Mesh Generation

Automatic Mesh Generation

1. Node Connection Approach
    1. Node Generation
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2. Topology Decomposition
3. Geometry Decomposition
    1. Recursive
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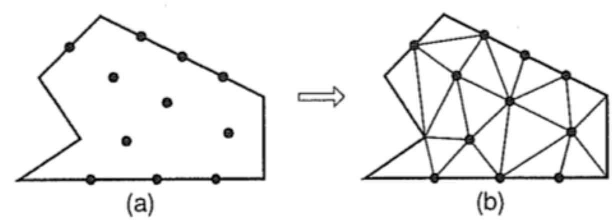
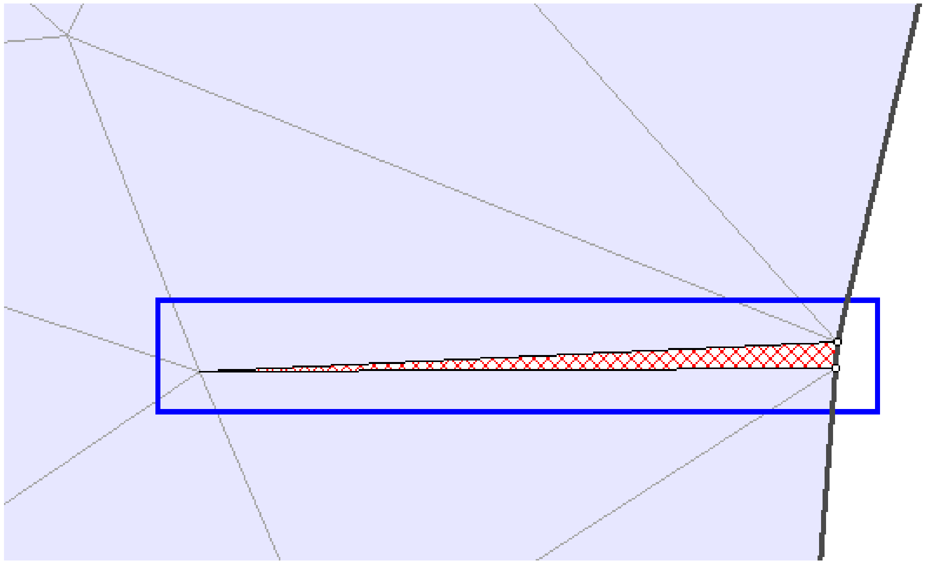
4. Grid-Based Approach
5. Mapped Element Approach



Mesh Quality

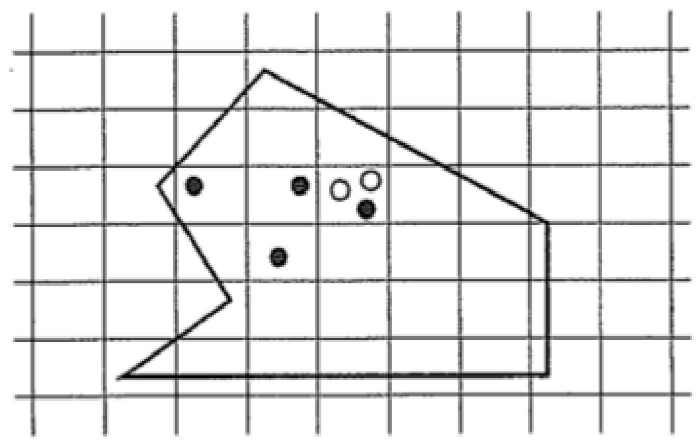
- Avoid high aspect ratio elements as they induce large numerical errors and sensitive results



1) Node Connection Approach Node Generation

Cavendish Method

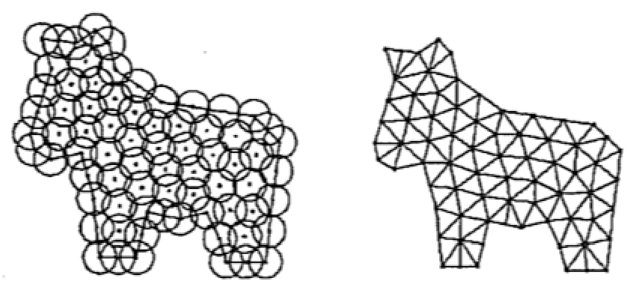
Divide the domain in zones. For each zone randomly pick a inner point, if for a fixed number of trials no point belonging to the domain (or too close to a predefined distance r(i) to another node or boundary) can be found, skip zone



1) Node Connection Approach
Node Generation

Shimada Method

Envision the domain filled with bubbles, each node is located at the center of eah bubble.



1) Node Connection Approach Element Generation

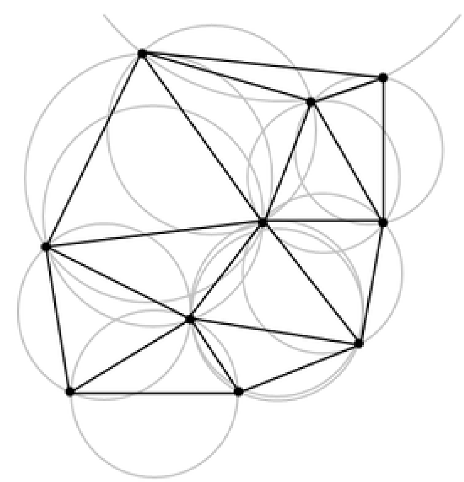
Lee's method

Used for quadrilateral elements

Delaunay Triangulation

A Delaunay triangulation for a set P of points in the plane is a triangulation DT(P) such that no point in P is inside the circumcircle of any triangle in DT(P). Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation; they tend to avoid skinny triangles.

Used Algorithms:
-Voronoi polygons
-Watson's algorithm

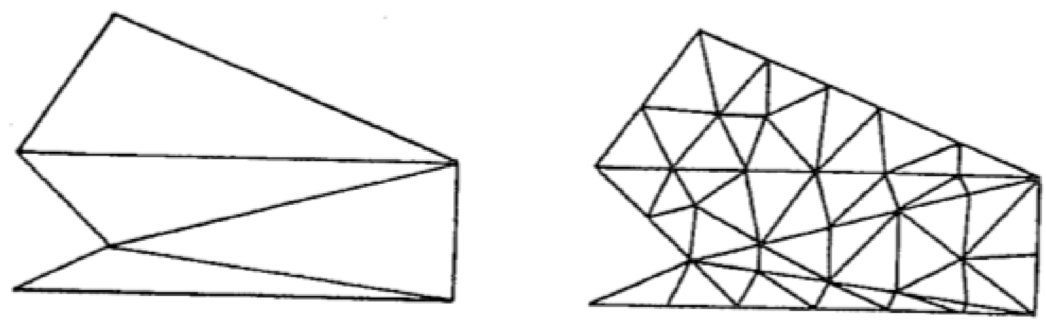


2) Topology Decomposition Approach

2D shown, 3D can be extrapolated.

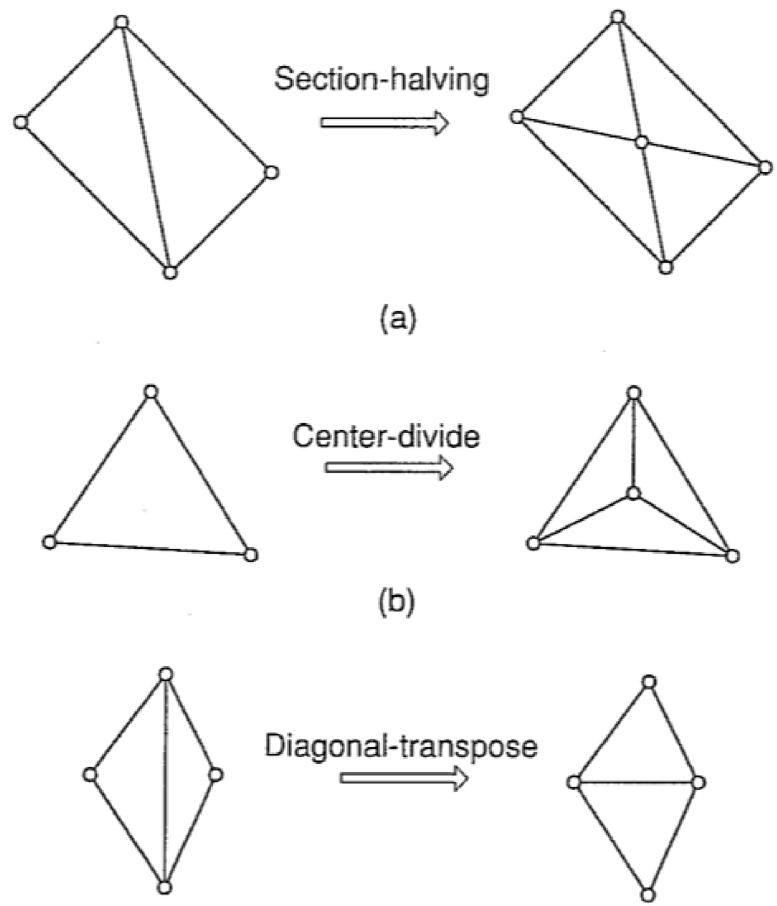
a)Object is approximated to be a polygon and it is decomposed into a set of gross elements (Triangles).

b)Each gross element is then refined.



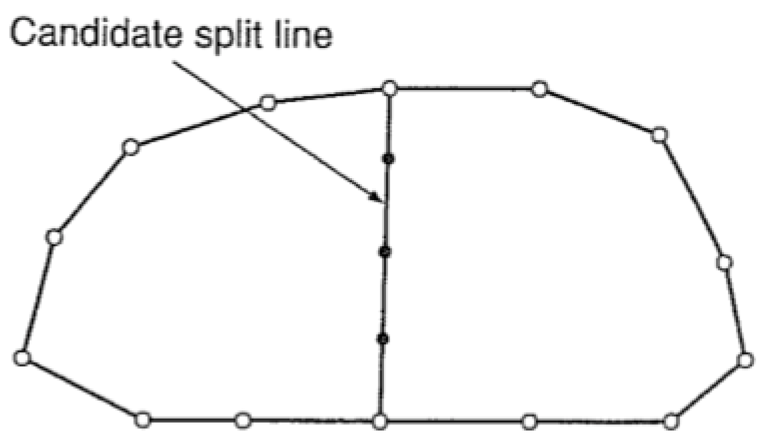
(a) (b)

Refining Methods



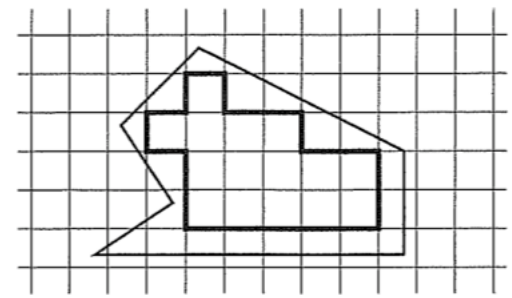
3) Geometry Decomposition Approach

- Recursive/iterative
  - Split the domain in convex subdomains
  - On each subdomain identify nodes on the boundaries at a distance to satisfy mesh distribution and convert boundaries to segments
  - Divide subdomain into 2 parts approximatly along the longest axis
  - Add nodes to dividing line.
  - Repeat operation until only triangles or quadrilaterals are left.



4) Grid-Based Approach

- Observing that a grid looks like a mesh:

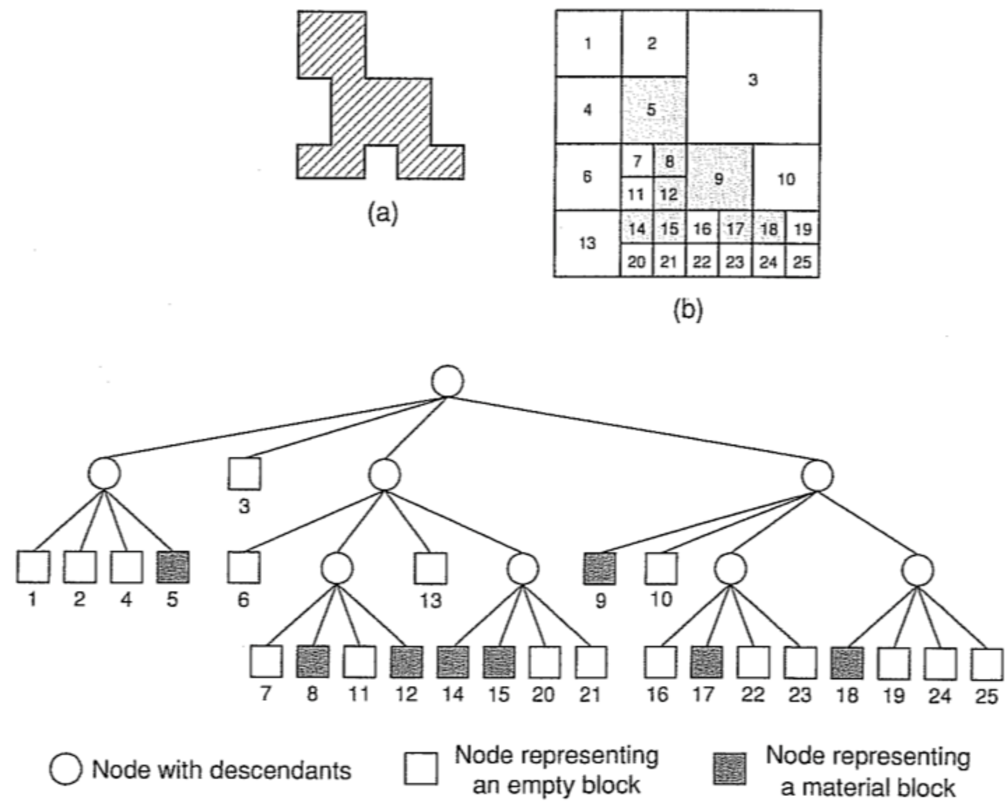


1)Enclose domain in a square

2)Subdivide it in 4 quadrants

3)Repeat the operation for each quadrant containing part of the domain until desired grid size is obtained

Grid Generation



(c)

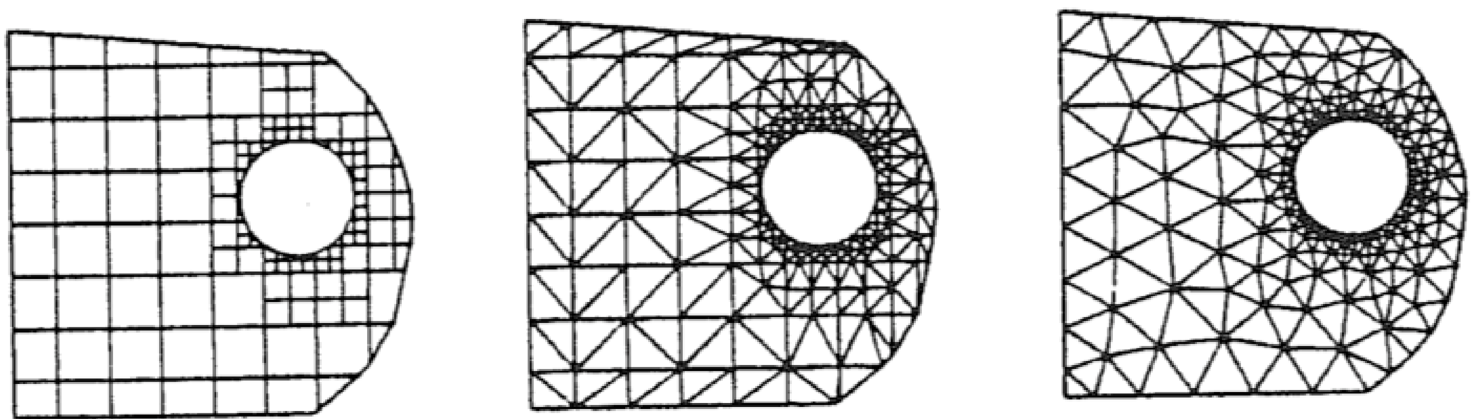
Mesh Generation

4) Convert Grid to mesh:

   a) Modify boundary elements to contain only domain portion

   b) Convert squares to triangle connecting diagonals

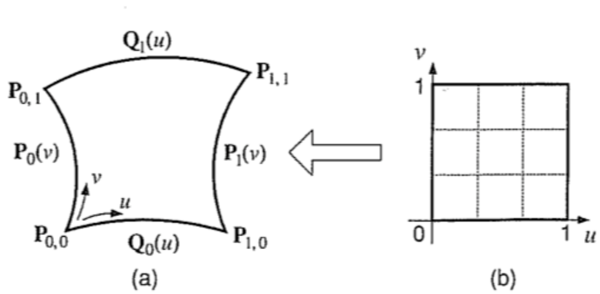
   c) Adopt mesh smothing for improving triangle quality.

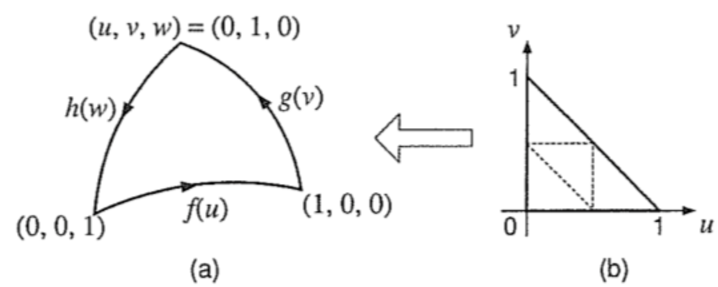


(a) (b) (c)

5) Mapped Element Approach

- Transfinite Mapping: it enables three or four-sided regions (2D) to be mapped to a regularized domain without geometric errors using parametric coordinates *u,v* for four-sided and *u,v,w* for three sided regions.Most commonly used by commercial packages.



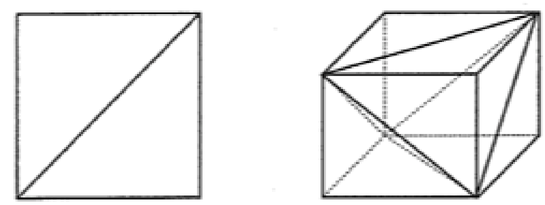


- Isoparametric Mapping: special case of transfinite mapping, where the geometric boundaries are discretized.

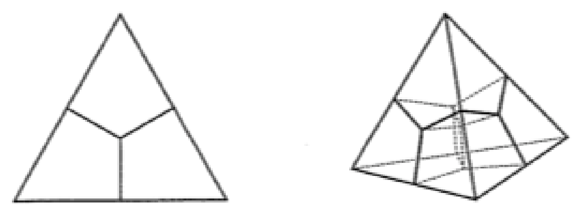
Improvement Mesh Quality

- Conversion of Element Type

Conversion of quadrilaterals and bricks into triangles and tetrahedra, respectively

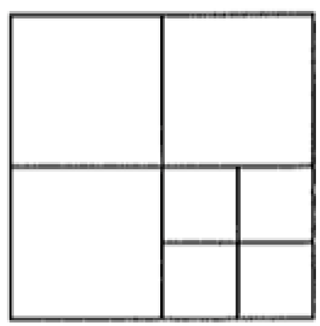


Conversion of a triangle and tetrahedron into quadrilaterals and bricks, respectively

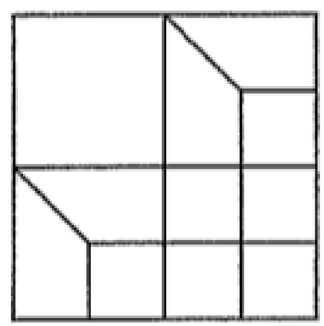


Improvement Mesh Quality

- Refinement of Meshes
  - Refinement may be desirable to satisfy generalized or localized mesh density
  - The refinement process might induce violation to comformity between neighboring elements (a)
  - Neighboring cells ar said to be comforming if they share an entire edge or face.



(a)



(b)

Improvement Mesh Quality

- Mesh Smoothing
  - Often automatic mesh generation can induce not well-shaped elements
  - Laplacian smoothing repositions the nodes so that each internal node is at the centroid of the polygon formed by its connecting neighbors.

Repositioning formula Herrmann [1976]:
w is a weighting factor, *w*=0 for Laplacian

$$\mathbf{P}_i = \frac{1}{N(2-w)} \sum_{n=1}^{N} (\mathbf{P}_{nj} + \mathbf{P}_{nl} - w\mathbf{P}_{nk})$$