The background features a stylized circuit board pattern with various traces and circular components. A dark horizontal band is positioned across the middle of the image, serving as a backdrop for the title text.

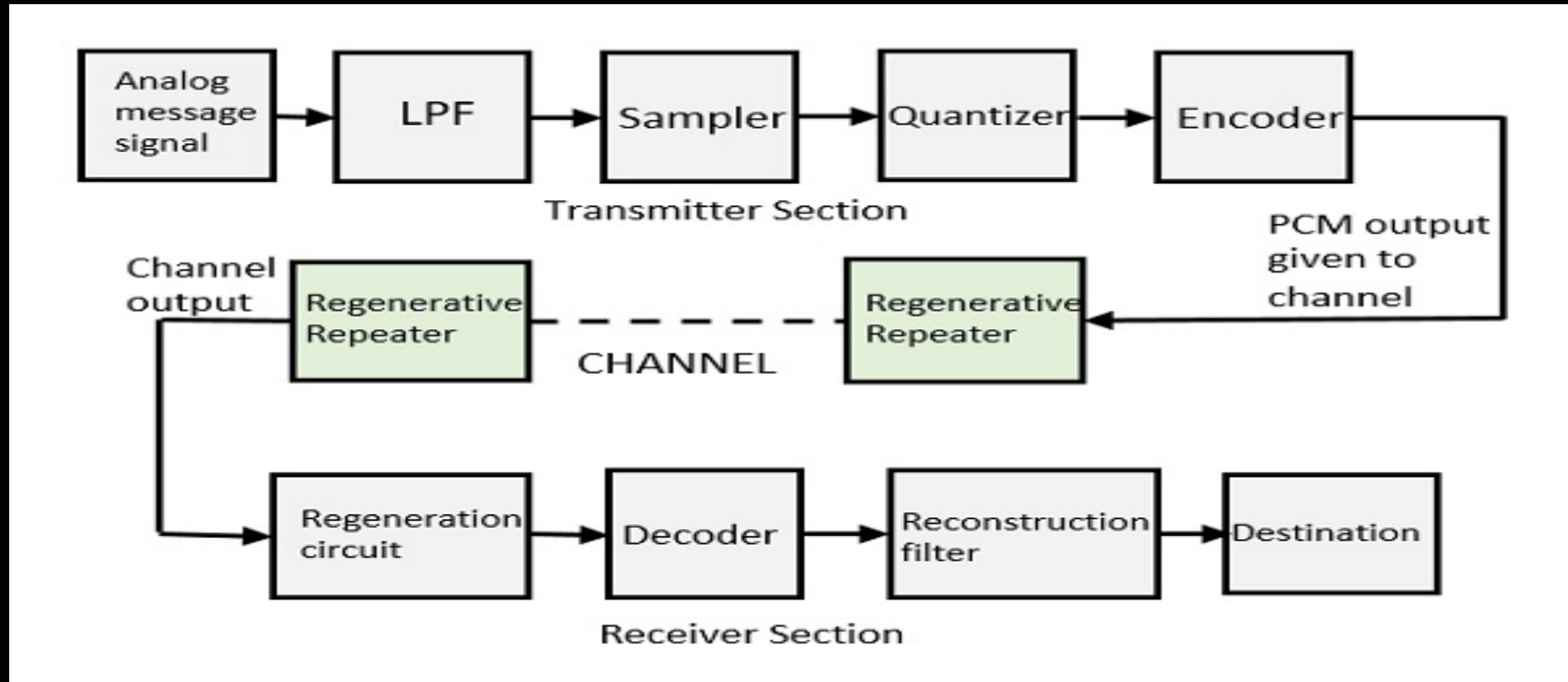
PULSE CODE MODULATION (PCM) NOISE ANALYSIS

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WHAT IS PCM??

- **Pulse-code modulation (PCM)** is a method used to digitally represent sampled analog signals.
- In a PCM , the amplitude of the analog signal is sampled regularly at uniform intervals, and each sample is quantized to the nearest value within a range of digital steps.
- The sampling rate must be greater than twice the highest frequency in the analog signal i.e. $f_s \geq 2f_m$.

BLOCK DIAGRAM of PCM



WHAT IS QUANTIZATION...?

- It is the process of converting sampled version of analog signal into discrete time and discrete amplitude signal.

WHAT IS QUANTIZATION ERROR...?

Quantization error is the difference between the analog signal and the closest available digital value at each sampling instant from the A/D converter. Quantization error also introduces noise, called quantization noise.

WHAT IS QUANTIZATION LEVEL...?

Approximating amplitude of signal to the nearest value from the set of discrete amplitude level is called quantization level.

$$\text{Quantizer level (L)} = 2^n$$

$$Q_e(\text{max}) = \Delta/2$$

$$Q_e(\text{min}) = -\Delta/2$$

PARAMETERS OF PCM

- Step size (Δ) = $\frac{v_{\max} - v_{\min}}{L}$
- Bit Duration (T_b) = T_s/n , T_s = sampling interval
- Bit rate (R_b) = n/T_s
- Sampling Rate (f_s) = $1/T_s$
- (B.W)_{max} = $1/T_b$ Hz
- (B.W)_{min} = $R_b/2$
- When $f_s = 2f_m$, then it is called Nyquist Rate

QUANTIZATION NOISE (N_q)

$$F(Q_e) = \frac{1}{\Delta}, \text{ where } -\Delta/2 \leq Q_e \leq \Delta/2 \\ = 0, \text{ otherwise}$$

N_q = quantization noise power = mean square value of Q_e

$$\begin{aligned} N_q &= \int_{-\infty}^{\infty} x^2 f(x) dx \\ &= \int_{-\Delta/2}^{\Delta/2} Q_e^2 f(Q_e) dQ_e \\ &= \int_{-\Delta/2}^{\Delta/2} Q_e \frac{1}{\Delta} dQ_e \end{aligned}$$

$$= \frac{1}{\Delta} \left[\frac{Qe^3}{3} \right]^{\frac{\Delta}{2}}$$
$$= \frac{1}{\Delta} \times \frac{1}{3} \left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8} \right]^{\frac{\Delta}{2}}$$

$$= \frac{1}{\Delta} * \frac{2\Delta^3}{24}$$

$$N_q = \frac{\Delta^2}{12}$$

Quantization Noise Power

- If the sampled quantization error waveform is represented as

$$e_s(t) = e(t)I \sum_{k=-\infty}^{k=\infty} \delta(t - kT_s)$$

- When $t = kT_s$

$$e_s(t) = I \sum_{k=-\infty}^{k=\infty} e(kT_s) \delta(t - kT_s)$$

- The power spectral density $G_{e_s}(f) = \frac{I^2}{T_s} \overline{e^2(kT_s)}$
- Quantization error = $\overline{e^2(t)} = \frac{\Delta^2}{12}$
- Quantization noise power = $N_q = \frac{I^2}{T_s} \left(\frac{\Delta^2}{12} \right)$

Thermal Noise Power

- The thermal noise is white and Gaussian, the probability of an error depends on the ratio E_b/N_0 . Where E_b is the signal energy transmitted during a bit and $N_0/2$ is the two sided power spectral density of the noise.
- Thermal noise power $\bar{N}_{th} = \frac{2^{2n} \Delta^2 p_e I^2}{3T_s^2}$.

Where p_e is probability error, I =impulse strength, T_s = sampling interval and Δ = quantization step size

- Probability error of PSK= $(P_e)_{psk} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$
- Probability error of FSK= $(P_e)_{fsk} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_0}}$

Output Signal Power

- The sampled signal which appears at the input to the baseband filter is given by $m_s(t)$.

$$m_s(t) = m(t)I \sum_{k=-\infty}^{k=\infty} \delta(t - KT_s)$$

- Impulse train is periodic it can be represented by a Fourier series. The impulse trains have a strength (area) I and are separated by a time T_s .
- Hence the signal $m_0(t)$ at the output of the baseband filter $m_0(t) = \frac{I}{T_s} m(t)$.
- Output signal power = $S_0 = \frac{I^2}{T_s^2} \overline{m^2(t)}$

Output Signal Power

- In order to calculate the value of $\overline{m^2(t)}$, let us assume the number of quantization levels= $L=l$ and step size Δ
- We assume that the signal can vary from $-\frac{l\Delta}{2}$ to $\frac{l\Delta}{2}$.
- The instantaneous value of $m(t)$ may fall anywhere in its allowable range of $l\Delta$ volts with equal likelihood.
- The probability density function of the instantaneous value of s is $f(s)$.

$$f(s) = \frac{1}{l\Delta}$$

- The variance of $m(t)$ is $\overline{m^2(t)}$.

$$\overline{m^2(t)} = \int_{-\frac{l\Delta}{2}}^{\frac{l\Delta}{2}} s^2 f(s) ds = \frac{1}{l\Delta} \int_{-\frac{l\Delta}{2}}^{\frac{l\Delta}{2}} s^2 ds$$

Output Signal Noise Power Ratio

- So, the variance of $m(t)$ is

$$\overline{m^2(t)} = \frac{L^2 \Delta^2}{12}$$

- The output signal power = $S_0 = \frac{I^2}{T^2} \frac{L^2 \Delta^2}{12}$

- The output signal to noise ratio (SNR) of PCM = $(SNR)_{\frac{o}{p}} = \frac{S_0}{N_q + N_{th}}$

Where N_q is quantization noise power and N_{th} is thermal noise power

$$(SNR)_{\frac{o}{p}} = \frac{S_0}{N_q + N_{th}} = \frac{S_0}{N_0} = \frac{\left(\frac{I^2}{T_s^2}\right) \left(\frac{L^2 \Delta^2}{12}\right)}{\left(\frac{I^2}{T_s^2}\right) \left(\frac{\Delta^2}{12}\right) + \left(\frac{I^2}{T_s^2}\right) \left(\frac{p_e 2^{2n} \Delta^2}{3}\right)}$$

OUTPUT SNR of PCM

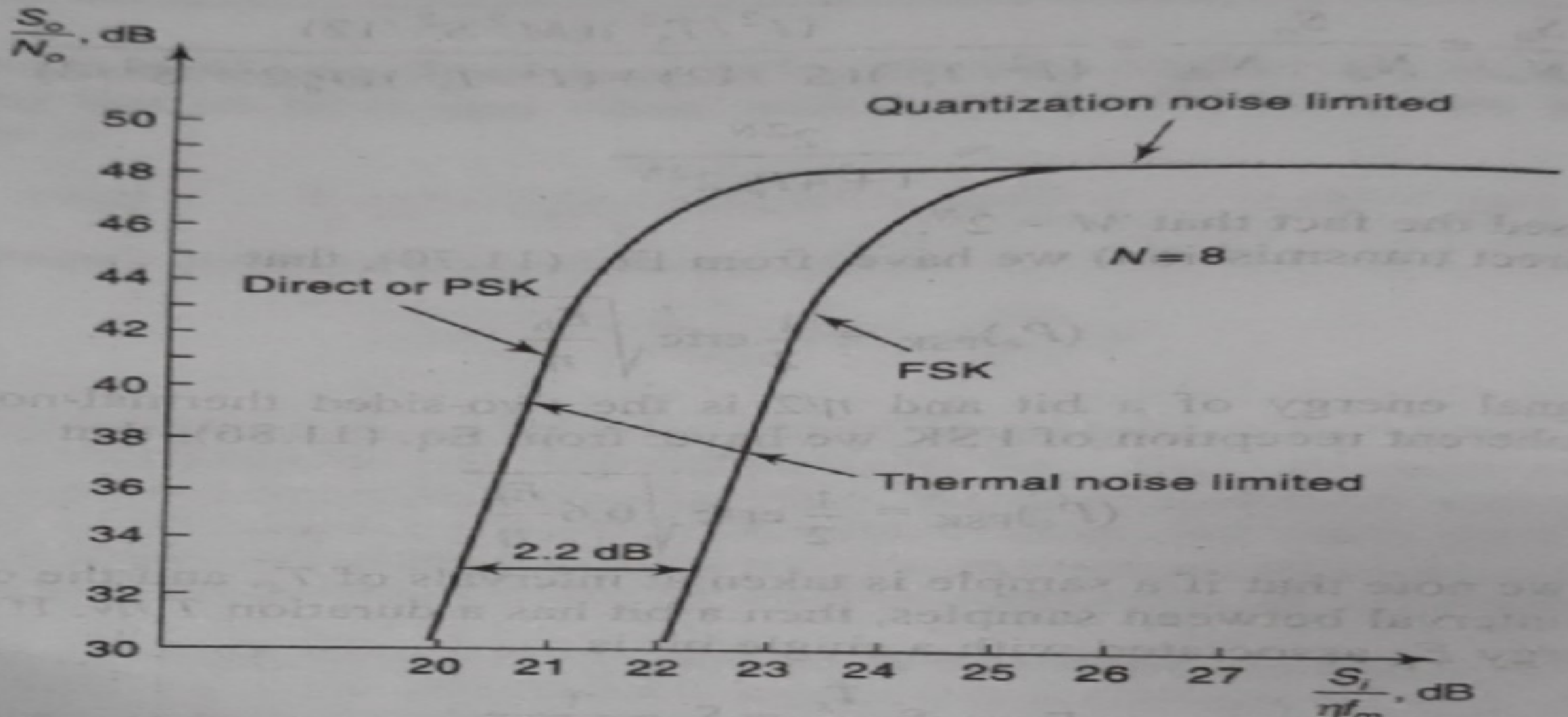
- General formula of SNR output is

$$\frac{S_o}{N_0} = \frac{2^{2n}}{1 + 4p_e 2^{2n}}$$

- Probability error of PSK = $(P_e)_{psk} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$
- Probability error of FSK = $(P_e)_{fsk} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_0}}$
- If the received signal power is S_i . The energy per bit $E_b = S_i \times (T_s/n) = S_i \times (1/2f_m n)$
- If η is the thermal – noise spectral density

$$\left(\frac{S_o}{N_0}\right)_{psk} = \frac{2^{2n}}{1 + 2^{2N+1} \operatorname{erfc} \sqrt{\left(\frac{1}{2n}\right) \left(\frac{S_i}{\eta f_m}\right)}}$$

Comparison of PCM Transmission Systems



THANK YOU..