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Unit 2. Fluid Dynamics



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Discharge (flow rate)

The amount of fluid passing a section of a stream in unit time is called the discharge. If v is the mean velocity and A is the cross sectional area, the discharge Q is defined by $Q = Av$ which is known as volume flow rate. Discharge is also expressed as mass flow rate and weight flow rate. Where:

Q = discharge in m^3/sec or ft^3/sec

A = cross-sectional area of flow in m^2 or ft^2

v = mean velocity of flow in m/sec or ft/sec

ρ = mass density of fluid in kg/m^3 or slugs/ft^3

γ = unit weight of fluid in N/m^3 or lb/ft^3

- **Euler's Equation of motion**

- Euler's equation of motion of an ideal fluid, for a steady flow along a stream line, is basically a relation between velocity, pressure and density of a moving fluid. Euler's equation of motion is based on the basic concept of Newton's second law of motion.

- When fluid will be in motion, there will be following forces associated as mentioned here.

1. Pressure force
2. Gravity force
3. Friction force due to viscosity
4. Force due to turbulence force
5. Force due to compressibility

- In Euler's equation of motion, we will consider the forces due to gravity and pressure only. Other forces will be neglected.

- **Assumptions**

- Euler's equation of motion is based on the following assumptions as mentioned here

1. The fluid is non-viscous. Frictional losses will be zero
2. The fluid is homogeneous and incompressible.
3. Fluid flow is steady, continuous and along the streamline.
4. Fluid flow velocity is uniform over the section
5. Only gravity force and pressure force will be under consideration.

- Let us consider that fluid is flowing from point A to point B and we have considered here one very small cylindrical section of this fluid flow of length dS and cross-sectional area dA as displayed here in following figure.

- ***Let us think about the forces acting on the cylindrical element***

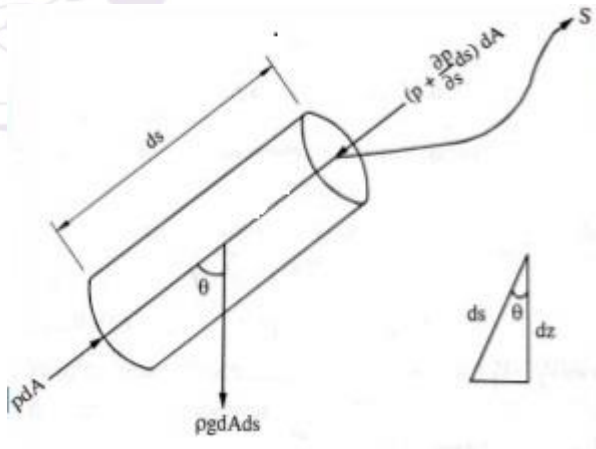
- Pressure force PdA , in the direction of fluid flow

- Pressure force $[P + (\partial P/\partial S) dS] dA$, in the opposite direction of fluid flow

- Weight of fluid element $(\rho g dA dS)$



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Let us consider that θ is the angle between the direction of fluid flow and the line of action of weight of the fluid element. As we have mentioned above that Euler's equation of motion is based on the basic concept of Newton's second law of motion.

Therefore, we can write here following equation as mentioned here

Net force over the fluid element in the direction of S = Mass of the fluid element x acceleration in the direction S.

$$P dA - \left(P + \frac{\partial P}{\partial S} ds \right) dA - \rho g dA ds \cos \theta = \rho dA ds a_s$$

a_s = Acceleration in direction of S.

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s} \frac{ds}{dt} + \frac{\partial V}{\partial t} = V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \quad \text{(Where V is a function of S and t)}$$

If the flow is steady $\frac{\partial V}{\partial t} = 0$,

$$\boxed{a_s = V \frac{\partial V}{\partial s}} \quad \& \quad \boxed{\cos \theta = \frac{dz}{ds}}$$

Simplify above equation by putting a_s & $\cos \theta$

$$\boxed{\frac{\partial P}{\rho} + V dV + g dZ = 0}$$



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BERNOULLI'S THEOREM

Bernoulli's theorem which is also known as Bernoulli's principle, states that an increase in the speed of moving air or a flowing fluid is accompanied by a decrease in the air or fluid's pressure or sum of the kinetic (velocity head), pressure (static head) and Potential energy of the fluid at any point remains constant, provided that the flow is steady, irrotational, and frictionless and the fluid is incompressible.

Bernoulli's equation from Euler's equation of motion could be derived by integrating the Euler's equation of motion.

According to Bernoulli's theorem.....

In an incompressible, ideal fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential energy will be constant along a stream line.

Assumptions

Assumptions made for deriving the Bernoulli's equation from Euler's equation of motion is as mentioned here.

1. Fluid is ideal, i.e. inviscid and incompressible.
2. Fluid flow is steady, one-dimensional and uniform
3. Fluid flow is irrotational
4. Forces which are considered are only pressure force and gravity force. Rest forces acting on fluid are neglected.

Let us recall the Euler's equation of motion. We will integrate the Euler's equation of motion in order to secure the Bernoulli's equation.

$$\frac{\partial P}{\rho} + VdV + gdZ = 0$$

Bernoulli's equation is obtained by integrating the Euler's equation of motion.

$$\int \frac{\partial P}{\rho} + \int VdV + \int gdZ = 0$$

$$\frac{P}{\rho} + \frac{V^2}{2} + gZ = \text{const.}$$

$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{const.}$$



$$\frac{P}{\rho g} = \text{Pressure head}$$

$$\frac{V^2}{2g} = \text{Kinetic head}$$

$$Z = \text{Potential or Elevation head}$$



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<https://www.youtube.com/watch?v=UJ3-Zm1wbIQ>