

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, ... 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, ... 7	No	No
Hexa- decimal	16	0, 1, ... 9, A, B, ... F	No	No

Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa- decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Quantities/Counting (2 of 3)

Decimal	Binary	Octal	Hexa- decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

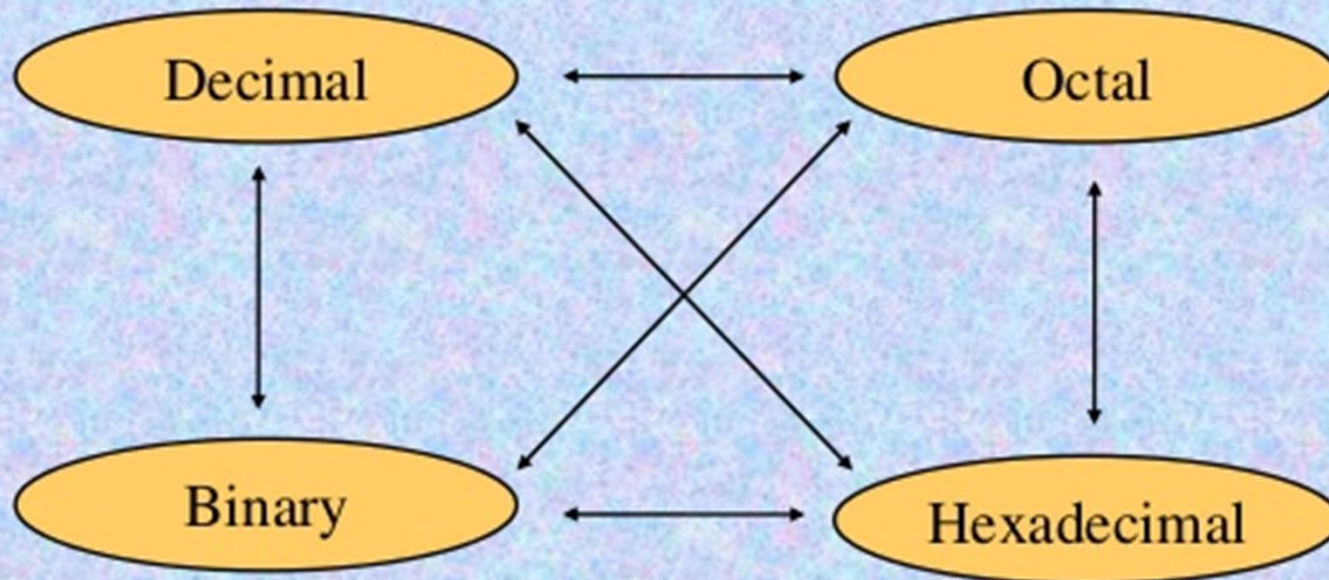
Quantities/Counting (3 of 3)

Decimal	Binary	Octal	Hexa- decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16

Etc.

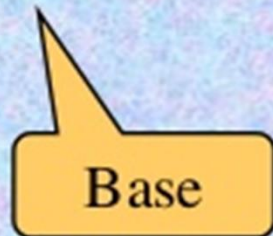
Conversion Among Bases

- The possibilities:



Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

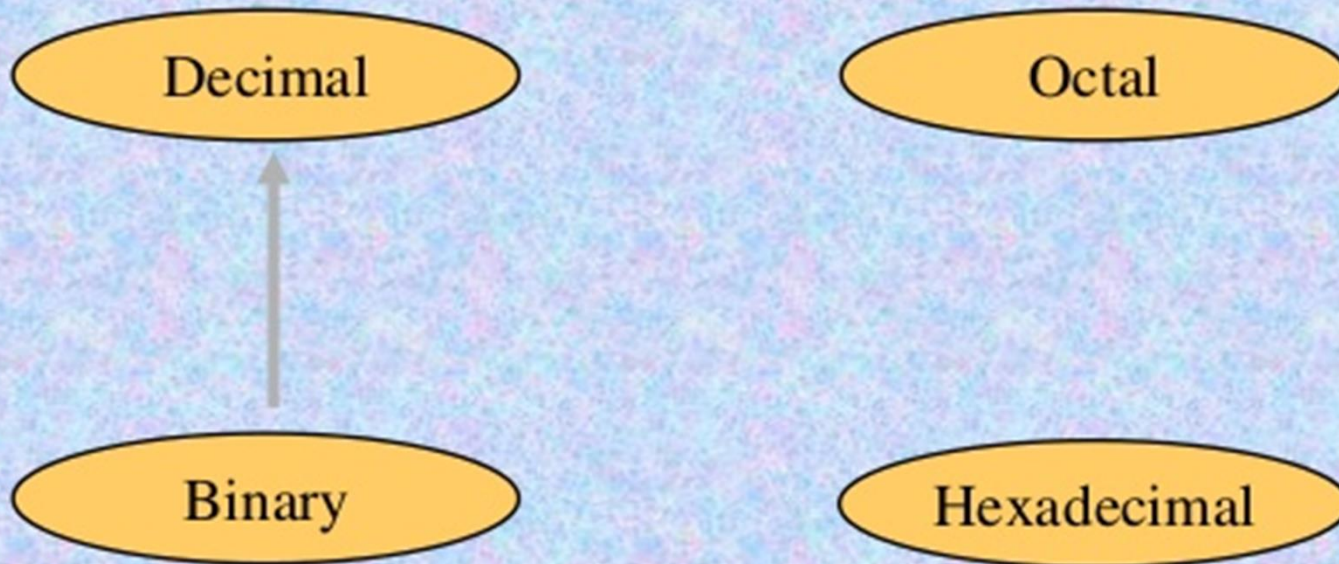


$$\begin{array}{r} 125_{10} \Rightarrow 5 \times 10^0 = 5 \\ 2 \times 10^1 = 20 \\ 1 \times 10^2 = 100 \\ \hline 125 \end{array}$$

Weight

Base

Binary to Decimal



Binary to Decimal

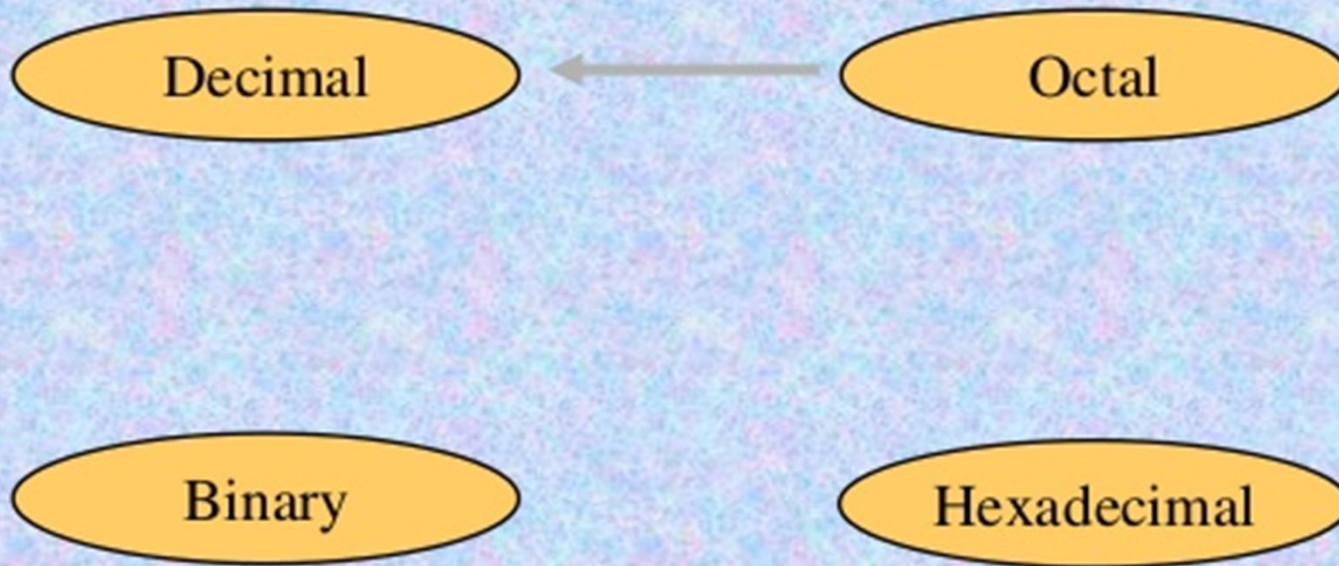
- Technique
 - Multiply each bit by 2^n , where n is the “weight” of the bit
 - The weight is the position of the bit, starting from 0 on the right
 - Add the results

Example

Bit "0"

$$\begin{array}{r} 101011_2 \Rightarrow \\ 1 \times 2^0 = 1 \\ 1 \times 2^1 = 2 \\ 0 \times 2^2 = 0 \\ 1 \times 2^3 = 8 \\ 0 \times 2^4 = 0 \\ 1 \times 2^5 = 32 \\ \hline 43_{10} \end{array}$$

Octal to Decimal



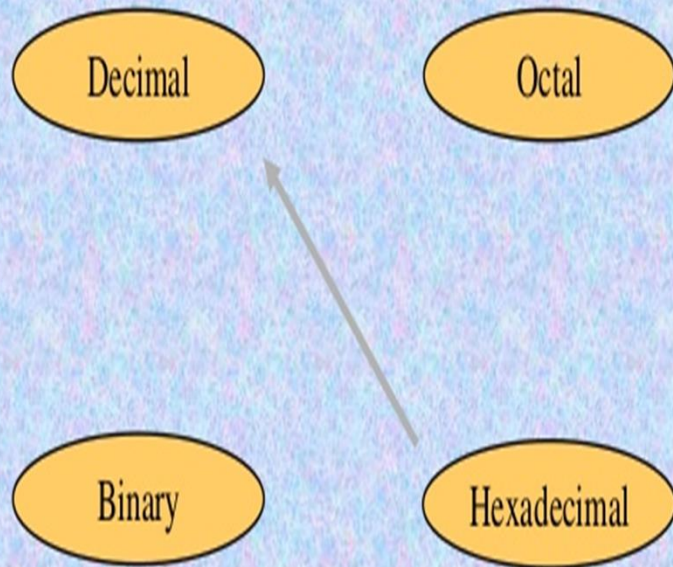
Octal to Decimal

- Technique
 - Multiply each bit by 8^n , where n is the “weight” of the bit
 - The weight is the position of the bit, starting from 0 on the right
 - Add the results

Example

$$\begin{array}{r} 724_8 \Rightarrow 4 \times 8^0 = 4 \\ 2 \times 8^1 = 16 \\ 7 \times 8^2 = 448 \\ \hline 468_{10} \end{array}$$

Hexadecimal to Decimal



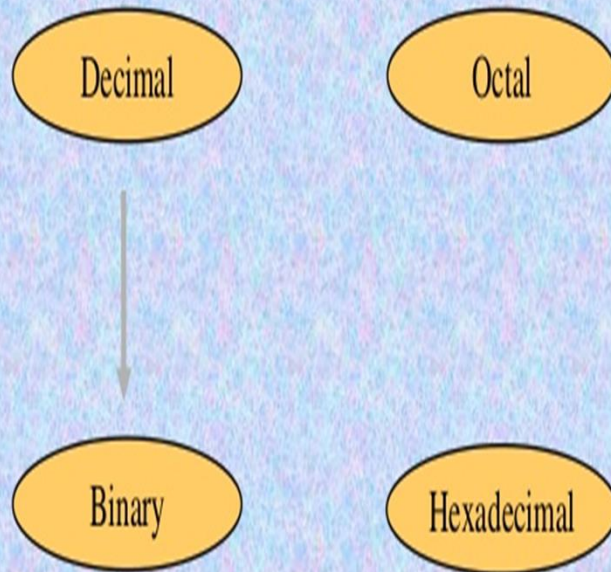
Hexadecimal to Decimal

- Technique
 - Multiply each bit by 16^n , where n is the “weight” of the bit
 - The weight is the position of the bit, starting from 0 on the right
 - Add the results

Example

$$\begin{array}{r} \text{ABC}_{16} \Rightarrow \\ \text{C} \times 16^0 = 12 \times 1 = 12 \\ \text{B} \times 16^1 = 11 \times 16 = 176 \\ \text{A} \times 16^2 = 10 \times 256 = \underline{2560} \\ \hline 2748_{10} \end{array}$$

Decimal to Binary



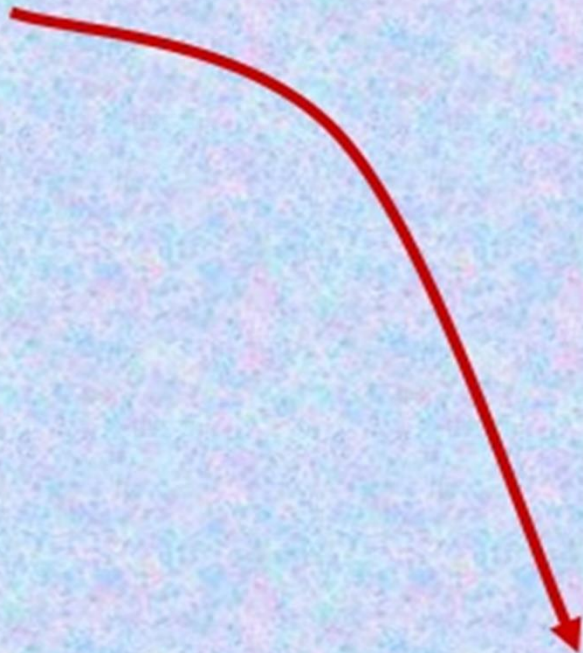
Decimal to Binary

- Technique
 - Divide by two, keep track of the remainder
 - First remainder is bit 0 (LSB, least-significant bit)
 - Second remainder is bit 1
 - Etc.

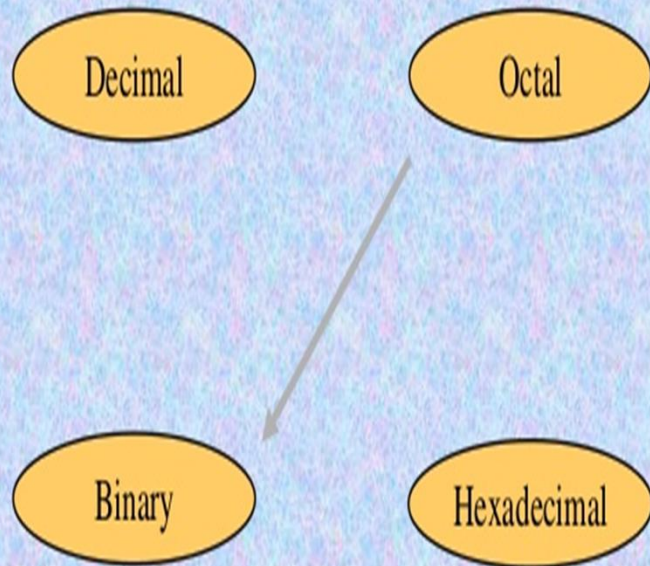
Example

$$125_{10} = ?_2$$

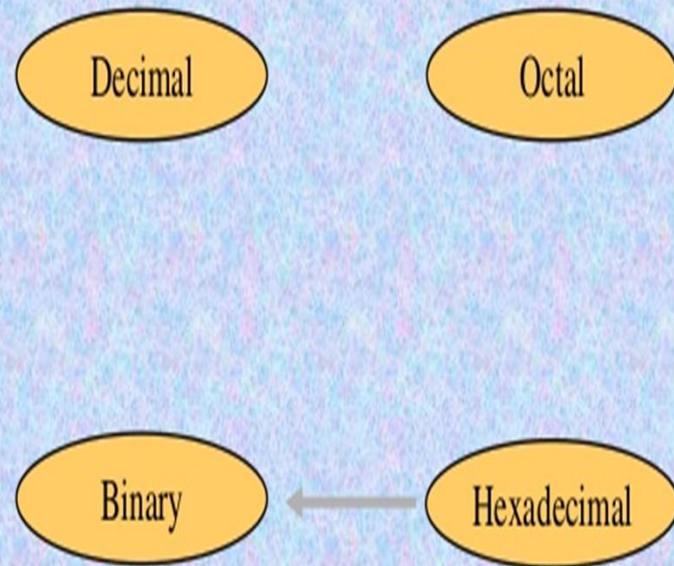
2		125	
2		62	1
2		31	0
2		15	1
2		7	1
2		3	1
2		1	1
		0	1


$$125_{10} = 1111101_2$$

Octal to Binary



Hexadecimal to Binary



Octal to Binary

- Technique
 - Convert each octal digit to a 3-bit equivalent binary representation

Hexadecimal to Binary

- Technique
 - Convert each hexadecimal digit to a 4-bit equivalent binary representation

Example

$$705_8 = ?_2$$

7	0	5
↓	↓	↓
111	000	101

$$705_8 = 111000101_2$$

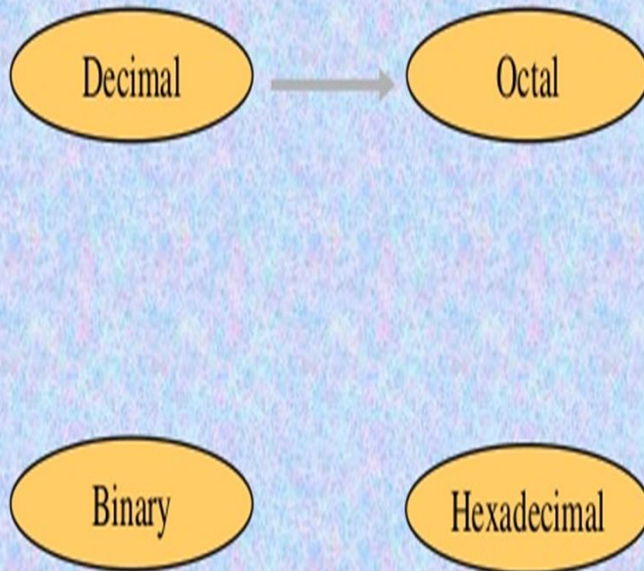
Example

$$10AF_{16} = ?_2$$

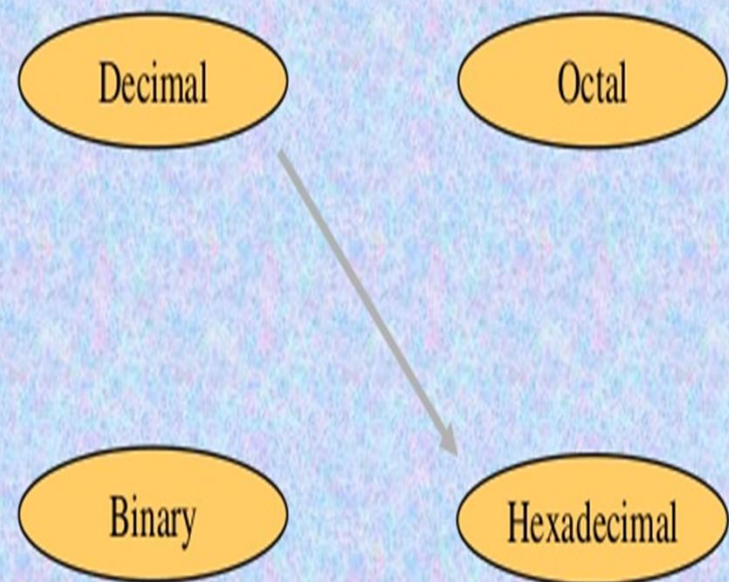
1	0	A	F
↓	↓	↓	↓
0001	0000	1010	1111

$$10AF_{16} = 0001000010101111_2$$

Decimal to Octal



Decimal to Hexadecimal



Decimal to Octal

- Technique
 - Divide by 8
 - Keep track of the remainder

Decimal to Hexadecimal

- Technique
 - Divide by 16
 - Keep track of the remainder

Example

$$1234_{10} = ?_8$$

$$\begin{array}{r} 8 \overline{) 1234} \\ 8 \overline{) 154} \quad 2 \\ 8 \overline{) 19} \quad 2 \\ 8 \overline{) 2} \quad 3 \\ \quad 0 \quad 2 \end{array}$$

$$1234_{10} = 2322_8$$

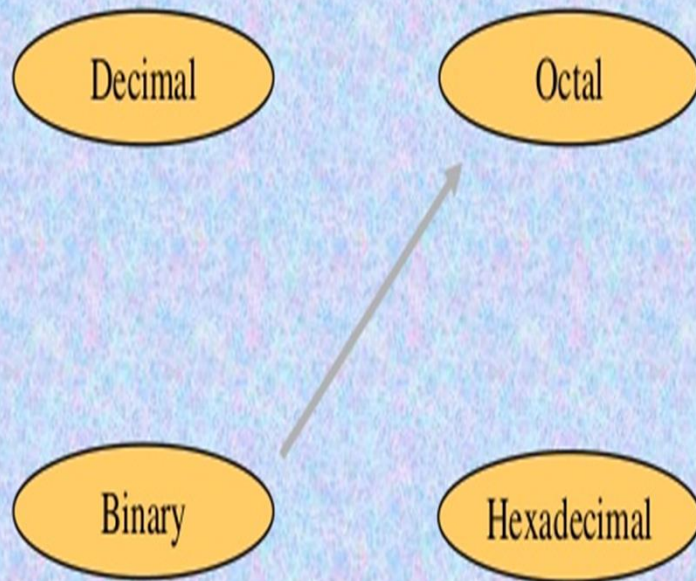
Example

$$1234_{10} = ?_{16}$$

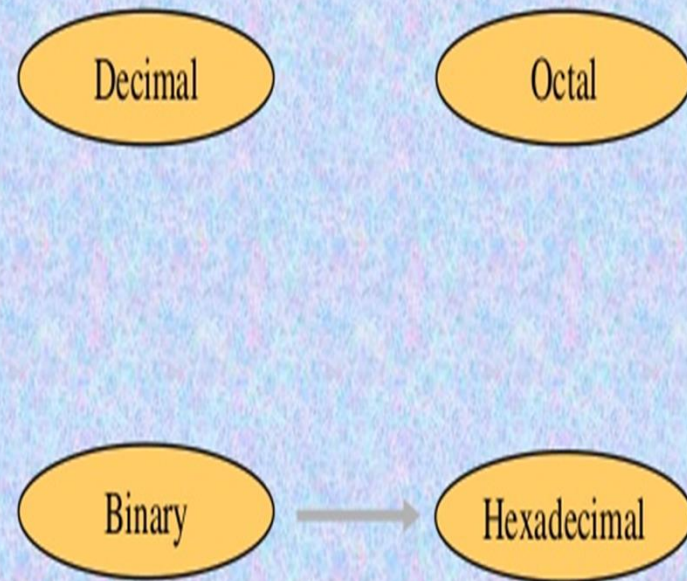
$$\begin{array}{r} 16 \overline{) 1234} \\ 16 \overline{) 77} \quad 2 \\ 16 \overline{) 4} \quad 13 = D \\ \quad 0 \quad 4 \end{array}$$

$$1234_{10} = 4D2_{16}$$

Binary to Octal



Binary to Hexadecimal



Binary to Octal

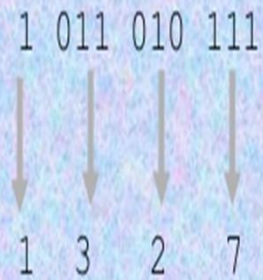
- Technique
 - Group bits in threes, starting on right
 - Convert to octal digits

Binary to Hexadecimal

- Technique
 - Group bits in fours, starting on right
 - Convert to hexadecimal digits

Example

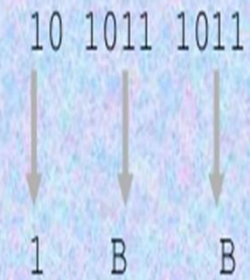
$$1011010111_2 = ?_8$$



$$1011010111_2 = 1327_8$$

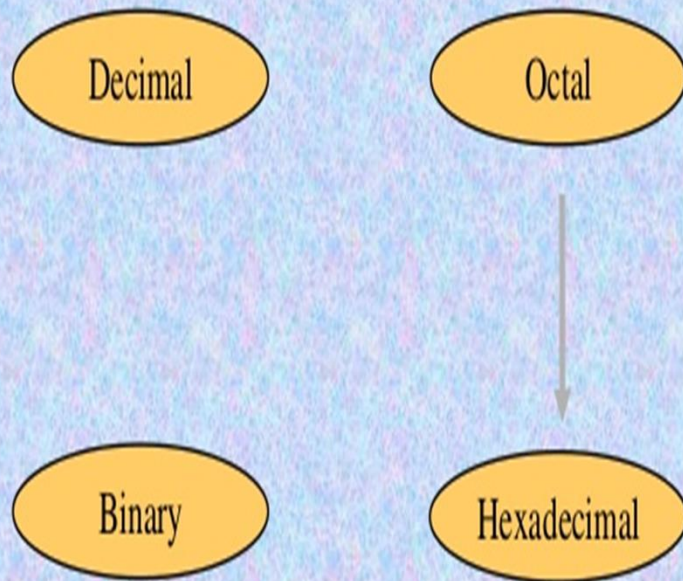
Example

$$1010111011_2 = ?_{16}$$

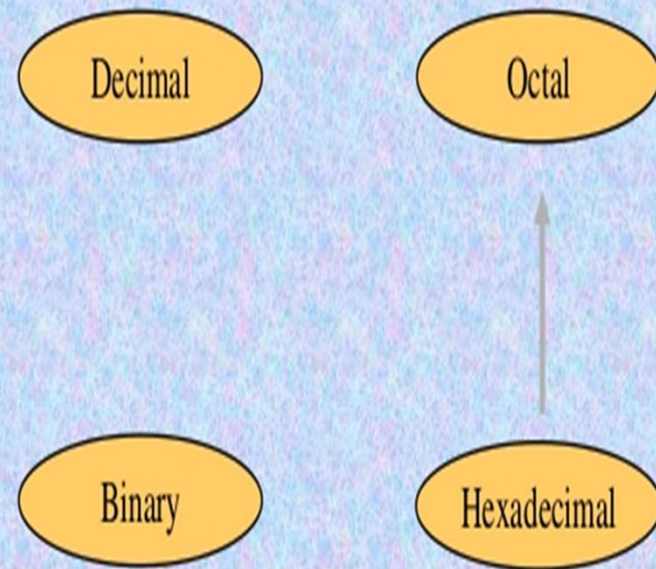


$$1010111011_2 = 2BB_{16}$$

Octal to Hexadecimal



Hexadecimal to Octal



Octal to Hexadecimal

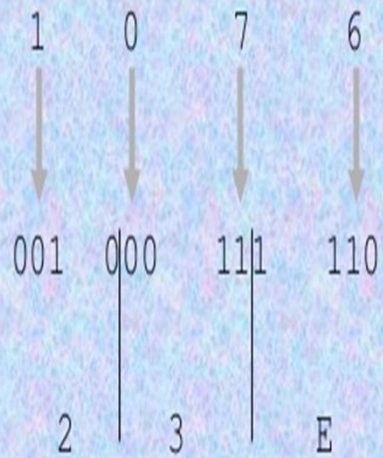
- Technique
 - Use binary as an intermediary

Hexadecimal to Octal

- Technique
 - Use binary as an intermediary

Example

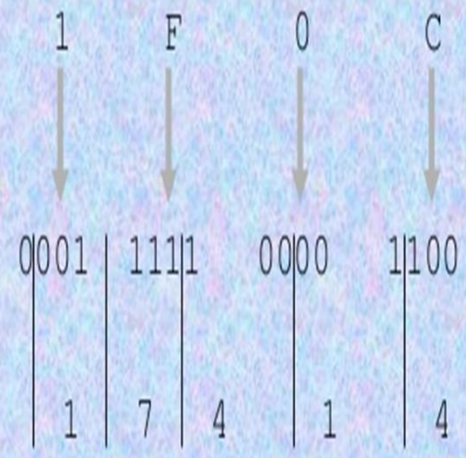
$$1076_8 = ?_{16}$$



$$1076_8 = 23E_{16}$$

Example

$$1F0C_{16} = ?_8$$



$$1F0C_{16} = 17414_8$$

Exercise – Convert ...

Answer

Decimal	Binary	Octal	Hexa- decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF