

# Chapter 11

## Relational Database Design Algorithms and Further Dependencies

# Chapter Outline

0. Designing a Set of Relations
1. Properties of Relational Decompositions
2. Algorithms for Relational Database Schema
3. Multivalued Dependencies and Fourth Normal Form
4. Join Dependencies and Fifth Normal Form
5. Inclusion Dependencies
6. Other Dependencies and Normal Forms

# DESIGNING A SET OF RELATIONS (1)

## The Approach of Relational Synthesis (Bottom-up Design) :

- Assumes that all possible functional dependencies are known.
- First constructs a minimal set of FDs
- Then applies algorithms that construct a target set of 3NF or BCNF relations.
- Additional criteria may be needed to ensure the the *set of relations* in a relational database are satisfactory (see Algorithms 11.2 and 11.4).

# DESIGNING A SET OF RELATIONS (2)

## Goals:

- Lossless join property (a must) – algorithm 11.1 tests for general losslessness.
- Dependency preservation property – algorithms 11.3 decomposes a relation into BCNF components by sacrificing the dependency preservation.
- Additional normal forms
  - 4NF (based on multi-valued dependencies)
  - 5NF (based on join dependencies)

# 1. Properties of Relational Decompositions (1)

## Relation Decomposition and Insufficiency of Normal Forms:

- **Universal Relation Schema:** a relation schema  $R = \{A_1, A_2, \dots, A_n\}$  that includes all the attributes of the database.
- **Universal relation assumption:** every attribute name is unique.
- **Decomposition:** The process of decomposing the universal relation schema  $R$  into a set of relation schemas  $D = \{R_1, R_2, \dots, R_m\}$  that will become the relational database schema by using the functional dependencies.

## Properties of Relational Decompositions (2)

### **Relation Decomposition and Insufficiency of Normal Forms (cont.):**

- **Attribute preservation condition:** Each attribute in  $R$  will appear in at least one relation schema  $R_i$  in the decomposition so that no attributes are “lost”.
- Another goal of decomposition is to have each individual relation  $R_i$  in the decomposition  $D$  be in BCNF or 3NF.
- Additional properties of decomposition are needed to prevent from generating spurious tuples

# Properties of Relational Decompositions (3)

## Dependency Preservation Property of a Decomposition :

### Definition:

- Given a set of dependencies  $F$  on  $R$ , the **projection** of  $F$  on  $R_i$ , denoted by  $p_{R_i}(F)$  where  $R_i$  is a subset of  $R$ , is the set of dependencies  $X \rightarrow Y$  in  $F^+$  such that the attributes in  $X \cup Y$  are all contained in  $R_i$ . *Hence, the projection of  $F$  on each relation schema  $R_i$  in the decomposition  $D$  is the set of functional dependencies in  $F^+$ , the closure of  $F$ , such that all their left- and right-hand-side attributes are in  $R_i$ .*

## Properties of Relational Decompositions (4)

### Dependency Preservation Property of a Decomposition (cont.):

- **Dependency Preservation Property:** a decomposition  $D = \{R_1, R_2, \dots, R_m\}$  of  $R$  is **dependency-preserving** with respect to  $F$  if the union of the projections of  $F$  on each  $R_i$  in  $D$  is equivalent to  $F$ ; that is,  $((\pi_{R_1}(F)) \cup \dots \cup (\pi_{R_m}(F)))^+ = F^+$

(See examples in Fig 10.12a and Fig 10.11)

**Claim 1:** It is always possible to find a dependency-preserving decomposition  $D$  with respect to  $F$  such that each relation  $R_i$  in  $D$  is in 3nf.



# Properties of Relational Decompositions (5)

## Lossless (Non-additive) Join Property of a Decomposition:

### Definition:

- **Lossless join property:** a decomposition  $D = \{R_1, R_2, \dots, R_m\}$  of  $R$  has the **lossless (nonadditive) join property** with respect to the set of dependencies  $F$  on  $R$  if, for *every* relation state  $r$  of  $R$  that satisfies  $F$ , the following holds, where  $*$  is the natural join of all the relations in  $D$ :

$$* (\pi_{R_1}(r), \dots, \pi_{R_m}(r)) = r$$

**Note:** The word loss in *lossless* refers to *loss of information*, not to loss of tuples. In fact, for “loss of information” a better term is “addition of spurious information”

# Properties of Relational Decompositions (6)

## Lossless (Non-additive) Join Property of a Decomposition (cont.):

### Algorithm 11.1: Testing for Lossless Join Property

**Input:** A universal relation  $R$ , a decomposition  $D = \{R_1, R_2, \dots, R_m\}$  of  $R$ , and a set  $F$  of functional dependencies.

1. Create an initial matrix  $S$  with one row  $i$  for each relation  $R_i$  in  $D$ , and one column  $j$  for each attribute  $A_j$  in  $R$ .
2. Set  $S(i,j) := b_{ij}$  for all matrix entries. (\* each  $b_{ij}$  is a distinct symbol associated with indices  $(i,j)$  \*).
3. For each row  $i$  representing relation schema  $R_i$   
{ for each column  $j$  representing attribute  $A_j$   
{ if (relation  $R_i$  includes attribute  $A_j$ ) then set  $S(i,j) := a_j$ ; }; };  
(\* each  $a_j$  is a distinct symbol associated with index  $(j)$  \*)

# Properties of Relational Decompositions (7)

## Lossless (Non-additive) Join Property of a Decomposition (cont.):

### Algorithm 11.1: Testing for Lossless Join Property (cont.)

4. Repeat the following loop until a *complete loop execution* results in no changes to  $S$ 
  - {for each functional dependency  $X \rightarrow Y$  in  $F$
  - {for all rows in  $S$  which have the same symbols in the columns corresponding to attributes in  $X$ 
    - {make the symbols in each column that correspond to an attribute in  $Y$  be the same in all these rows as follows: if any of the rows has an “ $a$ ” symbol for the column, set the other rows to that *same* “ $a$ ” symbol in the column. If no “ $a$ ” symbol exists for the attribute in any of the rows, choose one of the “ $b$ ” symbols that appear in one of the rows for the attribute and set the other rows to that same “ $b$ ” symbol in the column ;};};
5. If a row is made up entirely of “ $a$ ” symbols, then the decomposition has the lossless join property; otherwise it does not.

# Properties of Relational Decompositions (8)

Lossless (nonadditive) join test for  $n$ -ary decompositions.

(a) Case 1: Decomposition of EMP\_PROJ into EMP\_PROJ1 and EMP\_LOCS fails test. (b) A

decomposition of EMP\_PROJ that has the lossless join property.

- (a)  $R = \{\text{SSN, ENAME, PNUMBER, PNAME, PLOCATION, HOURS}\}$   $D = \{R_1, R_2\}$   
 $R_1 = \text{EMP\_LOCS} = \{\text{ENAME, PLOCATION}\}$   
 $R_2 = \text{EMP\_PROJ1} = \{\text{SSN, PNUMBER, HOURS, PNAME, PLOCATION}\}$

$F = \{\text{SSN} \rightarrow \text{ENAME}; \text{PNUMBER} \rightarrow \{\text{PNAME, PLOCATION}\}; \{\text{SSN, PNUMBER}\} \rightarrow \text{HOURS}\}$

	SSN	ENAME	PNUMBER	PNAME	PLOCATION	HOURS
$R_1$	$b_{11}$	$a_2$	$b_{13}$	$b_{14}$	$a_5$	$b_{16}$
$R_2$	$a_1$	$b_{22}$	$a_3$	$a_4$	$a_5$	$a_6$

(no changes to matrix after applying functional dependencies)

(b)

EMP		PROJECT			WORKS_ON		
SSN	ENAME	PNUMBER	PNAME	PLOCATION	SSN	PNUMBER	HOURS

# Properties of Relational Decompositions (8)

Lossless (nonadditive) join test for  $n$ -ary decompositions.

(c) Case 2:  
Decomposition of EMP\_PROJ into EMP, PROJECT, and WORKS\_ON satisfies test.

(c)  $R = \{SSN, ENAME, PNUMBER, PNAME, PLOCATION, HOURS\}$   $D = \{R_1, R_2, R_3\}$   
 $R_1 = EMP = \{SSN, ENAME\}$   
 $R_2 = PROJ = \{PNUMBER, PNAME, PLOCATION\}$   
 $R_3 = WORKS_ON = \{SSN, PNUMBER, HOURS\}$

$F = \{SSN \rightarrow \{ENAME, PNUMBER\}; PNUMBER \rightarrow \{PNAME, PLOCATION\}; \{SSN, PNUMBER\} \rightarrow HOURS\}$

	SSN	ENAME	PNUMBER	PNAME	PLOCATION	HOURS
$R_1$	$a_1$	$a_2$	$b_{13}$	$b_{14}$	$b_{15}$	$b_{16}$
$R_2$	$b_{21}$	$b_{22}$	$a_3$	$a_4$	$a_5$	$b_{26}$
$R_3$	$a_1$	$b_{32}$	$a_3$	$b_{34}$	$b_{35}$	$a_6$

(original matrix S at start of algorithm)

	SSN	ENAME	PNUMBER	PNAME	PLOCATION	HOURS
$R_1$	$a_1$	$a_2$	$b_{13}$	$b_{14}$	$b_{15}$	$b_{16}$
$R_2$	$b_{21}$	$b_{22}$	$a_3$	$a_4$	$a_5$	$b_{26}$
$R_3$	$a_1$	<del><math>b_{32}</math></del> $a_2$	$a_3$	<del><math>b_{34}</math></del> $a_4$	<del><math>b_{35}</math></del> $a_5$	$a_6$

(matrix S after applying the first two functional dependencies - last row is all "a" symbols, so we stop)

# Properties of Relational Decompositions (9)

## Testing Binary Decompositions for Lossless Join

### Property:

- **Binary Decomposition:** decomposition of a relation  $R$  into two relations.
- **PROPERTY LJ1 (lossless join test for binary decompositions):** A decomposition  $D = \{R_1, R_2\}$  of  $R$  has the lossless join property with respect to a set of functional dependencies  $F$  on  $R$  *if and only if* either
  - The f.d.  $((R_1 \cap R_2) \rightarrow (R_1 - R_2))$  is in  $F^+$ , or
  - The f.d.  $((R_1 \cap R_2) \rightarrow (R_2 - R_1))$  is in  $F^+$ .

# Properties of Relational Decompositions (10)

## Successive Lossless Join Decomposition:

- **Claim 2 (Preservation of non-additivity in successive decompositions):**

If a decomposition  $D = \{R_1, R_2, \dots, R_m\}$  of  $R$  has the lossless (non-additive) join property with respect to a set of functional dependencies  $F$  on  $R$ , and if a decomposition  $D_i = \{Q_1, Q_2, \dots, Q_k\}$  of  $R_i$  has the lossless (non-additive) join property with respect to the projection of  $F$  on  $R_i$ , then the decomposition  $D_2 = \{R_1, R_2, \dots, R_{i-1}, Q_1, Q_2, \dots, Q_k, R_{i+1}, \dots, R_m\}$  of  $R$  has the non-additive join property with respect to  $F$ .

## 2. Algorithms for Relational Database Schema Design (1)

### **Algorithm 11.2: Relational Synthesis into 3NF with Dependency Preservation (*Relational Synthesis Algorithm*)**

**Input:** A universal relation  $R$  and a set of functional dependencies  $F$  on the attributes of  $R$ .

1. Find a minimal cover  $G$  for  $F$  (use Algorithm 10.2);
2. For each left-hand-side  $X$  of a functional dependency that appears in  $G$ , create a relation schema in  $D$  with attributes  $\{X \cup \{A_1\} \cup \{A_2\} \dots \cup \{A_k\}\}$ , where  $X \rightarrow A_1, X \rightarrow A_2, \dots, X \rightarrow A_k$  are the only dependencies in  $G$  with  $X$  as left-hand-side ( $X$  is the *key* of this relation) ;
3. Place any remaining attributes (that have not been placed in any relation) in a single relation schema to ensure the attribute preservation property.

**Claim 3:** *Every relation schema created by Algorithm 11.2 is in 3NF.*



# Algorithms for Relational Database Schema Design (2)

## **Algorithm 11.3: Relational Decomposition into BCNF with Lossless (non-additive) join property**

**Input:** A universal relation  $R$  and a set of functional dependencies  $F$  on the attributes of  $R$ .

1. Set  $D := \{R\}$ ;
2. While there is a relation schema  $Q$  in  $D$  that is not in BCNF do {  
    choose a relation schema  $Q$  in  $D$  that is not in BCNF;  
    find a functional dependency  $X \rightarrow Y$  in  $Q$  that violates BCNF;  
    replace  $Q$  in  $D$  by two relation schemas  $(Q - Y)$  and  $(X \cup Y)$ ;  
};

**Assumption:** *No null values are allowed for the join attributes.*

# Algorithms for Relational Database Schema Design (3)

## **Algorithm 11.4 Relational Synthesis into 3NF with Dependency Preservation and Lossless (Non-Additive) Join Property**

**Input:** A universal relation  $R$  and a set of functional dependencies  $F$  on the attributes of  $R$ .

1. Find a minimal cover  $G$  for  $F$  (*Use Algorithm 10.2*).
2. For each left-hand-side  $X$  of a functional dependency that appears in  $G$ , create a relation schema in  $D$  with attributes  $\{X \cup \{A_1\} \cup \{A_2\} \dots \cup \{A_k\}\}$ , where  $X \rightarrow A_1, X \rightarrow A_2, \dots, X \rightarrow A_k$  are the only dependencies in  $G$  with  $X$  as left-hand-side ( $X$  is the *key* of this relation).
3. If none of the relation schemas in  $D$  contains a key of  $R$ , then create one more relation schema in  $D$  that contains attributes that form a key of  $R$ . (*Use Algorithm 11.4a to find the key of  $R$* )

# Algorithms for Relational Database Schema Design (4)

## Algorithm 11.4a Finding a Key $K$ for $R$ Given a set $F$ of Functional Dependencies

**Input:** A universal relation  $R$  and a set of functional dependencies  $F$  on the attributes of  $R$ .

1. Set  $K := R$ .
2. For each attribute  $A$  in  $K$  {  
    compute  $(K - A)^+$  with respect to  $F$ ;  
    If  $(K - A)^+$  contains all the attributes in  $R$ ,  
    then set  $K := K - \{A\}$ ; }

# Algorithms for Relational Database Schema Design (5)

Issues with null-value joins. (a) Some EMPLOYEE tuples have null for the join attribute

DNUM.

(a)

<b>EMPLOYEE</b>				
ENAME	<u>SSN</u>	BDATE	ADDRESS	DNUM
Smith, John B.	123456789	1965-01-09	731 Fondren, Houston, TX	5
Wong, Franklin T.	333445555	1955-12-08	638 Voss, Houston, TX	5
Zelaya, Alicia J.	999887777	1968-07-19	3321 Castle, Spring, TX	4
Wallace, Jennifer S.	987654321	1941-06-20	291 Berry, Bellaire, TX	4
Narayan, Ramesh K.	666884444	1962-09-15	975 Fire Oak, Humble, TX	5
English, Joyce A.	453453453	1972-07-31	5631 Rice, Houston, TX	5
Jabbar, Ahmad V.	987987987	1969-03-29	980 Dallas, Houston, TX	4
Borg, James E.	888665555	1937-11-10	450 Stone, Houston, TX	1
Berger, Anders C.	999775555	1965-04-26	6530 Braes, Bellaire, TX	null
Benitez, Carlos M.	888664444	1963-01-09	7654 Beech, Houston, TX	null

## DEPARTMENT

DNAME	<u>DNUM</u>	DMGRSSN
Research	5	333445555
Administration	4	987654321
Headquarters	1	888665555

# Algorithms for Relational Database Schema Design (5)

Issues with null-value joins. (b) Result of applying NATURAL JOIN to the EMPLOYEE and DEPARTMENT relations. (c) Result of applying LEFT OUTER JOIN to EMPLOYEE and DEPARTMENT.

(b)

ENAME	<u>SSN</u>	BDATE	ADDRESS	DNUM	DNAME	DMGRSSN
Smith, John B.	123456789	1965-01-09	731 Fondren, Houston, TX	5	Research	333445555
Wong, Franklin T.	333445555	1955-12-08	638 Voss, Houston, TX	5	Research	333445555
Zelaya, Alicia J.	999887777	1968-07-19	3321 Castle, Spring, TX	4	Administration	987654321
Wallace, Jennifer S.	987654321	1941-06-20	291 Berry, Bellaire, TX	4	Administration	987654321
Narayan, Ramesh K.	666884444	1962-09-15	975 Fire Oak, Humble, TX	5	Research	333445555
English, Joyce A.	453453453	1972-07-31	5631 Rice, Houston, TX	5	Research	333445555
Jabbar, Ahmad V.	987987987	1969-03-29	980 Dallas, Houston, TX	4	Administration	987654321
Borg, James E.	888665555	1937-11-10	450 Stone, Houston, TX	1	Headquarters	888665555

(c)

ENAME	<u>SSN</u>	BDATE	ADDRESS	DNUM	DNAME	DMGRSSN
Smith, John B.	123456789	1965-01-09	731 Fondren, Houston, TX	5	Research	333445555
Wong, Franklin T.	333445555	1955-12-08	638 Voss, Houston, TX	5	Research	333445555
Zelaya, Alicia J.	999887777	1968-07-19	3321 Castle, Spring, TX	4	Administration	987654321
Wallace, Jennifer S.	987654321	1941-06-20	291 Berry, Bellaire, TX	4	Administration	987654321
Narayan, Ramesh K.	666884444	1962-09-15	975 Fire Oak, Humble, TX	5	Research	333445555
English, Joyce A.	453453453	1972-07-31	5631 Rice, Houston, TX	5	Research	333445555
Jabbar, Ahmad V.	987987987	1969-03-29	980 Dallas, Houston, TX	4	Administration	987654321
Borg, James E.	888665555	1937-11-10	450 Stone, Houston, TX	1	Headquarters	888665555
Berger, Anders C.	999775555	1965-04-26	6530 Braes, Bellaire, TX	<b>null</b>	<b>null</b>	<b>null</b>
Benitez, Carlos M.	888664444	1963-01-09	7654 Beech, Houston, TX	<b>null</b>	<b>null</b>	<b>null</b>

# Algorithms for Relational Database Schema Design (6)

The “dangling tuple” problem. (a) The relation EMPLOYEE\_1 (includes all attributes of EMPLOYEE from figure 11.2a except DNUM).

(a) **EMPLOYEE\_1**

ENAME	<u>SSN</u>	BDATE	ADDRESS
Smith, John B.	123456789	1965-01-09	731 Fondren, Houston, TX
Wong, Franklin T.	333445555	1955-12-08	638 Voss, Houston, TX
Zelaya, Alicia J.	999887777	1968-07-19	3321 Castle, Spring, TX
Wallace, Jennifer S.	987654321	1941-06-20	291 Berry, Bellaire, TX
Narayan, Ramesh K.	666884444	1962-09-15	975 Fire Oak, Humble, TX
English, Joyce A.	453453453	1972-07-31	5631 Rice, Houston, TX
Jabbar, Ahmad V.	987987987	1969-03-29	980 Dallas, Houston, TX
Borg, James E.	888665555	1937-11-10	450 Stone, Houston, TX
Berger, Anders C.	999775555	1965-04-26	6530 Braes, Bellaire, TX
Benitez, Carlos M.	888664444	1963-01-09	7654 Beech, Houston, TX

# Algorithms for Relational Database Schema Design (6)

The “dangling tuple” problem. (b) The relation EMPLOYEE\_2 (includes DNUM attribute with null values). (c) The relation EMPLOYEE\_3 (includes DNUM attribute but does not include tuples for which DNUM has null values).

(b) **EMPLOYEE\_2**

<u>SSN</u>	DNUM
123456789	5
333445555	5
999887777	4
987654321	4
666884444	5
453453453	5
987987987	4
888665555	1
999775555	null
888664444	null

(c) **EMPLOYEE\_3**

<u>SSN</u>	DNUM
123456789	5
333445555	5
999887777	4
987654321	4
666884444	5
453453453	5
987987987	4
888665555	1

# Algorithms for Relational Database Schema Design (7)

## Discussion of Normalization Algorithms:

### Problems:

- The database designer must first specify *all* the relevant functional dependencies among the database attributes.
- These algorithms are *not deterministic* in general.
- It is not always possible to find a decomposition into relation schemas that preserves dependencies and allows each relation schema in the decomposition to be in BCNF (instead of 3NF as in Algorithm 11.4).



# Algorithms for Relational Database Schema Design (8)

**Table 11.1 Summary of some of the algorithms discussed above**

Algorithm	Input	Output	Properties/Purpose	Remarks
11.1	A decomposition D of R and a set F of functional dependencies	Boolean result: yes or no for lossless join property	Testing for non-additive join decomposition	See a simpler test in Section 11.1.4 for binary decompositions
11.2	Set of functional dependencies F	A set of relations in 3NF	Dependency preservation	No guarantee of satisfying lossless join property
11.3	Set of functional dependencies F	A set of relations in BCNF	Lossless join decomposition	No guarantee of dependency preservation
11.4	Set of functional dependencies F	A set of relations in 3NF	Lossless join <b>and</b> dependency preserving decomposition	May not achieve BCNF
11.4a	Relation schema R with a set of functional dependencies F	Key K of R	To find a key K (which is a subset of R)	The entire relation R is always a default superkey

# 3. Multivalued Dependencies and Fourth Normal Form (1)

(a) The EMP relation with two MVDs: ENAME  $\twoheadrightarrow$  PNAME and ENAME  $\twoheadrightarrow$  DNAME. (b) Decomposing the EMP relation into two 4NF relations EMP\_PROJECTS and

EMP\_DEPENDENTS.

(a)

**EMP**

<u>ENAME</u>	PNAME	<u>DNAME</u>
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Smith	X	John
Smith	Y	Anna
Smith	X	Anna
Smith	Y	John

(b)

**EMP\_PROJECTS**

<u>ENAME</u>	<u>PNAME</u>
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Smith	X
Smith	Y

**EMP\_DEPENDENTS**

<u>ENAME</u>	<u>DNAME</u>
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Smith	John
Smith	Anna

# 3. Multivalued Dependencies and Fourth Normal Form (1)

- (c) The relation SUPPLY with no MVDs is in 4NF but not in 5NF if it has the JD(R1, R2, R3).  
 (d) Decomposing the relation SUPPLY into the 5NF relations R1, R2, and R3.

(c) **SUPPLY**

SNAME	PARTNAME	PROJNAME
Smith	Bolt	ProjX
Smith	Nut	ProjY
Adamsky	Bolt	ProjY
Walton	Nut	ProjZ
Adamsky	Nail	ProjX
Adamsky	Bolt	ProjX
Smith	Bolt	ProjY

(d)

**R1**

SNAME	PARTNAME
Smith	Bolt
Smith	Nut
Adamsky	Bolt
Walton	Nut
Adamsky	Nail

**R2**

SNAME	PROJNAME
Smith	ProjX
Smith	ProjY
Adamsky	ProjY
Walton	ProjZ
Adamsky	ProjX

**R3**

PARTNAME	PROJNAME
Bolt	ProjX
Nut	ProjY
Bolt	ProjY
Nut	ProjZ
Nail	ProjX

# Multivalued Dependencies and Fourth Normal Form (2)

## Definition:

- A **multivalued dependency (MVD)**  $X \twoheadrightarrow Y$  specified on relation schema  $R$ , where  $X$  and  $Y$  are both subsets of  $R$ , specifies the following constraint on any relation state  $r$  of  $R$ : If two tuples  $t_1$  and  $t_2$  exist in  $r$  such that  $t_1[X] = t_2[X]$ , then two tuples  $t_3$  and  $t_4$  should also exist in  $r$  with the following properties, where we use  $Z$  to denote  $(R - (X \cup Y))$ :
  - $t_3[X] = t_4[X] = t_1[X] = t_2[X]$ .
  - $t_3[Y] = t_1[Y]$  and  $t_4[Y] = t_2[Y]$ .
  - $t_3[Z] = t_2[Z]$  and  $t_4[Z] = t_1[Z]$ .
- An MVD  $X \twoheadrightarrow Y$  in  $R$  is called a **trivial MVD** if (a)  $Y$  is a subset of  $X$ , or (b)  $X \cup Y = R$ .

# Multivalued Dependencies and Fourth Normal Form (3)

## Inference Rules for Functional and Multivalued Dependencies:

IR1 (reflexive rule for FDs): If  $X \supseteq Y$ , then  $X \rightarrow Y$ .

IR2 (augmentation rule for FDs):  $\{X \rightarrow Y\} \mid = XZ \rightarrow YZ$ .

IR3 (transitive rule for FDs):  $\{X \rightarrow Y, Y \rightarrow Z\} \mid = X \rightarrow Z$ .

IR4 (complementation rule for MVDs):  $\{X \twoheadrightarrow Y\} \mid = X \twoheadrightarrow (R - (X \cup Y))$ .

IR5 (augmentation rule for MVDs): If  $X \twoheadrightarrow Y$  and  $W \supseteq Z$  then  $WX \twoheadrightarrow YZ$ .

IR6 (transitive rule for MVDs):  $\{X \twoheadrightarrow Y, Y \twoheadrightarrow Z\} \mid = X \twoheadrightarrow (Z \sqcup Y)$ .

IR7 (replication rule for FD to MVD):  $\{X \rightarrow Y\} \mid = X \twoheadrightarrow Y$ .

IR8 (coalescence rule for FDs and MVDs): If  $X \twoheadrightarrow Y$  and there exists  $W$  with the properties that (a)  $W \cap Y$  is empty, (b)  $W \rightarrow Z$ , and (c)  $Y \supseteq Z$ , then  $X \rightarrow Z$ .

# Multivalued Dependencies and Fourth Normal Form (4)

## Definition:

- A relation schema  $R$  is in **4NF** with respect to a set of dependencies  $F$  (that includes functional dependencies and multivalued dependencies) if, for every *nontrivial* multivalued dependency  $X \twoheadrightarrow Y$  in  $F^+$ ,  $X$  is a superkey for  $R$ .

Note:  $F^+$  is the (complete) set of all dependencies (functional or multivalued) that will hold in every relation state  $r$  of  $R$  that satisfies  $F$ . It is also called the **closure** of  $F$ .

# Multivalued Dependencies and Fourth Normal Form (5)

Decomposing a relation state of EMP that is not in 4NF. (a) EMP relation with additional tuples. (b) Two corresponding 4NF relations EMP\_PROJECTS and EMP\_DEPENDENTS.

(a) **EMP**

<u>ENAME</u>	PNAME	<u>DNAME</u>
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Smith	X	John
Smith	Y	Anna
Smith	X	Anna
Smith	Y	John
Brown	W	Jim
Brown	X	Jim
Brown	Y	Jim
Brown	Z	Jim
Brown	W	Joan
Brown	X	Joan
Brown	Y	Joan
Brown	Z	Joan
Brown	W	Bob
Brown	X	Bob
Brown	Y	Bob
Brown	Z	Bob

(b) **EMP\_PROJECTS**

<u>ENAME</u>	<u>PNAME</u>
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Smith	X
Smith	Y
Brown	W
Brown	X
Brown	Y
Brown	Z

**EMP\_DEPENDENTS**

<u>ENAME</u>	<u>DNAME</u>
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Smith	Anna
Smith	John
Brown	Jim
Brown	Joan
Brown	Bob

# Multivalued Dependencies and Fourth Normal Form (6)

## Lossless (Non-additive) Join Decomposition into 4NF

### Relations:

- **PROPERTY LJ1'**

The relation schemas  $R_1$  and  $R_2$  form a lossless (non-additive) join decomposition of  $R$  with respect to a set  $F$  of functional *and* multivalued dependencies if and only if

$$(R_1 \cap R_2) \twoheadrightarrow (R_1 - R_2)$$

or by symmetry, if and only if

$$(R_1 \cap R_2) \twoheadrightarrow (R_2 - R_1).$$



# Multivalued Dependencies and Fourth Normal Form (7)

## **Algorithm 11.5: Relational decomposition into 4NF relations with non-additive join property**

**Input:** A universal relation  $R$  and a set of functional and multivalued dependencies  $F$ .

1. Set  $D := \{ R \}$ ;
2. While there is a relation schema  $Q$  in  $D$  that is not in 4NF do
  - { choose a relation schema  $Q$  in  $D$  that is not in 4NF;
  - find a nontrivial MVD  $X \twoheadrightarrow Y$  in  $Q$  that violates 4NF;
  - replace  $Q$  in  $D$  by two relation schemas  $(Q - Y)$  and  $(X \vee Y)$ ;
  - };

# 4. Join Dependencies and Fifth Normal Form

## (1)

### Definition:

- A **join dependency (JD)**, denoted by  $JD(R_1, R_2, \dots, R_n)$ , specified on relation schema  $R$ , specifies a constraint on the states  $r$  of  $R$ . The constraint states that every legal state  $r$  of  $R$  should have a non-additive join decomposition into  $R_1, R_2, \dots, R_n$ ; that is, for every such  $r$  we have

$$* (\pi_{R_1}(r), \pi_{R_2}(r), \dots, \pi_{R_n}(r)) = r$$

*Note: an MVD is a special case of a JD where  $n = 2$ .*

- A join dependency  $JD(R_1, R_2, \dots, R_n)$ , specified on relation schema  $R$ , is a **trivial JD** if one of the relation schemas  $R_i$  in  $JD(R_1, R_2, \dots, R_n)$  is equal to  $R$ .

# Join Dependencies and Fifth Normal Form (2)

## Definition:

- A relation schema  $R$  is in **fifth normal form (5NF)** (or **Project-Join Normal Form (PJNF)**) with respect to a set  $F$  of functional, multivalued, and join dependencies if, for every nontrivial join dependency  $JD(R_1, R_2, \dots, R_n)$  in  $F^+$  (that is, implied by  $F$ ), every  $R_i$  is a superkey of  $R$ .

# Relation SUPPLY with Join Dependency and conversion to Fifth Normal Form

(c) The relation SUPPLY with no MVDs is in 4NF but not in 5NF if it has the JD(R1, R2, R3).

(d) Decomposing the relation SUPPLY into the 5NF relations R1, R2, and R3.

(c) **SUPPLY**

SNAME	PARTNAME	PROJNAME
Smith	Bolt	ProjX
Smith	Nut	ProjY
Adamsky	Bolt	ProjY
Walton	Nut	ProjZ
Adamsky	Nail	ProjX
Adamsky	Bolt	ProjX
Smith	Bolt	ProjY

(d) **R1**

SNAME	PARTNAME
Smith	Bolt
Smith	Nut
Adamsky	Bolt
Walton	Nut
Adamsky	Nail

**R2**

SNAME	PROJNAME
Smith	ProjX
Smith	ProjY
Adamsky	ProjY
Walton	ProjZ
Adamsky	ProjX

**R3**

PARTNAME	PROJNAME
Bolt	ProjX
Nut	ProjY
Bolt	ProjY
Nut	ProjZ
Nail	ProjX

## 5. Inclusion Dependencies (1)

### Definition:

- An **inclusion dependency**  $R.X < S.Y$  between two sets of attributes— $X$  of relation schema  $R$ , and  $Y$  of relation schema  $S$ —specifies the constraint that, at any specific time when  $r$  is a relation state of  $R$  and  $s$  a relation state of  $S$ , we must have

$$\pi_X(r(R)) \supseteq \pi_Y(s(S))$$

**Note:** The  $\supseteq$  (subset) relationship does not necessarily have to be a proper subset. The sets of attributes on which the inclusion dependency is specified— $X$  of  $R$  and  $Y$  of  $S$ —must have the same number of attributes. In addition, the domains for each pair of corresponding attributes should be compatible.

# Inclusion Dependencies (2)

## Objective of Inclusion Dependencies:

To formalize two types of interrelational constraints which cannot be expressed using F.D.s or MVDs:

- Referential integrity constraints
- Class/subclass relationships

## ● Inclusion dependency inference rules

**IDIR1 (reflexivity):**  $R.X < R.X$ .

**IDIR2 (attribute correspondence):** If  $R.X < S.Y$

where  $X = \{A_1, A_2, \dots, A_n\}$  and  $Y = \{B_1, B_2, \dots, B_n\}$  and  $A_i$  Corresponds-to  $B_i$ , then  $R.A_i < S.B_i$   
for  $1 \leq i \leq n$ .

**IDIR3 (transitivity):** If  $R.X < S.Y$  and  $S.Y < T.Z$ , then  $R.X < T.Z$ .

## 6. Other Dependencies and Normal Forms (1)

### Template Dependencies:

- Template dependencies provide a technique for representing constraints in relations that typically have no easy and formal definitions.
- The idea is to specify a template—or example—that defines each constraint or dependency.
- There are two types of templates: tuple-generating templates and constraint-generating templates.
- A template consists of a number of **hypothesis tuples** that are meant to show an example of the tuples that may appear in one or more relations. The other part of the template is the **template conclusion**.

# Other Dependencies and Normal Forms (2)

Templates for some common types of dependencies.

(a) Template for functional dependency  $X \rightarrow Y$ .

(b) Template for the multivalued dependency  $X \twoheadrightarrow Y$ .

(c) Template for the inclusion dependency  $R.X \subset S.Y$ .

(a)	$R = \{ A, B, C, D \}$		
hypothesis	$\begin{array}{ccc} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_2 \end{array}$ <hr style="width: 50%; margin-left: 0;"/>	$X = \{ A, B \}$ $Y = \{ C, D \}$	
conclusion	$c_1 = c_2 \text{ and } d_1 = d_2$		
(b)	$R = \{ A, B, C, D \}$		
hypothesis	$\begin{array}{cccc} a_1 & b_1 & 1 & d_1 \\ a_1 & b_1 & 2 & d_2 \end{array}$ <hr style="width: 50%; margin-left: 0;"/>	$X = \{ A, B \}$ $Y = \{ C \}$	
conclusion	$\begin{array}{cccc} a_1 & b_1 & 2 & d_1 \\ a_1 & b_1 & 1 & d_2 \end{array}$		
(c)	$R = \{ A, B, C, D \}$	$S = \{ E, F, G \}$	
hypothesis	$\begin{array}{cccc} a_1 & b_1 & c_1 & d_1 \end{array}$ <hr style="width: 50%; margin-left: 0;"/>		$X = \{ C, D \}$ $Y = \{ E, F \}$
conclusion		$c_1 \quad d_1 \quad g$	



# Other Dependencies and Normal Forms (3)

Templates for the constraint that an employee's salary must be less than the supervisor's salary.

EMPLOYEE = { NAME , SSN , ... , SALARY , SUPERVISORSSN }

	a	b	c	d
hypothesis	e	d	f	g
conclusion	<hr/>			
	c < f			

# Other Dependencies and Normal Forms (4)

## Domain-Key Normal Form (DKNF):

- **Defintion:**A relation schema is said to be in **DKNF** if all constraints and dependencies that should hold on the valid relation states can be enforced simply by enforcing the domain constraints and key constraints on the relation.
- The **idea** is to specify (theoretically, at least) the “*ultimate normal form*” that takes into account all possible types of dependencies and constraints. .
- For a relation in DKNF, it becomes very straightforward to enforce all database constraints by simply checking that each attribute value in a tuple is of the appropriate domain and that every key constraint is enforced.
- The practical utility of DKNF is limited