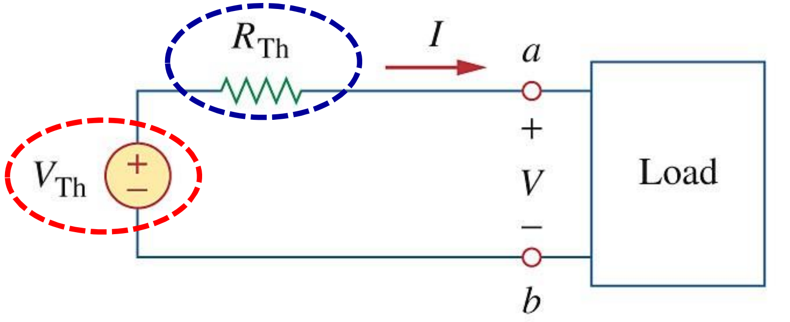
5.4 Thevenin's Theorem (1)

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where

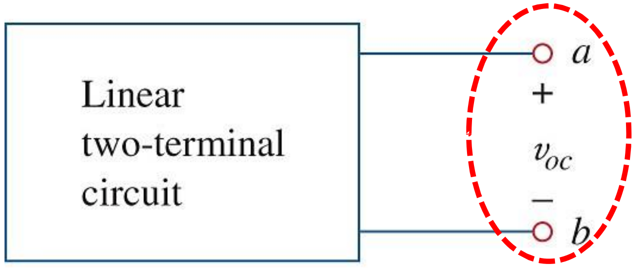
- ✓ V_{Th} is the open-circuit voltage at the terminals.
- ✓ R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



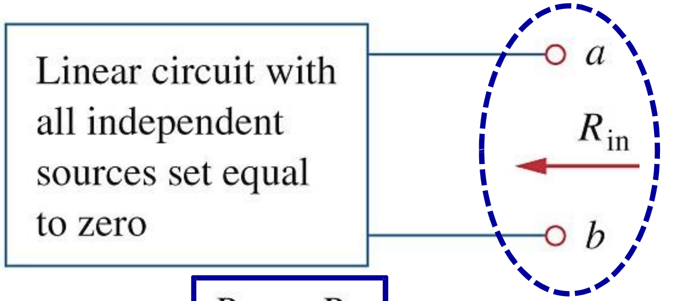
(a)



(b)



$\boxed{V_{Th} = v_{oc}}$



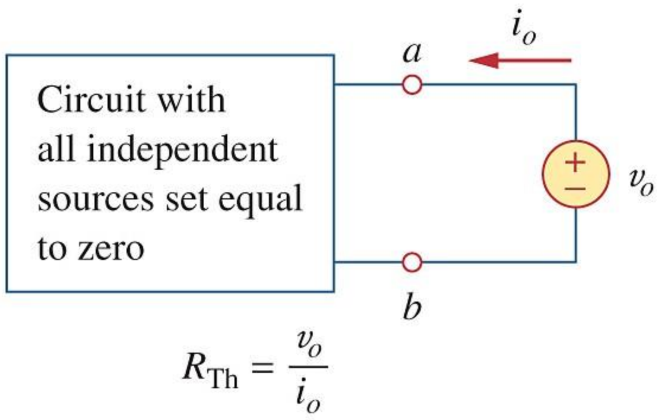
$\boxed{R_{Th} = R_{in}}$

5.4 Thevenin's Theorem (2)

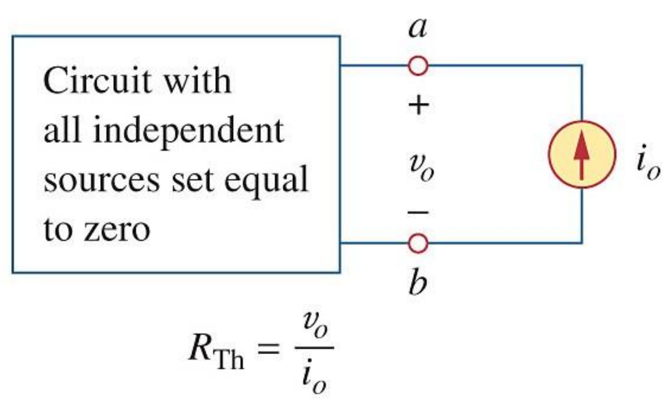
To find R_{Th}:

- ✓ ***Case 1***: If the network has no dependent sources, we turn off all indep. Source. R_{Th} is the input resistance of the network looking btw terminals a & b.
- ✓ ***Case 2***: If the network has depend. Sources. Depend. sources are not to be turned off because they are controlled by circuit variables. (a) Apply v_o at a & b and determine the resulting i_o. Then $R_{Th} = v_o/i_o$. Alternatively, (b) insert i_o at a & b and determine v_o. Again $R_{Th} = v_o/i_o$.

(a)

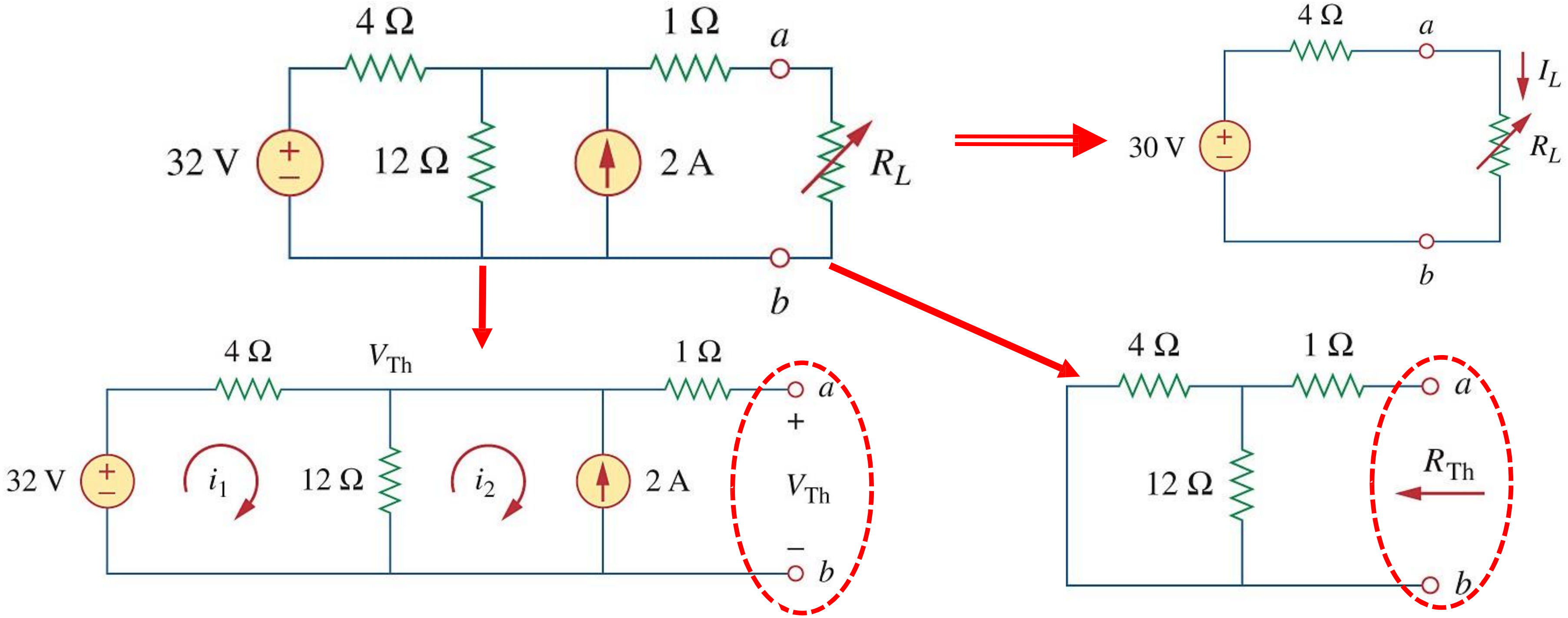


(b)



5.4 Thevenin’s Theorem (3)

Example: Find the *Thevenin equivalent circuit* at the terminals a & b.



Indep. voltage source as a short circuit & the current source as an open circuit.

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2 \text{ A}$$

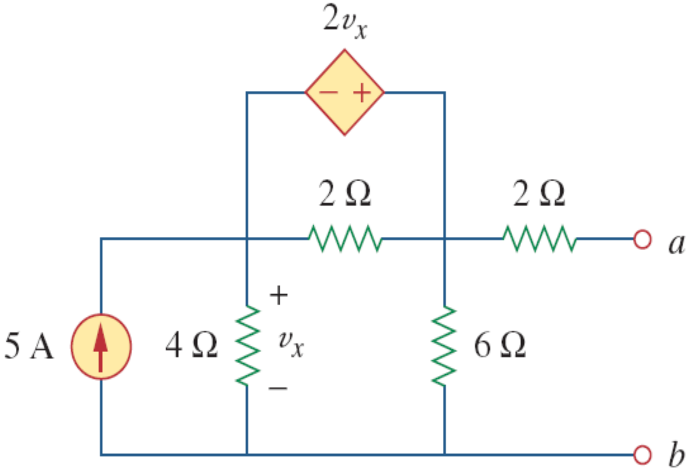
for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30 \text{ V}$$

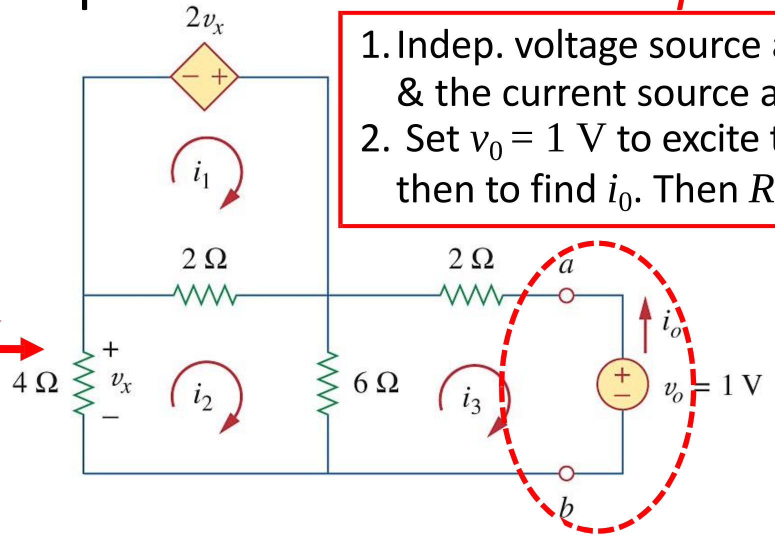
$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

5.4 Thevenin's Theorem (4)

Example: Find the Thevenin equivalent circuit with *dep. source*.



R_{Th}



1. Indep. voltage source as a short circuit & the current source as an open circuit.
2. Set $v_0 = 1$ V to excite the circuit, and then to find i_0. Then $R_{\text{Th}} = v_0 / i_0$.

$$i_o = -i_3 = 1/6\ \text{A}.$$

$$\boxed{R_{\text{Th}} = \frac{1\ \text{V}}{i_o} = 6\ \Omega}$$

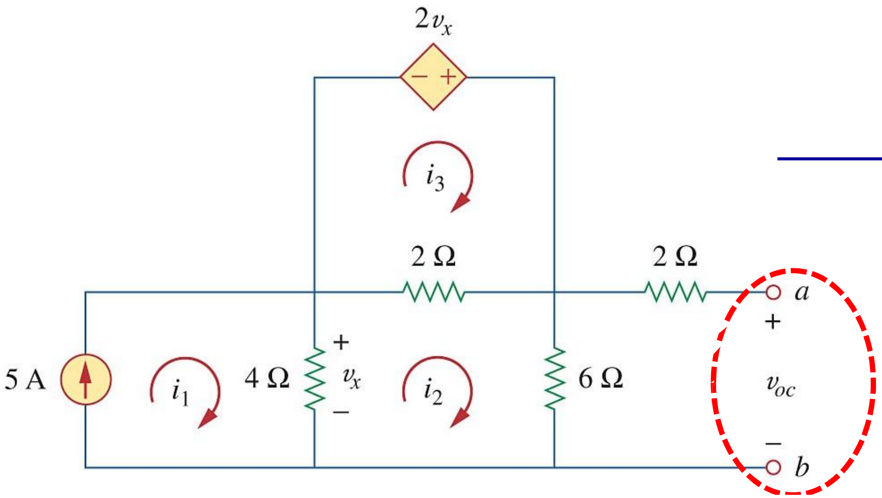
$$-2v_x + 2(i_1 - i_2) = 0 \quad \text{or} \quad v_x = i_1 - i_2$$

$$\text{But } -4i_2 = v_x = i_1 - i_2; \text{ hence,} \quad i_1 = -3i_2$$

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0 \qquad i_3 = -\frac{1}{6}\ \text{A}$$

V_{Th}



$$i_1 = 5$$

$$-2v_x + 2(i_3 - i_2) = 0 \quad \Rightarrow \quad v_x = i_3 - i_2$$

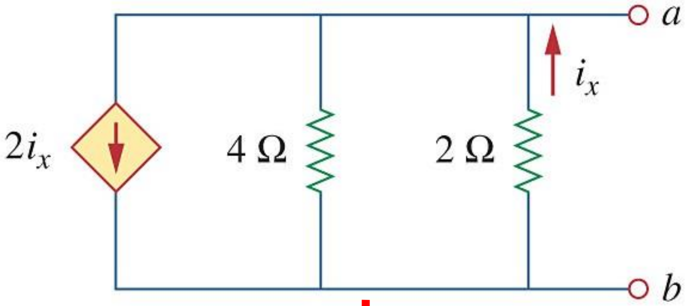
$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

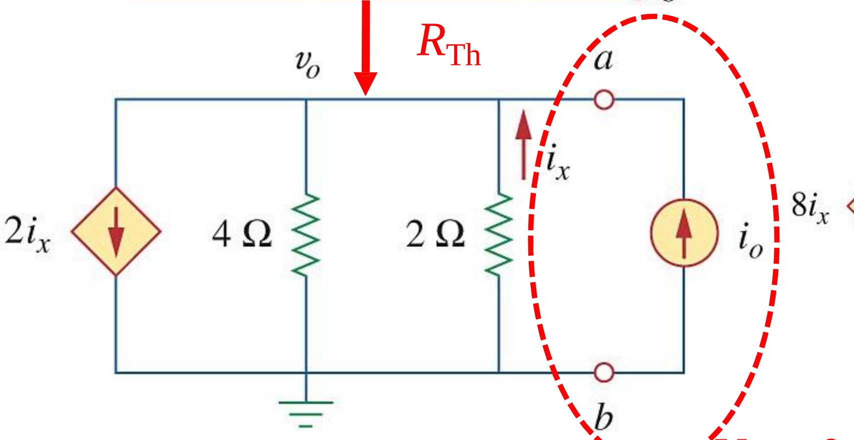
$$\text{But } 4(i_1 - i_2) = v_x. \quad \text{leads to } i_2 = 10/3.$$

$$\boxed{V_{\text{Th}} = v_{oc} = 6i_2 = 20\ \text{V}}$$

5.4 Thevenin's Theorem (5)

Example: Find the Thevenin equivalent circuit with *only dep. source*.





$$i_x = (0 - v_o)/2 = -v_o/2$$

$$2(-v_o/2) + (v_o - 0)/4 + (v_o - 0)/2 + (-1) = 0$$

$$= (-1 + \tfrac{1}{4} + \tfrac{1}{2})v_o \; - 1 \qquad \text{or} \qquad v_o = -4 \text{ V}$$

$$v_o = 1 \times R_{Th}, \text{ then } R_{Th} = v_o/1 = \mathbf{-4\ \Omega}.$$

$$(4 + 2 - 8)i_1 + (-2 + 8)i_2 = 0$$

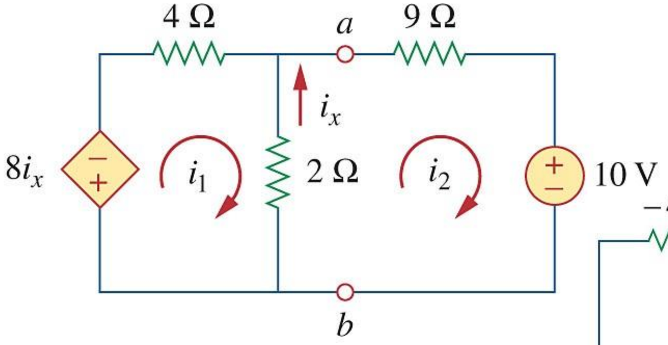
or

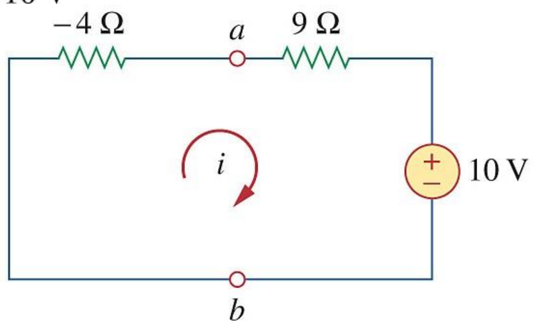
$$-2i_1 + 6i_2 = 0 \qquad \text{or} \qquad i_1 = 3i_2$$

$$-2i_1 + 11i_2 = -10$$

Substituting the first equation into the second gives

$$-6i_2 + 11i_2 = -10 \qquad \text{or} \qquad i_2 = -10/5 = \mathbf{-2\ A}$$





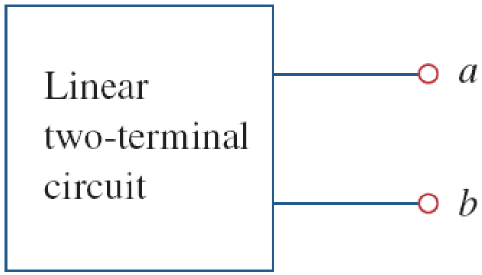
Using the Thevenin equivalent is quite easy since we have only one loop, as shown in Fig.

$$-4i + 9i + 10 = 0 \qquad \text{or} \qquad i = -10/5 = \mathbf{-2\ A}$$

5.5 Norton's Theorem (1)

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a current source I_N in parallel with a resistor R_N,

(a)



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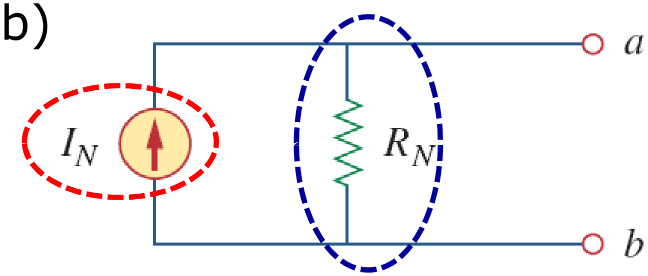
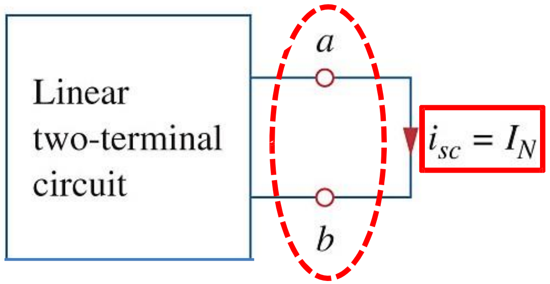
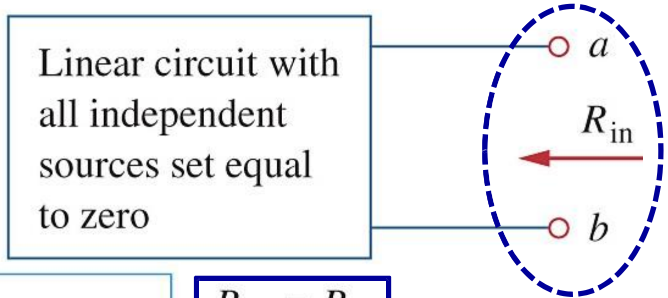


where

- ✓ I_N is the short-circuit current through the terminals.
- ✓ R_N is the input or equivalent resistance at the terminals when the indepen. sources are turned off.





$R_N = R_{Th}$

$R_{Th} = R_{in}$

5.5 Norton's Theorem (2)

The Thevenin and Norton equivalent circuits are related by a *source transformation*.

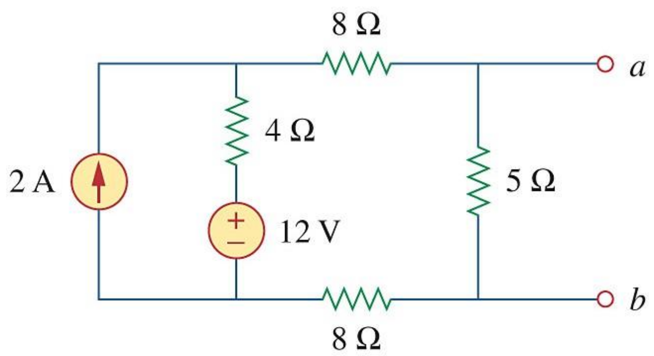
$$I_N = \frac{V_{Th}}{R_{Th}}$$

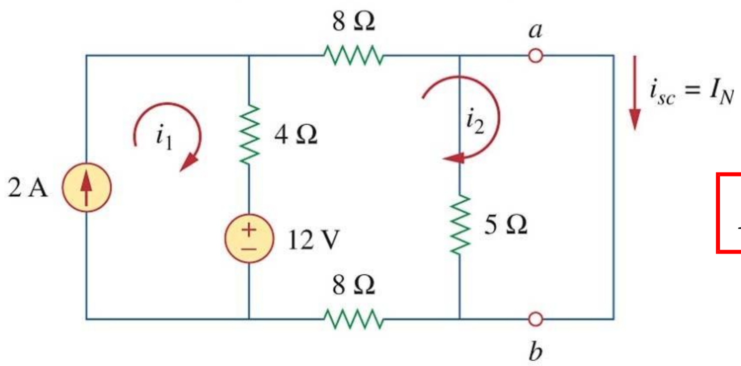
$$V_{TH} = v_{oc}$$

$$I_N = i_{sc}$$

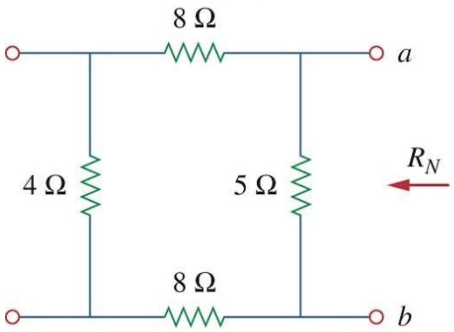
$$R_{TH} = v_{oc} / i_{sc} = R_N$$

Example: Find the *Norton equivalent circuit* at the terminals a & b.



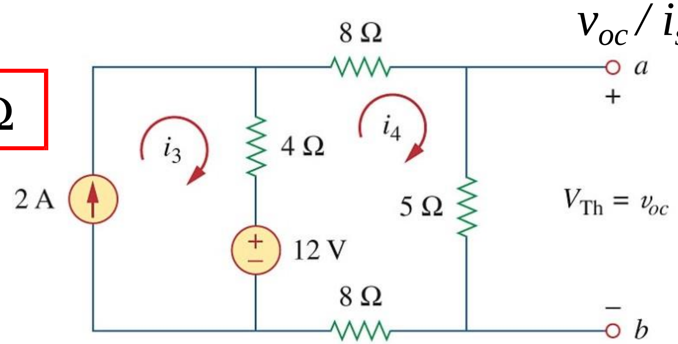


$$I_N = i_{sc} = 1\text{A}$$



$$R_{TH} = R_N = 4\,\Omega$$

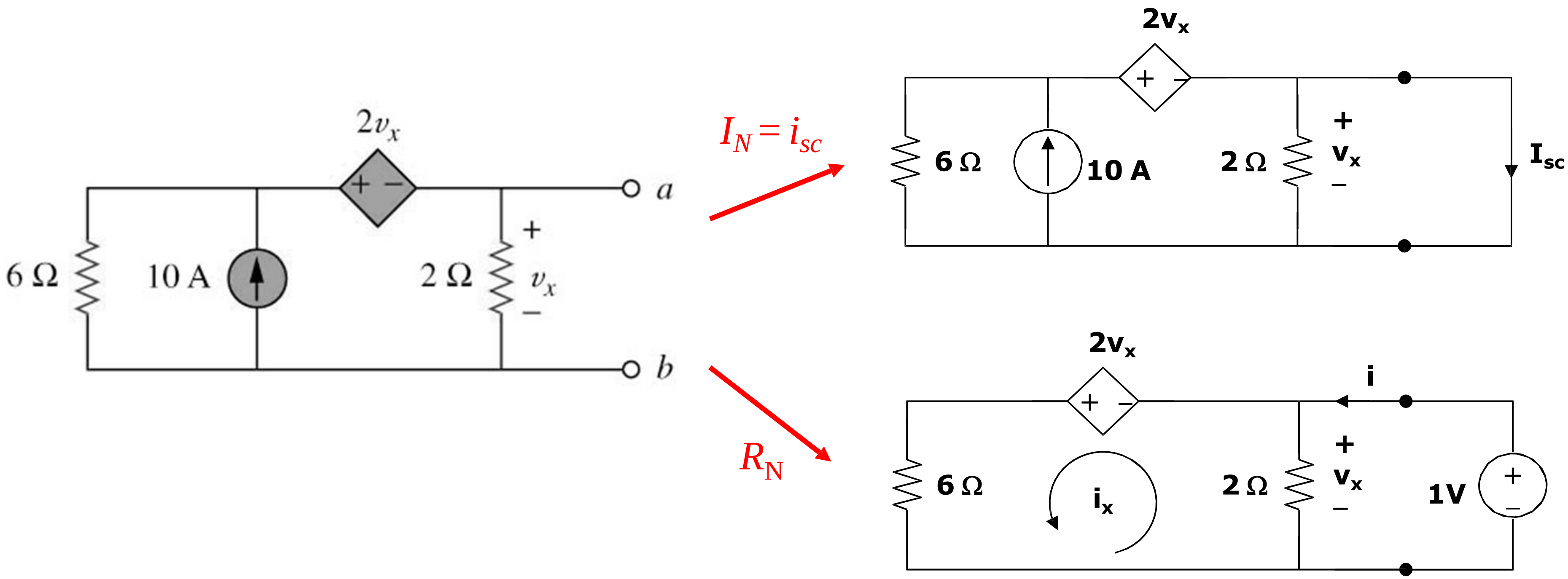
$$v_{oc} / i_{sc} = 4\,\Omega$$



$$V_{TH} = v_{oc} = 4\text{V}$$

5.5 Norton's Theorem (3)

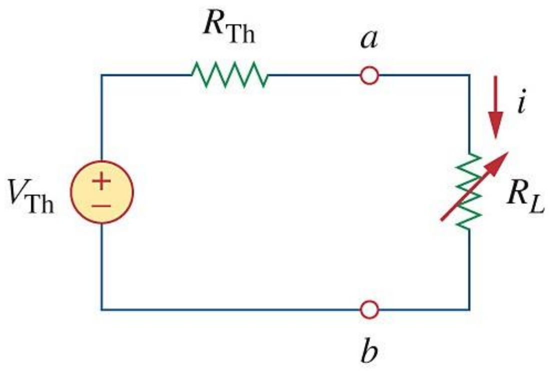
Example: Find the Norton equivalent circuit with *dep. source*.



5.6 Maximum Power Transfer (1)

- There are applications where it is desirable to maximize the power delivered to a load. Also, power utility systems are designed to transport the power to the load with the greatest efficiency by reducing the losses on the power lines.

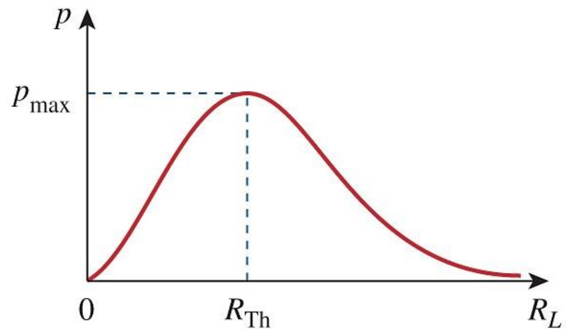
- If the entire circuit is replaced by its Thevenin equivalent except for the load, the power delivered to the load is:



$$\boxed{P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L}$$

- **Maximum power** is transferred to the load resistance equals the Thevenin resistance as seen from the load.

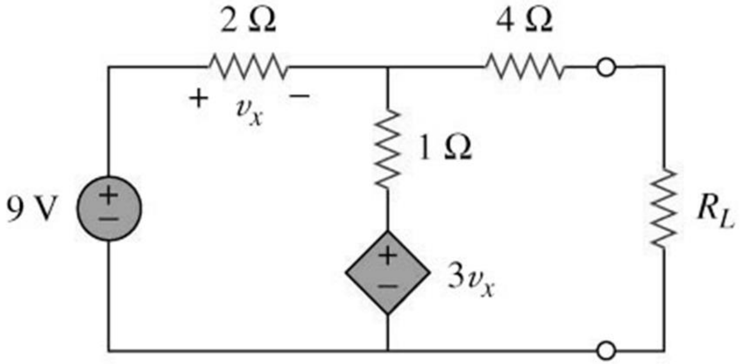
$$\boxed{R_L = R_{Th} \Rightarrow P_{max} = \frac{V_{Th}^2}{4R_L}}$$



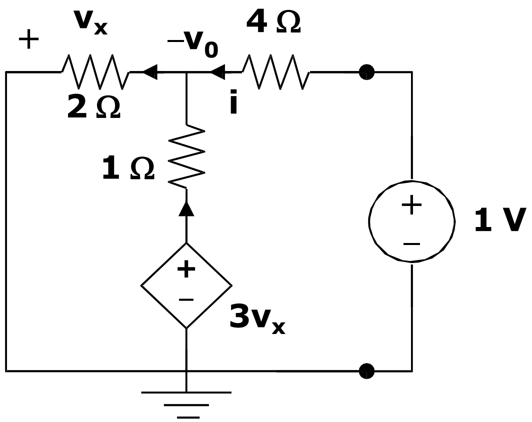
The power transfer profile with different R_L

5.6 Maximum Power Transfer (2)

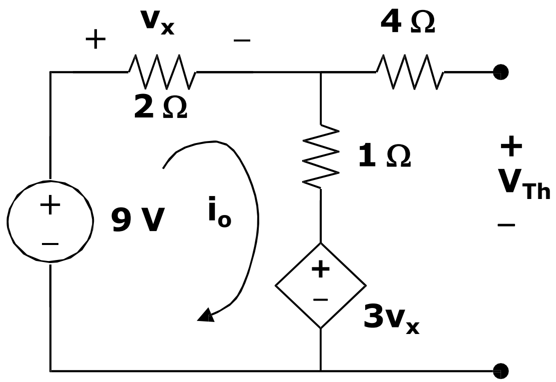
Example: Determine the value of R_L that will draw the maximum power. Calculate the maximum power.



$$\boxed{R_L = R_{\text{Th}} \quad \Rightarrow \quad P_{\max} = \frac{V_{\text{Th}}^{\ 2}}{4R_L}}$$



(a)



(b)

Fig. (a)

=> To determine R_{Th}

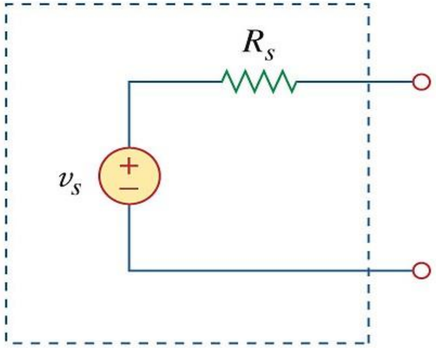
Fig. (b)

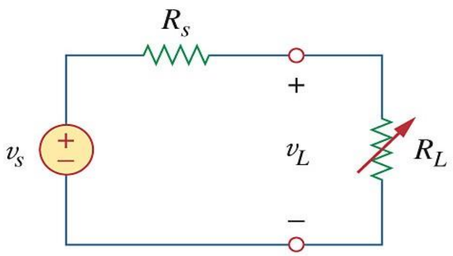
=> To determine V_{Th}

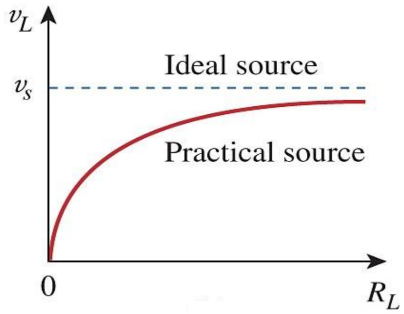
*Refer to in-class illustration, textbook, R_L = 4.22W, P_m = 2.901W

5.6 Maximum Power Transfer (3)

- Practical voltage

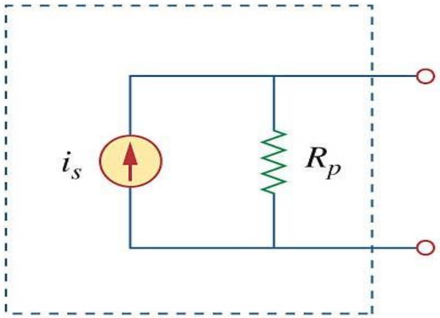


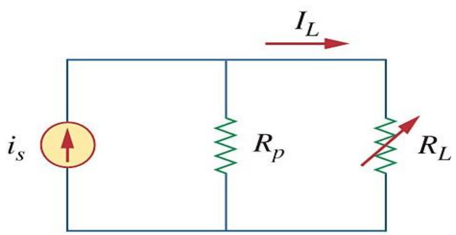


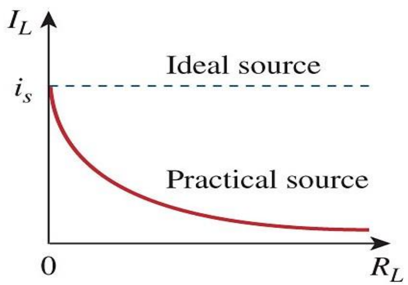


$$v_L = \frac{R_L}{R_s + R_L} v_s$$

- Practical current

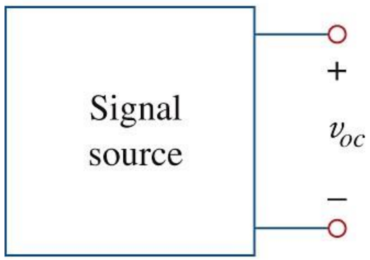


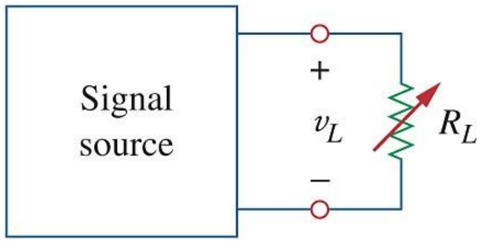




$$i_L = \frac{R_p}{R_p + R_L} i_s$$

- To measure v_s and R_s:





5.7 Summary

Table 5.11-1 Source Transformations

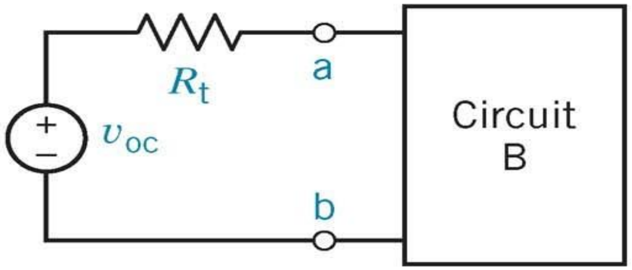
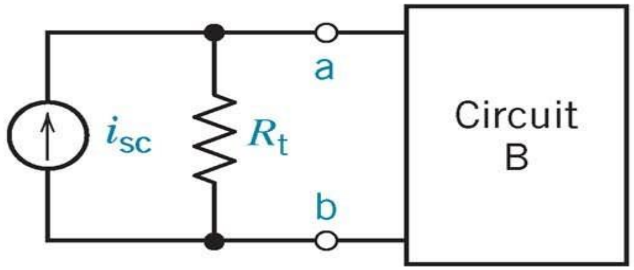
THÉVENIN CIRCUIT	NORTON CIRCUIT
	

Table 5.11-2 Thévenin and Norton Equivalent Circuits

ORIGINAL CIRCUIT	THÉVENIN CIRCUIT	NORTON EQUIVALENT CIRCUIT