

Fluid Kinematics

Kinematics of flow:

Kinematics is a branch of science which deals with the motion of the particles without considering the forces causing the motion.

The velocity at any point in a flow field at any time is studied in the branch of fluid mechanics. Once, the velocity will be known then pressure distribution and hence force acting on the fluid can be determined.

The fluid motion will be described by 2 methods

i) Lagrangian method - In this method a single fluid particle is followed during its motion and its velocity, accn, density etc are described

ii) Eulerian method (EM): In case of EM the velocity, accn, pressure, density etc are described at a point in flow field. It is used generally in fluid mechanics.

Types of fluid flow

- i) steady and unsteady flow
- ii) uniform and non-uniform flow
- iii) laminar and turbulent flow
- iv) compressible & incompressible flow.
- v) one, two, three dimensional flow.

i) steady flow is defined as type of flow in which the fluid's characteristics like velocity, pressure, density etc do not change with respect to time at any time point.

Mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

Unsteady flow is the type of flow in which the velocity, pressure, density etc changes w.r.t time at any given point.
Mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

ii) Uniform flow is the type of flow in which the velocity at any given time will not change w.r.t space (length of direction of flow).
Mathematically

$$\left(\frac{\partial v}{\partial s}\right)_t = 0 \quad \text{ss - change of length of flow in direction of } s$$

Non-uniform flow is the type of flow in which the velocity at any given time will change w.r.t space.

Mathematically $\left(\frac{\partial v}{\partial s}\right)_t \neq 0$

iii) Laminar and Turbulent flow
 $Re = \frac{\rho v D}{\mu}$ μ - Kinematic viscosity
 D - Diameter of fluid.

(Re) < 2000 , then the flow is laminar flow
(Re) > 4000 , then flow is turbulent flow
→ If the Re no. is b/w 2000 to 4000 the flow is transition flow.

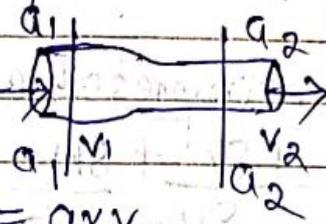
iv) Compressible and incompressible flow -
($\rho \neq \text{const}$) ($\rho = \text{const}$)

v) one, two, three dimensional flow →

Rate of Flow & Rate of discharge (Q)

$Q = m^3/sec. = m^2 \cdot m/s = \text{Area} \times \text{Velocity}$
 $= AXV = axv.$

Continuity equation:

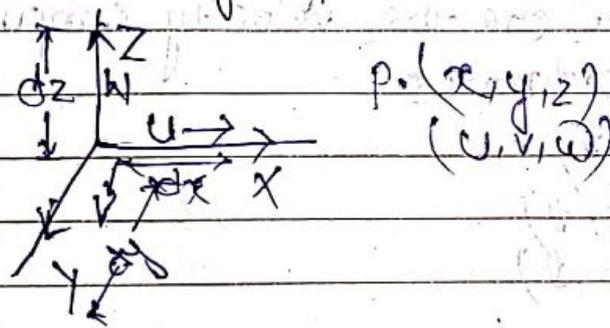


\dot{v} (volume flow rate) $= axv$

- $\rightarrow \dot{m} = \rho \dot{v} = \rho axv$
- $\rightarrow \dot{m} = \rho_1 a_1 v_1 = \rho_2 a_2 v_2$
- $\rightarrow \rho_1 = \rho_2 = \rho = \text{const.}$
- $\rightarrow \dot{m} = \rho a_1 v_1 = \rho a_2 v_2$
 $= a_1 v_1 = a_2 v_2$

$\boxed{\dot{m} = a_1 v_1 = a_2 v_2}$ One dimensional incompressible flow.

Continuity eqn in 3-Dimension



$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} = 0 \quad \text{--- (i)}$$

consider a fluid element of length dx, dy, dz in the dir. of x, y & z and let u, v, w are the velocity component in x, y & z direction.

Eqn (i) is called as continuity eqn in cartesian co-ordinate. This eqn is applicable for steady and unsteady flow, uniform & non-uniform flow, compressible & incompressible flow.

(i) For steady flow, $\frac{\delta p}{\delta t} = 0$ $\left[\frac{\delta(pu)}{\delta x} + \frac{\delta(pv)}{\delta y} + \frac{\delta(pw)}{\delta z} \right] = 0$

(ii) For incompressible flow

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0$$

(iii) For two dimensional flow, $\frac{\delta w}{\delta z} = 0$

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 0$$

Velocity and Acceleration of the fluid

Let, v is the resultant velocity of a point in a fluid flow. Let u, v & w are the velocity components in x, y & z co-ordinates.

$$u = F_1(x, y, z, t)$$

$$v = F_2(x, y, z, t)$$

$$w = F_3(x, y, z, t)$$

So, the resultant velocity, $v = u\hat{i} + v\hat{j} + w\hat{k}$
 $= \sqrt{u^2 + v^2 + w^2}$

Let, us consider a_x, a_y & a_z are the total accn in x, y & z direction respectively

$$a_x = \frac{du}{dt} = \frac{\delta u}{\delta x} \frac{\delta x}{\delta t} + \frac{\delta u}{\delta y} \frac{\delta y}{\delta t} + \frac{\delta u}{\delta z} \frac{\delta z}{\delta t} + \frac{\delta u}{\delta t}$$

$$a_x = u \frac{\delta u}{\delta x} + v \frac{\delta u}{\delta y} + w \frac{\delta u}{\delta z} + \frac{\delta u}{\delta t}$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial x} \frac{dx}{dt} + \frac{\partial v}{\partial y} \frac{dy}{dt} + \frac{\partial v}{\partial z} \frac{dz}{dt} + \frac{\partial v}{\partial t}$$

$$= u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt} + \frac{\partial w}{\partial t}$$

$$= u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Local accn = $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$, rest are convective accn

$$A = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$= \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Local accn is defined as the rate of increase of velocity w.r.t time at a given point in a flow field.

Convective accn - It is defined as the rate of change of velocity due to the change of posn of fluid particle in a fluid flow. The expression other than $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ are called convective accn.

Ques: The following cases represent the 2 velocity components & determine the 3rd component of velocity such that they satisfy the continuity eqn

i) $u = x^2 + y^2 + z^2, v = xy^2 - yz^2 + xy$

ii) $v = 2yz, w = 2xyz$

Soln - Continuity eqn

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$i) \frac{\delta(x^2 + y^2 + z^2)}{\delta x} + \frac{\delta(xy^2 - yz^2 + xy)}{\delta y} + \frac{\delta w}{\delta z} = 0$$

$$\Rightarrow 2x + 2xy - z^2 + x + \frac{\delta w}{\delta z} = 0$$

$$\Rightarrow 3x + 2xy - z^2 + \frac{\delta w}{\delta z} = 0$$

$$\Rightarrow \frac{\delta w}{\delta z} = -(3x + 2xy - z^2)$$

$$\Rightarrow \int \delta w = (-3x - 2xy + z^2) \delta z$$

$$\Rightarrow w = -3xz - 2xyz + \frac{z^3}{3} + C$$

$$ii) \frac{\delta u}{\delta x} + \frac{\delta(2y^2)}{\delta y} + \frac{\delta(2xy^2)}{\delta z} = 0$$

$$\Rightarrow \frac{\delta u}{\delta x} + 4xy + 2xy = 0$$

$$\Rightarrow \frac{\delta u}{\delta x} = -4xy - 2xy$$

$$\Rightarrow \int \delta u = (-4xy - 2xy) \delta x$$

$$\Rightarrow u = -6xy^2 + C \quad \text{(Ans)}$$

$$\Rightarrow \int \delta u = (-4y - 2xy) \delta x$$

$$\Rightarrow u = -4xy - x^2y + C$$

$$\Rightarrow u = -4xy - x^2y + C \quad \text{(Ans)}$$

Velocity potential Function & Stream function
 It is defined as scalar function of space & time so that the derivative in any direction will give the velocity in that direction. It is denoted by ϕ .

$$\phi = f(x, y, z)$$

$$-\left(\frac{\partial \phi}{\partial x}\right) = u \quad , \quad -\left(\frac{\partial \phi}{\partial z}\right) = w$$

$-\left(\frac{\partial \phi}{\partial y}\right) = v$ It is applicable for irrotational, 3-Dimensional and incompressible fluid.

Continuity eqn for 3 dimensional fluid

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$= \frac{\partial}{\partial x} \left(-\frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(-\frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left(-\frac{\partial \phi}{\partial z} \right) = 0$$

$$= -\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$\Rightarrow \boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0}$$

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

→ Stream function It is defined as the scalar function of space and time such that its partial derivative w.r.t any direction gives the velocity component at 90° angle to that direction.

$$\frac{\partial \psi}{\partial x} = v, \quad \frac{\partial \psi}{\partial y} = -u, \quad \frac{\partial \psi}{\partial z} = 0$$

→ Equipotential line : $\phi = \text{const.}$
 $\phi = F(x, y)$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$\Rightarrow 0 = -u dx - v dy$$

$$\Rightarrow u dx = -v dy$$

$$\Rightarrow \left| \frac{dy}{dx} = -\left(\frac{u}{v}\right) \right|$$

Slope of equipotential line or line of con

Line of constant stream function
 $\psi = \text{const.}, \quad d\psi = 0$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$\Rightarrow 0 = v dx - u dy$$

$$\Rightarrow v dx = u dy$$

$$\Rightarrow \left| \frac{dy}{dx} = \frac{v}{u} \right|$$

Slope of stream line.

Relation b/w velocity potential & Stream Function

$$\begin{aligned}
 u &= -\frac{\partial \phi}{\partial x} & u &= -\frac{\partial \psi}{\partial y} & \Rightarrow & -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} \\
 v &= -\frac{\partial \phi}{\partial y} & v &= \frac{\partial \psi}{\partial x} & \Rightarrow & \left[\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \right] \\
 & & & & & \Rightarrow \boxed{\begin{matrix} -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x} \end{matrix}}
 \end{aligned}$$

Q) The velocity potential function $\phi = -xy^3 - x^2 + \frac{x^3y}{3} + y^2$.
 Show that the above component will satisfy the continuity eqn.

Sol:
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3y}{3} + y^2$$

$$\frac{\partial \phi}{\partial x} = \frac{-y^3}{3} - 2x + \frac{3x^2y}{3} + 0$$

$$\Rightarrow \frac{\partial^2 \phi}{\partial x^2} = 0 - 2 + \frac{2xy}{1} = -2 + 2xy$$

$$\frac{\partial \phi}{\partial y} = -xy^2 + x^3 + 2y$$

$$\frac{\partial^2 \phi}{\partial y^2} = -2xy + 2$$

$$\therefore \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2 + 2xy - 2xy + 2 = 0 \quad \text{proved}$$

Hence, the ^{given} eqn satisfies the continuity eqn.

Que: If for a two-d potential flow the velocity potential is given by $\phi = x(2y-1)$. Determine the velocity at the point $P(4,5)$ (b) Determine the value of stream function ψ at the point P .

Sol: i) $v = \sqrt{u^2 + v^2}$

$u = \frac{\partial \phi}{\partial x} = (2y-1)$ at $x=4, y=5$

~~$u = 8 - 5 = 3 - 9 = -6$~~ $u = -(10-1) = -9$

~~$v = \frac{\partial \phi}{\partial y} = -2x = -8$~~

~~$\therefore v = \sqrt{(-9)^2 + (-8)^2} = \sqrt{81+64} = \sqrt{145}$~~

~~$= 10 \text{ m/s}$~~

$= \sqrt{81+64} = \sqrt{145} = 12.05 \text{ Unit.}$

~~$v = \frac{\partial \psi}{\partial x} = (2y-1)$ at $x=4, y=5$~~

~~$u = -\frac{\partial \psi}{\partial y} = -2x = -8$~~

~~ii) $\psi = u = +\frac{\partial \psi}{\partial y} = -(-2y+1)$~~

~~$\int \frac{\partial \psi}{\partial y} = \int (2y+1)$~~

~~$\rightarrow \psi = (2y+1)y$~~

~~$\psi = y^2 + y + c$~~

$$\frac{\delta \psi}{\delta x} = \frac{\delta K}{\delta x}$$

$$\Rightarrow \frac{\delta \psi}{\delta x} = v = -2x = \frac{\delta K}{\delta x}$$

$$\Rightarrow \frac{\delta K}{\delta x} = -2x \Rightarrow \delta K = (-2x) \delta x$$

$$\Rightarrow K = \frac{-2x^2}{2} \Rightarrow K = -x^2$$

putting $K = -x^2$ in (1)

$$\psi = y^2 - y - x^2$$

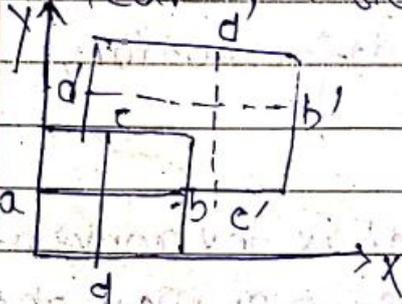
$$\text{at } p(4, 5)$$

$$\psi = 25 - 5 - 16 = 4$$

Types of motion

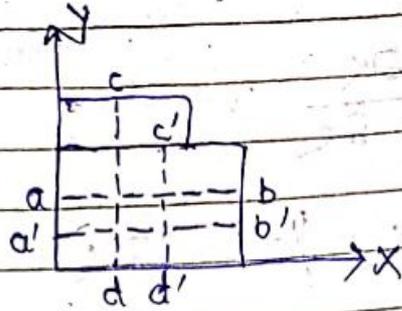
- 1) Linear Translation or pure translation
- 2) Linear Deformation
- 3) Angular Deformation
- 4) Rotation

① Linear Translation ①



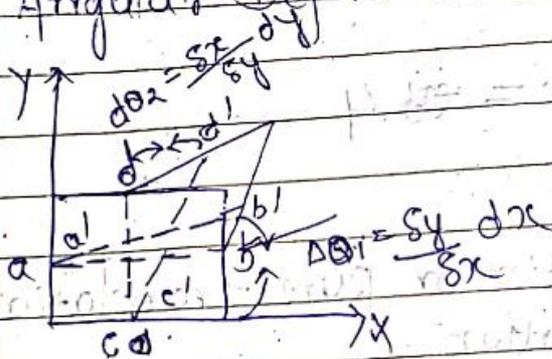
It is defined as the movement of a fluid element in such a way that it moves linearly from one position to another position and the x axis ab and cd are represented in a new position $a'b''$ and $c'd''$.

② Linear Deformation



The deformation of the fluid element in linear direction when the element moves the axis of the deformed position and undeformed position are $a'b'$, $c'd'$ and ab , cd respt.

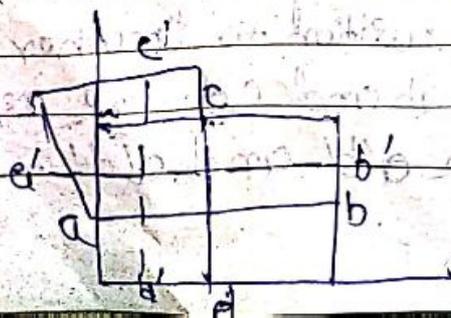
③ Angular Deformation



It is defined as the average changes in angle contained by 2 adjacent sides. Let $d\theta_1$ and $d\theta_2$ is the change in the angle b/w 2 adjacent sides of the fluid element also the angular or shear deformation,

$$= \frac{1}{2}(d\theta_1 + d\theta_2)$$

④ Rotation - It is defined as the movement of fluid element in such a way that both of its horizontal and vertical rotate in the same direction.



Expression of rotation in

x-y direction = ω_z , y-z direction = ω_x

z-x direction = ω_y

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \quad \omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_y = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \quad \omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \quad y-z \text{ dir.}$$

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \quad \omega_y = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \quad z-x \text{ dir.}$$

These are the components of rotation.

→ Vorticity It is defined as the value twice of the rotation and hence $\text{Vorticity} = 2\omega$.

Que:- A fluid is given by $v = 8x^3i - 10x^2yj$. Find the shear strain rate and state whether the flow is rotational or irrotational.

Sol:- Given, $v = 8x^3i - 10x^2yj$
 $u = 8x^3, \frac{\partial u}{\partial x} = 24x^2, \frac{\partial u}{\partial y} = 0$

and $v = -10x^2y, \frac{\partial v}{\partial x} = -20xy, \frac{\partial v}{\partial y} = -10x^2$

(i) Shear strain rate is given by eqn

$$= \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) = \frac{1}{2} (-20xy + 0) = -10xy \text{ Ans}$$

Rotation in x-y plane is given by

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{1}{2} (-20xy - 0) = -10xy$$

As, $\omega_z \neq 0$. Hence, flow is rotational.

VORTEX FLOW is defined as the flow of a fluid along a curve path or the flow of a rotation mass of fluid is known as vortex flow.

Types of vortex flow :-

- 1) Forced vortex
- 2) Free vortex

1) Forced vortex is defined as the type of vortex flow in which some external torque is required to rotate the fluid mass. The fluid mass rotate at a const angular velocity.

$$v = \omega \times r \text{ where } r, \text{ radius of fluid particle}$$

e.g. A vertical cylinder containing liquid which is rotated about its central axis with angular velocity ω .

(b) Flow of liquid inside the impeller of centrifugal pump.

2) Free vortex when no external torque is required to rotate the fluid mass that rotation is called free vortex flow.

e.g. A whirlpool in a river, flow of liquid around a circular bend in a pipe, flow of fluid in a centrifugal pump casing.

Dynamic of fluid flow

(19) The velocity components in a 2-D flow are $u = y^3/3 + 2x - x^2y$ and $v = xy^2 - 2y - x^3/3$. Show that these components represents a possible case of irrotational flow.

Dynamic of flow fluid flow

Solved

$$\text{Sol: } u = \frac{y^3}{3} + 2x - x^2y, \quad v = xy^2 - 2y - \frac{x^3}{3}$$

But we know continuity eqn for a rotational & irrotational flow is $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$ in x-y dir. is

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$= \frac{1}{2} (y^2 - x^2 - y^2 + x^2) = \frac{1}{2} \times 0 = 0$$

$\therefore \omega_z = 0$, Hence, it is a case of irrotational flow.

16/07/19 Dynamic of fluid flow

* Dynamic of fluid flow It includes the study of forces causing the motion of the fluid.

It is the study of fluid motion due to the forces acting on it. It is analysed by Newton's law of motion:

$$F_x = ma$$

- 1) Force due to gravity (F_g)
- 2) Force due to pressure (F_p)
- 3) Force due to viscosity (F_v)
- 4) Force due to turbulent (F_t)
- 5) Force due to compressibility (F_c).

All the above forces acting on fluid when it is in motion.

$$\text{The net force } (F_{net}) = F_g + F_p + F_v + F_t + F_c$$

→ Reynold's eqn of motion

$$F_x = F_g + F_p + F_v + F_t \quad (\because F_c = 0) \quad \text{--- (1)}$$

When the force due to compressibility is zero the net force will be F_x (1) which is called Reynold's eqn of motion.

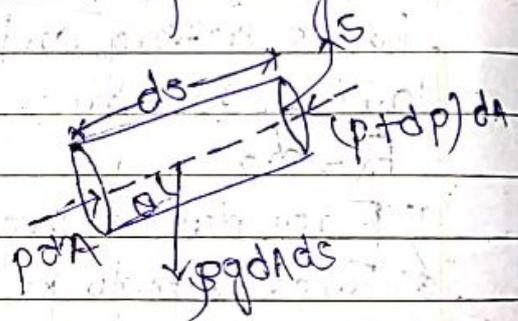
→ When the force due to compressibility and turbulent is equal to zero - the net force will be given by $F_x = F_g + F_p + F_v$ - (1) which is called Navier stroke eqn.

→ When $F_v = 0, F_t = 0, F_c = 0$
 $F_x = F_g + F_p$ - This eqn is called Euler's equation of motion.

Euler's equation of motion: - In this eqn of motion the forces due to gravity and pressure force are taken into consideration. This is derived by considering the motion of the fluid element along a stream line.

Consider a fluid which is flowing in S direction.

Consider a cylindrical element of cross-section DA and length ds .



The net force acting on the element are pDA in the dir of flow. $(p + \frac{\rho ds}{\rho} dp) DA$ is the force which is acting opposite to the dir of motion of fluid.
 weight of the element = $\rho g DA ds$

$$\frac{\rho}{\rho} dp + v dv + g dz = 0$$

Bernoulli's Equation of Motion: - Bernoulli's eqn of motion will be derived by integrating the Euler's eqn.

$$\int \left(\frac{\rho}{\rho} + \right) v dr + \int g dz = \text{const}$$

$$\rightarrow \frac{\rho}{\rho} + \frac{v^2}{2} + gz = \text{const}$$

Dividing g both sides

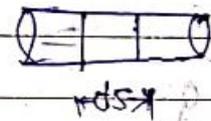
$$\rightarrow \frac{\rho}{\rho g} + \frac{v^2}{2g} + z = \text{const}$$

$\frac{\rho}{\rho g}$ = pressure energy per unit weight of the fluid which is called as pressure head.

$\frac{v^2}{2g}$ = kinetic energy per unit weight of the fluid particle which is termed as kinetic head.

z = potential energy per unit weight of the fluid particle or it is called as potential or datum head.

$$28/07/19 \quad \left(\frac{\rho}{\rho g} + \frac{v^2}{2g} + z = \text{const} \right)$$



$$W = F \times d \times s$$

$$= p \times A \times d \times s$$

$$= p \times v = p \times \frac{m}{\rho}$$

$$= \frac{p \times m}{\rho \times g} = \frac{p}{\rho g}$$

* Assumption for Bernoulli's eqn.

i) The fluid is ideal i.e. viscosity = 0

ii) The flow must be steady

iii) The fluid must be incompressible

iv) The fluid is irrotational

* Bernoulli's principle states that for a ideal incompressible and steady flow the total energy or total head remain constant of a flowing fluid.

Q) Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 and with mean velocity of 2 m/s . Find the total head of the water and Corres. of a cross section which is 5 m above the datum line.

Sol/s - Given $D = 5 \text{ cm}$ $p = 29.43 \text{ N/cm}^2$
 $\phantom{\text{Sol/s - Given}} = 0.05 \text{ m}$ $= 29.43 \times 10^4 \text{ N/m}^2$

$v = 2 \text{ m/s}$, $z = 5 \text{ m}$

$\frac{p}{\rho g} = \frac{29.43 \times 10^4}{10^3 \times 9.81} = 29.88730$

$\frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204$

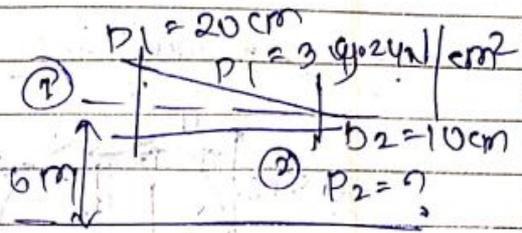
$z = 5$

Total head = $\frac{p}{\rho g} + \frac{v^2}{2g} + z = 29.88730 + 0.204 + 5 = 35.09130$

Q) Water is flowing through a pipe having diameters 20 cm and 10 cm at section 1 and 2 respectively. The rate of flow through the pipe is 35 l/sec . The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm^2

find the intensity of pressure at section 2.

Sol:- $Q = 85 \text{ l/sec} =$
 $85 \times 0.001 \text{ m}^3/\text{sec}$
 $= 0.085 \text{ m}^3/\text{sec}$



At section 1
 $D_1 = 0.20 \text{ m}$
 $p_1 = 34.24 \text{ N/cm}^2$
 $= 34.24 \times 10^4 \text{ N/m}^2$
 $z_1 = 6 \text{ m}$

At section 2
 $D_2 = 0.10 \text{ m}$
 $z_2 = 4 \text{ m}$

$Q = a_1 v_1 = a_2 v_2$

$85 \times 10^{-3} = \frac{\pi}{4} (0.20)^2 \times v_1 = \frac{\pi}{4} (0.10)^2 \times v_2$

$\Rightarrow 0.085 = 0.00785 \times v_2$

$v_2 = 10.82 \text{ m/s}$

$v_1 = \frac{0.085}{0.0314} = 2.707 \text{ m/s}$

$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$

$\frac{34.24 \times 10^4}{10^3 \times 9.81} + \frac{2.707^2}{2 \times 9.81} + 6 = \frac{p_2}{10^3 \times 9.81} + \frac{10.82^2}{2 \times 9.81} + 4$

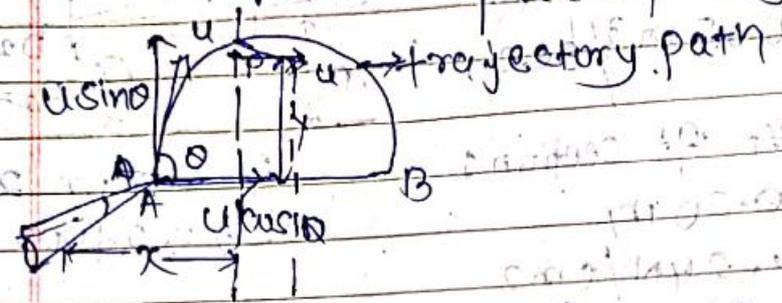
$\Rightarrow 40 + 0.063 + 6 = \frac{p_2}{9810} + 1.009 + 4$

$\Rightarrow 46.063 - 5.009 = \frac{p_2}{9810}$

$\Rightarrow p_2 = 41.054 \times 9810 = 402739.7 \text{ N/m}^2$

$\approx 40.274 \text{ N/cm}^2$ (Ans)

Free liquid jet



Free liquid jet is defined as the jet of water coming out from nozzle in atmosphere the path travelled by the free jet is parabolic. Let the jet at A makes an angle θ with the horizontal direction if u is the velocity of jet of water then, horizontal and vertical components are $u \cos \theta$ and $u \sin \theta$.

Let us consider a point P ^{from A} and the co-ordinates of P from A are x and y and velocity of jet at point P are u and v in the x and y direction.

Let, a liquid particle takes time t to reach from A to B , then horizontal & vertical distance travelled by the liquid particle in time period t are

$x =$ velocity component \times time taken in x -direction

$x = u \cos \theta \times t$
 $\Rightarrow t = x / u \cos \theta$

$y = u \sin \theta \times t - \frac{1}{2} g t^2$

$S = ut + \frac{1}{2} at^2$
 $= ut + \frac{1}{2} gt^2$

$= u \sin \theta \times \frac{x}{u \cos \theta} - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$

$$= x \tan \theta - \frac{1}{2} \frac{g x^2}{u^2 \cos^2 \theta}$$

Let y is the vertical dist. covered from A to B.
 • Maximum ^{height} attained by the jet.

$$v_2^2 - v_1^2 = 2as$$

$$\Rightarrow v_2^2 - v_1^2 = -2gs$$

$$-(u \sin \theta)^2 = -2gs$$

$$\Rightarrow u^2 \sin^2 \theta = 2gs$$

$$\Rightarrow \boxed{s = \frac{u^2 \sin^2 \theta}{2g}}$$

Time of flight is the time taken of the fluid particle in reaching from A to B.

Let, T is the time of flight.

$$\theta = T$$

$$y = 0$$

$$y = u \sin \theta T - \frac{1}{2} g T^2$$

$$0 = u \sin \theta T - \frac{1}{2} g T^2$$

$$\Rightarrow \frac{1}{2} g T^2 = u \sin \theta T$$

$$\Rightarrow \boxed{T = \frac{2u \sin \theta}{g}}$$

Time to reach the highest point is half of the time of flight

$$T^* = T/2$$

$$T^* = \frac{2u \sin \theta}{2g} = \frac{u \sin \theta}{g}$$

$$\boxed{T^* = \frac{u \sin \theta}{g}}$$

* Horizontal range of the jet
 The total horizontal distance travelled by the fluid particle is called horizontal range of the jet

$$x^* = u \cos \theta \times T$$

$$= u \cos \theta \times \frac{2u \sin \theta}{g}$$

$$x^* = \frac{u^2}{g} 2 \sin \theta \cos \theta$$

* value of θ for maximum range

$$\sin 2\theta = 1$$

$$\sin 2\theta = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 90/2 = 45^\circ$$

$$x^* = \frac{u^2}{g} \sin 2\theta$$

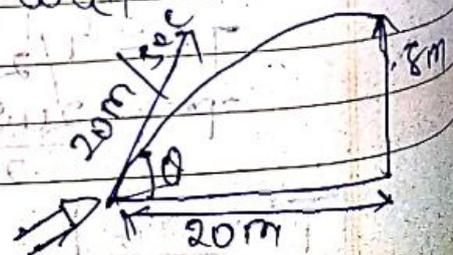
$$x^*_{\text{max}} = \frac{u^2}{g}$$

Q) A vertical wall of 8m in height. A jet of water is coming out from a nozzle with a velocity of 20 m/s. The nozzle is situated at a distance of 20m from the vertical wall. Find the angle of projection of the nozzle to the horizontal so that the jet of water clears the top of the wall.

$$x = 20\text{m}$$

$$y = 8\text{m}$$

$$u = 20\text{m/s}, \theta = ?$$



$$y = x \tan \theta - \frac{1}{2} g \frac{x^2}{v^2 \cos^2 \theta}$$

$$8 = 20 \tan \theta - \frac{1}{2} \cdot 9.8 \times \frac{(20)^2}{(20)^2 (\cos^2 \theta)}$$

$$\Rightarrow 8 = 20 \tan \theta - \frac{4 \cdot 9}{\cos^2 \theta}$$

$$\Rightarrow 8 = 20 \tan \theta - 4 \cdot 9 \sec^2 \theta$$

$$\Rightarrow 8 = 20 \tan \theta - 4 \cdot 9 (1 + \tan^2 \theta)$$

$$\Rightarrow 8 = 20 \tan \theta - 4 \cdot 9 - 4 \cdot 9 \tan^2 \theta$$

$$12 \cdot 9 = 20 \tan \theta - 4 \cdot 9 \tan^2 \theta$$

$$\Rightarrow 4 \cdot 9 \tan^2 \theta - 20 \tan \theta + 12 \cdot 9 = 0$$

$$\tan \theta = \frac{20 \pm \sqrt{400 - 4 \times 4 \cdot 9 \times 12 \cdot 9}}{2 \times 4 \cdot 9}$$

$$= \frac{20 \pm 12 \cdot 13}{19 \cdot 8 \cdot 9 \cdot 8}$$

$$= \frac{32 \cdot 13}{9 \cdot 8}, \quad \frac{7 \cdot 87}{9 \cdot 8}$$

$$= 3.29$$

$$= 0.8$$

$$\tan \theta = \frac{18.74}{3.29}$$

$$\frac{18.76}{0.8}$$

$$\theta = \tan^{-1}(18.74)$$

$$= 86.94$$

$$\theta = \tan^{-1}(0.8)$$

$$= 38.65^\circ$$

$$\theta = \tan^{-1}(3.29)$$

$$72.99^\circ$$

(6) Given Dist. of window from the ground = 30m
Dia. of nozzle = 0.05m

$$A = \frac{\pi}{4} (0.05)^2 = 0.001963 \text{ m}^2$$

$$Q = 3.5 \text{ m}^3/\text{min}, \quad x = ?, \quad y = 30 - 1 = 29 \text{ m}$$
$$= 0.058 \text{ m}^3/\text{sec}$$

$$Q = AV$$

$$0.058 = 0.001963 \times V$$

$$V = 29.54 \text{ m/s}$$

$$y = x \tan \theta - \frac{gx^2 \sec^2 \theta}{2u^2}$$

$$29 = x \tan \theta - \frac{9.81 \cdot x^2 \sec^2 \theta}{2(29.54)^2}$$

$$29 = x \tan \theta - 0.00562 x^2 (1 + \tan^2 \theta)$$

$$= x \tan \theta - 0.00562 x^2 - 0.00562 x^2 \tan^2 \theta$$

$$\Rightarrow 0.00562 x^2 \tan^2 \theta - x \tan \theta + 29.00562 x^2 = 0$$

$$29 = x \tan \theta - 0.00562 x^2 - 0.00562 x^2 \tan^2 \theta$$

$$0.00562 x^2 \tan^2 \theta - x \tan \theta + 0.00562 x^2 + 29 = 0$$

$$\rightarrow x \tan \theta - \frac{0.00562 x^2}{\cos^2 \theta} - 29 = 0$$

$$\left(\frac{dx}{d\theta} \right) = 0$$

$$\frac{d(x \tan \theta)}{d\theta} - 0.00562 x^2 \frac{d(\sec^2 \theta)}{d\theta} - \frac{d(29)}{d\theta}$$

$$x(\sec^2 \theta - 0.00562 x^2 \tan \theta) = 0$$

$$\frac{dx}{d\theta} \cdot \tan \theta + \frac{d(\tan \theta)}{d\theta} x - 0.00562 \frac{d(x^2)}{d\theta} \tan \theta$$

$$\frac{d(x \tan \theta)}{d\theta} - 0.00562 \frac{d(x^2 \sec^2 \theta)}{d\theta} - 2g = 0$$

$$\Rightarrow \frac{dx}{d\theta} \tan \theta + \frac{d(\tan \theta)}{d\theta} x - 0.00562 \left(\frac{d(x^2)}{d\theta} \sec^2 \theta + \frac{d(\sec^2 \theta)}{d\theta} x^2 \right)$$

$$\Rightarrow 0 + x \sec^2 \theta - 0.00562 \tan \theta x^2 = 0$$

$$(x^2 \sec^2 \theta \tan \theta) = 0$$

$$\Rightarrow x \sec^2 \theta - 0.01124 x^2 \sec^2 \theta \tan \theta = 0$$

$$\Rightarrow \frac{x - 0.01124 \tan \theta x^2}{\cos^2 \theta} = 0$$

$$\Rightarrow x = 0.01124 x^2 \tan \theta$$

$$\Rightarrow \frac{1}{0.01124} = x^2 \tan \theta$$

$$x = \frac{1}{0.01124 \tan \theta} = \frac{88.96}{\tan \theta}$$

$$x = \frac{88.96}{\tan \theta} \quad \text{--- (ii)}$$

Substituting x in (i)

$$0 = \frac{x \tan \theta \cdot 88.96}{\tan \theta} - 0.00562 \frac{(88.96)^2}{\cos^2 \theta} - 2g$$

$$88.96 - \frac{44.47}{\sin^2 \theta} - 2g = 0$$

$$59.96 - \frac{44.47}{\sin^2 \theta} = 0$$

$$\cancel{\sin^2 \theta} \sin^2 \theta = \frac{44.47}{59.96}$$

$$\cancel{\sin} = 8.0322 \times 10^{-3}$$

$$\cancel{\sin} \theta = 0.009$$

$$\theta =$$

$$59.96 = \frac{44.47}{\sin^2 \theta}$$

$$\theta = 59.93^\circ$$

Substituting θ in (i)

$$x \tan(59.93^\circ) - \frac{0.00562 x^2}{\cos^2(59.93^\circ)} - 2g = 0$$

$$x = \frac{88.96}{\tan(59.93^\circ)} = 51.72 \text{ m (Ans)}$$

Module III Momentum Equation

The momentum equation is based on law of conservation of momentum or on the momentum principle which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

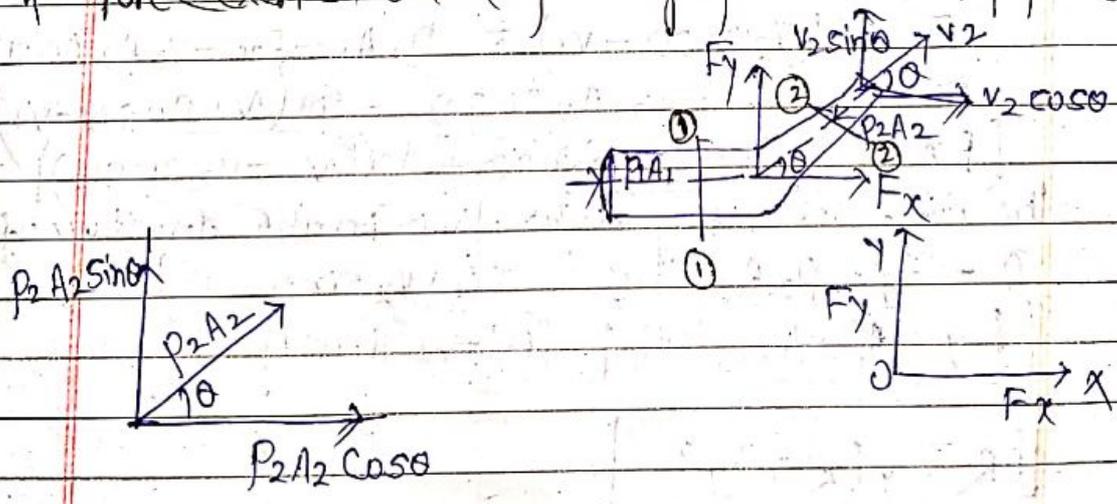
The force acting on a fluid mass is given by Newton's 2nd law of motion, $F = ma$.

$$\Rightarrow F = m \cdot \frac{dv}{dt}$$

$$\Rightarrow \boxed{F_x dt = m_x dv = d(mv)} \text{ Impulse momentum equation.}$$

This equation is known as momentum principle. This states the impulse of a force acting on a fluid of mass m within a short interval of time dt is equal to change of momentum $d(mv)$ in the direction of force.

* Force exerted on a flowing fluid on a pipe bend



The impulse momentum equation is to determine the resultant force acting by a flowing fluid on a pipe bend.

Consider 2 sections 1 and 2, velocity of flow, pressure intensity & cross-sectional area of pipe at section 1 and 2 are v_1, P_1, A_1 and v_2, P_2, A_2 respectively.

Let, F_x and F_y are the component of force exerted by flowing fluid on the bend pipe in x and y direction respectively.

The force exerted by the bend pipe on the fluid in the direction of x and y are F_x and F_y respectively but, in the opposite direction hence F_x will be $-F_x$, F_y will be $-F_y$, another external force acting on the fluid are $p_1 A_1$ and $p_2 A_2$ respectively

Then, the momentum equation in x -direction will be equal to Net force acting on the fluid in the direction of $x =$ Rate of change of momentum in x -direction.

the net force acting in the fluid in the direction of x will be $p_1 A_1 - F_x - p_2 A_2 \cos \theta =$ rate of change of momentum in x -direction
 $= \dot{m} \times \frac{dv}{dt} = \rho Q \times (dv \text{ in } x\text{-direction})$

$$= \rho Q (v_2 \cos \theta - v_1) = p_1 A_1 - F_x - p_2 A_2 \cos \theta$$

$$F_x = p_1 A_1 - p_2 A_2 \cos \theta - \rho Q (v_2 \cos \theta - v_1)$$

$$F_x = p_1 A_1 - p_2 A_2 \cos \theta + \rho Q (v_1 - v_2 \cos \theta)$$

the net force acting in the fluid in the direction of y

$$0 - F_y - p_2 A_2 \sin \theta = \rho Q (v_2 \sin \theta - 0)$$

$$F_y = -p_2 A_2 \sin \theta + \rho Q (-v_2 \sin \theta)$$

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$\tan \theta = \frac{F_y}{F_x}$$

$$\theta = \tan^{-1} \left(\frac{F_y}{F_x} \right)$$

- Q) A 45° reducing bend is connected in a pipe line the dia at the inlet and outlet of the bend being 600 and 300 mm resp. Find the force exerted by the water on the bend if the intensity of pressure at the inlet to the bend is 8.82 gN/cm^2 and rate of flow of water is 600 l/sec.

$$\text{Sol: } p_1 = 8.82 \text{ gN/cm}^2$$

$$Q = 600 \text{ liter/sec}$$

$$D_1 = 600 \text{ mm}$$

$$D_2 = 300 \text{ mm}$$

$$\theta = 45^\circ$$

$$A_1 = \frac{\pi}{4} (0.6)^2 = 0.2826 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} (0.3)^2 = 0.0706 \text{ m}^2$$

$$Q = A_1 v_1 = A_2 v_2$$

$$0.6 = 0.282 v_1 = 0.0706 v_2$$

$$v_1 = \frac{0.6}{0.282} = 2.127 \text{ m/s}$$

$$v_2 = \frac{0.6}{0.0706}$$

$$= 8.49 \text{ m/s}$$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \quad (z_1 = z_2)$$

$$\frac{8.82 \text{ g} \times 10^4}{10^3 \times 9.81} + \frac{(2.127)^2}{2 \times 9.81} = \frac{p_2}{10^3 \times 9.81} + \frac{(8.49)^2}{2 \times 9.81}$$

$$9 + 0.229 = \frac{p_2}{10^3 \times 9.81} + 3.673$$

$$p_2 = 5.556 \times 10^3 \times 9.81$$

$$= 5.44 \text{ N/cm}^2$$

$$= 5.44 \times 10^4 \text{ N/m}^2$$

$$F_x = P_1 A_1 - P_2 A_2 \cos 50^\circ - \rho Q (V_2 \cos 50^\circ - V_1)$$

$$= 8.829 \times 10^4 \times 0.2826 - 5.44 \times 10^4 \times 0.07 \times \cos 45^\circ - 10^3 \times 0.6 (8.49 \cos 45^\circ - 2.12)$$

$$2.495 \times 10^4 - 0.380 \times 10^4 \times 0.269 \times 10^4$$

$$- 10^3 \times 2.326$$

$$10^3 (2.495 - 0.269 - 2.326)$$

$$= 19.934 \times 10^3 \text{ N}$$

$$F_y = -P_2 A_2 \sin 50^\circ - \rho Q (V_2 \sin 50^\circ)$$

$$= -5.44 \times 10^4 \times 0.07 \times \sin 45^\circ - 10^3 \times 0.6 \times 8.49 \times \sin 45^\circ$$

$$= 10^3 (-0.269 \times 10 - 36.014)$$

$$= -33.32 \times 10^3 \text{ N} \quad 6322.4 \text{ N}$$

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{408386.756}$$

$$\sqrt{437337.0978}$$

$$= 20912.606$$

$$\theta = \tan^{-1} \left(\frac{6322.4}{19934} \right)$$

$$= 17.58^\circ$$

Q) A 300mm dia. pipe carries water under a head of 20m with a velocity of 3.5m/s. If the axis of the pipe turn through 45° find the magnitude and direction of resultant force at the bend.

$$D_1 = D_2 = 0.3 \text{ m}$$

$$v_1 = v_2 = 3.5 \text{ m/s}$$

$$A_1 = A_2 = \frac{\pi}{4} (0.3)^2 \\ = 0.0706 \text{ m}^2$$

$$p = 20 \text{ m} \rightarrow \rho = 20 \times 10^3 \times 9.81$$

$$= 196200 \text{ N/m}^2$$

$$Q = A_1 v_1 = 0.247 \text{ m}^3/\text{sec}$$

$$F_x = p_1 A_1 - p_2 A_2 \cos \theta + \rho Q (v_1 - v_2 \cos \theta)$$

$$196200 \times 0.0706 - 196200 \cos 45^\circ + 10^3 \times 0.247 (3.5 - 3.5 \cos 45^\circ)$$

$$= 13871.34 + 253.2985$$

$$= 14124.6385 \text{ N}$$

$$= 14124.64 \text{ N}$$

$$F_y = -p_2 A_2 \sin \theta - \rho Q (v_2 \sin \theta)$$

$$= -196200 \times 0.0706 \times 0.707 - 10^3 \times 0.247 (3.5 \times \frac{1}{\sqrt{2}})$$

$$= -1061.15751 - 611.2015$$

$$= -1672.359 \text{ N} = -1041.8535 \text{ N}$$

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{14124.64^2 + 1672.359^2}$$

$$= 14231.9785 \text{ N} \quad \sqrt{127188473.3593}$$

$$= 11277.7867 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{1672.359}{14124.64} \right)$$

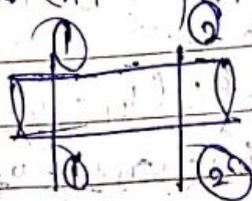
$$= 7.8^\circ$$

$$\theta = \tan^{-1} \left(\frac{1041.8535}{4317.7259} \right)$$

$$= \tan^{-1}(2.412)$$

$$\rightarrow \theta = 67.48^\circ \text{ (Ans)}$$

* Moment of Momentum equation
 This is derived from momentum principle which states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.



V_1 is the velocity of fluid at section 1, V_2 is vel. of fluid at section 2, r_1, r_2 are radius of curvature of section 1 and 2 resp. Q is the rate of flow, ρ is equal to density of fluid.
 Moment of fluid at section 1 = $\rho Q V_1 r_1$
 Moment of momentum at section 2 = $\rho Q V_2 r_2$
 " " " " " " section 2 = $\rho Q V_2 r_2$

Rate of change of moment of momentum
 $= \rho Q V_2 r_2 - \rho Q V_1 r_1$
 $T = \rho Q (V_2 r_2 - V_1 r_1)$

- This equation is applied for analysis of flow in turbine and centrifugal pump.
- For finding torque created by water on sprayer or sprinkler.

* Laminar & turbulent flow:

(Re) Reynold's number = $\frac{\rho V D}{\mu}$

→ In laminar flow the fluid particles move along straight and parallel path in layers, such that the path of individual fluid particles do not cross the neighbouring or adjacent particle.

Laminar flow is possible only at low velocity and when the fluid is highly viscous but when the velocity increase and fluid is less viscous the fluid particles don't move in straight path they move in random manner resulting in general mixing of particles this type of flow is called turbulent flow.

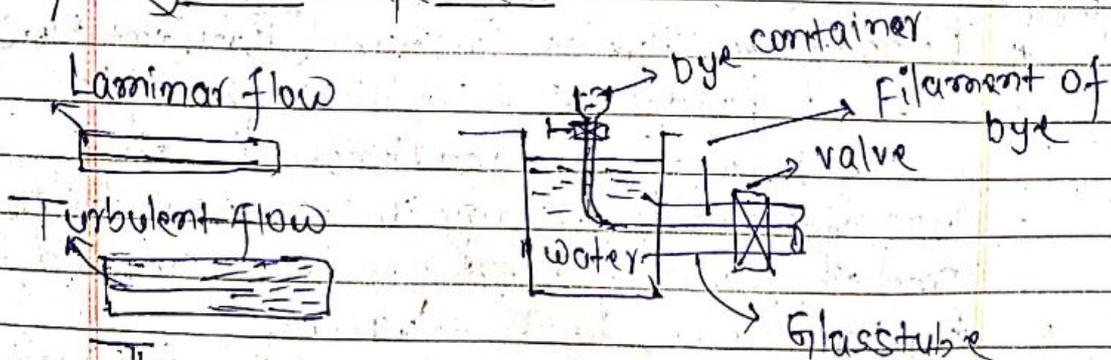
→ A laminar flow changes to turbulent flow when velocity is increased and diameter of the pipe is increased, the viscosity of the fluid decreased.

→ $Re < 2000$ - Laminar flow

$Re > 4000$ - Turbulent

$Re = 2000 - 4000$ - Transition flow

Reynold's Experiment



This experiment is used to determine the Reynold's number and the type of flow. The apparatus consist of a tank containing water at a constant head, a small tank containing some dye, a glass

tube having a regulating valve at both ends. The water from the tank was allowed to flow through the glass tube the velocity of flow was varied by the regulating valve. A liquid dye having same specific weight as water was introduced to the glass tube.

The following observations were made.

① When the velocity of flow was low the dye filament in the glass tube was in the form of straight line and the straight line of the dye filament was parallel to the glass tube.

② With the increase of velocity of flow the dye filament was no longer a straight line but, it became a wavy form.

③ With further increase of velocity of flow the wavy dye filament broke up and finally diffused in water. This means that the fluid particles of the dye at the higher velocity are moving in random which is turbulent flow.

Thus, in case of turbulent flow the mixing of dye filament and water are irregular, random, disorder.

20/08/19

Loss of head due to Friction (Turbulent flow)

When a fluid is flowing through a pipe having diameter 'D' and length 'L', then, loss of head or energy arises due to friction and this loss of head can be calculated by Darcy Weisbach equation

$$h_f = \frac{4fLV^2}{2gD}$$

where h_f = loss of head due to friction

f = coefficient of friction

L = length of pipe

D = Diameter of pipe

V = velocity of fluid

$$f = \frac{16}{Re} \quad \text{Re Laminar flow} \\ Re < 2000$$

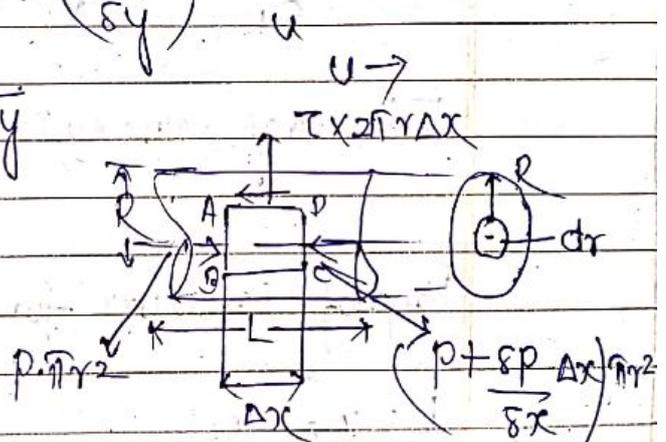
$$f = \frac{0.0791}{(Re)^{1/4}} \quad (Re < 4000 \rightarrow 10^5) \quad \text{turbulent flow}$$

$$Re = \frac{\rho v D}{\mu}$$

When a fluid is flowing with very low velocity the flow is considered to be laminar. In the laminar flow one layer of fluid slides over adjacent layer of the fluid due to which a velocity gradient ($\frac{\delta v}{\delta y}$). So shear stress will be developed.

$$\text{Shear stress } (\tau) \propto \left(\frac{\delta v}{\delta y} \right)$$

$$\tau = \mu \frac{\delta v}{\delta y}$$



Consider a viscous fluid is flowing through a pipe having radius R and length L .

Let us consider a small fluid element ABCD having radius r and length Δx .

The following forces will be developed

Pressure Force = $p \pi r^2$ on face AB and

$$" \quad " = \left[p + \left(\frac{\delta p}{\delta x} \Delta x \right) \right] \pi r^2 \quad \text{on face CD}$$

Another force, shear force = $\tau \cdot 2\pi r \Delta x$

$$p \pi r^2 - \left[p + \left(\frac{\delta p}{\delta x} \right) \Delta x \right] \pi r^2 - \tau \times 2 \pi r \Delta x = 0$$

$$\tau p \pi r^2 - p \pi r^2 - \left(\frac{\delta p}{\delta x} \right) \Delta x \pi r^2 - \tau \times 2 \pi r \Delta x = 0$$

$$\Rightarrow - \left(\frac{\delta p}{\delta x} \right) r = 2 \tau$$

$$\Rightarrow \tau = - \left(\frac{\delta p}{\delta x} \right) \frac{r}{2}$$

28/08/19 Velocity distribution

$$u = \frac{1}{4\mu} \left(\frac{\delta p}{\delta x} \right) [R^2 - r^2] \quad \mu = \text{dynamic viscosity.}$$

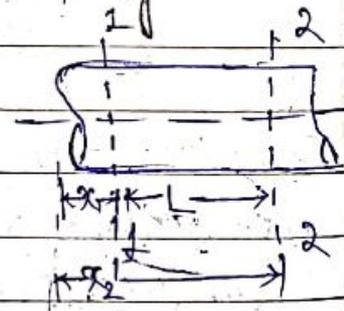
$$u_{\text{max.}} = \frac{1}{4\mu} \left(\frac{\delta p}{\delta x} \right) R^2$$

$$Q = \frac{\pi}{8\mu} \left(\frac{\delta p}{\delta x} \right) R^4$$

$$\bar{u} \text{ (Average velocity)} = \frac{1}{8\mu} \left(\frac{\delta p}{\delta x} \right) R^2$$

Drop of pressure for a given length L

$$L = x_2 - x_1$$



$$\bar{u} = \frac{1}{8\mu} \left(\frac{\delta p}{\delta x} \right) R^2$$

$$- \left(\frac{\delta p}{\delta x} \right) = \frac{8\mu \bar{u}}{R^2}$$

$$- \int_1^2 \delta p = \left(\frac{8\mu \bar{u}}{R^2} \right) \int_1^2 \delta x$$

$$\Rightarrow - (p_1 - p_2) = \frac{8\mu \bar{u}}{R^2} (x_2 - x_1)$$

23/08/19

$$\Rightarrow p_1 - p_2 = \frac{8\mu\bar{u}}{R^2} (x_2 - x_1)$$

$$\Rightarrow p_1 - p_2 = \frac{8\mu\bar{u}}{R^2} L$$

$$= \frac{8\mu\bar{u}}{(D/2)^2} L$$

$$= \frac{8\mu\bar{u}}{\frac{D^2}{4}} L = \frac{32\mu\bar{u}L}{D^2}$$

$$\Rightarrow \boxed{\frac{p_1 - p_2}{\rho g} = \frac{32\mu\bar{u}L}{\rho g D^2}} \quad \text{(Laminar or viscous flow)} \\ \text{Hagen's Poiseuille Formula}$$

① An oil of viscosity 0.1 N s/m^2 and relative density 0.9 is flowing through a circular pipe of dia 50 mm and length 300 m . The rate of flow through the pipe is 3.5 l/s . Find the pressure drop in a length of 300 m and also the shear stress developed at the wall of pipe.

Sol: Given

$$\mu = 0.1 \text{ N s/m}^2,$$

$$\text{relative density} = 0.9$$

$$\rho = 0.9 \times 10^3 = 900 \text{ kg/m}^3$$

$$D = 0.05 \text{ m}, L = 300 \text{ m}$$

$$Q = 3.5 \text{ l/s} = \frac{3.5}{1000} = 0.0035 \text{ m}^3/\text{s}$$

$$V = 1.78 \text{ m/s}, \quad V = \frac{0.0035 \times 10^3}{1.9625} = 1.78$$

$$Re = \frac{\rho V D}{\mu} = \frac{900 \times 1.78 \times 0.05}{0.1} = 890 < 2000$$

$$Q = \frac{\pi}{4} D^2 \times V$$

Hence, the flow is laminar flow

$$\text{pressure drop } (P_1 - P_2) = \frac{32 \mu \bar{u} L}{D^2}$$

$$= \frac{32 \times 0.1 \times 1.78 \times 300}{(0.05)^2}$$

$$= 683520 \text{ N/m}^2 \text{ (Ans)}$$

Shear stress developed

$$\tau = - \left(\frac{\delta p}{\delta x} \right) \frac{r}{2}$$

$$= - \left(\frac{\delta p}{\delta x} \right) = - \left(\frac{P_2 - P_1}{\delta x} \right) = \left(\frac{P_1 - P_2}{\delta x} \right)$$

$$= \frac{683520}{300} = 2278.4 \text{ N/m}^2$$

$$\tau = \frac{2278.4 \times 0.025}{2}$$

$$= 28.48 \text{ N/m}^2$$

② A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100mm and length 10m. Calculate the difference of pressure at the 2 ends of the pipe if 100kg of the oil is collected in a tank in 60 sec. 80 sec.

Sol:- Given $\mu = 0.97 \text{ poise} = 9.7 \text{ N s/m}^2 = 0.097 \text{ N s/m}^2$
 relative density = 0.9
 $\rho = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$$\text{Dia.} = 100 \text{ mm} \quad L = 10 \text{ m} \\ = 0.1 \text{ m}$$

We know,

$$p_1 - p_2 = \frac{32 \bar{U} \mu L}{D^2}$$

AIQ Mass of oil per second = $\frac{100 \text{ kg}}{305} = 3.333 \text{ kg/s}$

$$\therefore \rho \times Q = 3333$$

$$Q = \frac{3.333}{900} = 0.0037 \text{ m}^3/\text{s}$$

And, $\bar{U} = \frac{Q}{A} = \frac{0.0037 \times 10^3}{7.85} = 0.471 \text{ m/s}$

$Re = \rho \frac{v_0}{\mu} = 437 < 200$ so, flow is laminar.

Now,

$$p_1 - p_2 = \frac{32 \mu \bar{U} L}{D^2}$$

$$= \frac{32 \times 0.987 \times 0.471 \times 10}{0.01^2}$$

$$= 1461.984 \text{ N/m}^2$$

27/09/18 @ $p_1 - p_2$

* Loss of head due to friction in viscous flow

$$h_f = \frac{4fLV}{2gD}$$

The loss of pressure head h_f in a pipe of dia. D and length L and dynamic viscosity μ is

flowing with a velocity \bar{U} is given by Hagen's poiseuille equation

$$\frac{P_1 - P_2}{\rho g} = \frac{32 \mu V L}{\rho g D^2} \quad \text{--- (1)}$$

The loss of head due to friction

$$h_f = \frac{4fLV^2}{2gD} \quad \text{--- (2)}$$

Equating eqn (1) and (2)

$$\frac{32 \mu V L}{\rho g D^2} = \frac{4fLV^2}{2gD}$$

$$\frac{16 \mu V}{\rho D} = F V \quad \frac{16 \mu}{\rho D} = F V$$

$$F = \frac{16 \mu}{\rho D V}$$

$$\left[\because Re = \frac{\rho V D}{\mu} \right]$$

$$\left[F = \frac{16}{Re} \right] \text{ for Laminar flow}$$

Q) Water is flowing thru a 200 mm dia. pipe with coefficient of friction $f = 0.04$, the shear stress at a point 40 mm from the pipe axis is 0.00981 N/cm^2 then calculate the shear stress at the pipe wall.

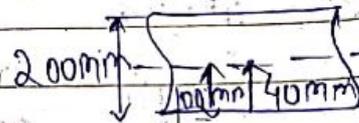
Sol: Given

$$D = 200 \text{ mm} \quad f = 0.04$$

$$= 0.2 \text{ m}$$

$$R = 40 \text{ mm}$$

$$\tau = 0.00981 \text{ N/cm}^2$$



Assuming shear stress at the wall of the pipe is τ_0 .

$$\frac{\rho}{\mu} = \dots$$

$$f = \frac{16}{Re} \Rightarrow 0.04 = \frac{16}{Re}$$

$\Rightarrow Re = \frac{16}{0.04} = 400 < 2000$ then the flow is laminar or viscous flow.

$$\tau = -\left(\frac{\Delta p}{\Delta x}\right) \frac{r}{2}$$

$\tau \Delta x$

$$\frac{\tau}{r} = \frac{\tau_0}{R} \Rightarrow \frac{0.0098}{40} = \frac{\tau_0}{1000}$$

$$\tau_0 = 0.0245 \text{ N/cm}^2 \text{ (Ans)}$$

Q) An oil of specific gravity 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 l/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m.

Take $\mu = 0.2 \text{ g stroke} = 0.2 \times 10^{-4} \text{ N s/m}^2$

Sol: Sp. gravity = 0.7

Dia = 300 mm = 0.3 m, $A = 0.070695 \text{ m}^2$

$Q = 500 \text{ l/s}$

$= 0.5 \text{ m}^3/\text{s}$

sp. gravity = $\frac{\rho_{oil}}{\rho_{water}}$

$\rho_{oil} = 700 \text{ kg/m}^3$

$v = \frac{0.5}{0.070695} = 7.077 \text{ m/s}$

$Re = \frac{\rho v D}{\mu} = \frac{700 \times 7.077 \times 0.3}{0.2 \times 10^{-4}}$

$= \frac{v D}{\nu} = \frac{7.077 \times 0.3}{0.29 \times 10^{-4}} = 7321.03$

Turbulent flow

laminar flow

$$f = \frac{0.0791}{(73218034)^{1/4}} = \frac{0.0791}{16.449} = 0.0048$$

$$h_f = \frac{4fLv^2}{2gD} = 4fL$$

$$= \frac{4 \times 0.0048 \times 10^3 \times (7.077)^2}{2 \times 9.81 \times 0.3}$$

$$= \frac{961.61}{5.886} = 163.37 \text{ m (Ans)}$$

$$P = \frac{\rho g Q h_f}{1000} = \frac{700 \times 9.81 \times 0.5 \times 163.37}{10^3}$$

$$= 560.93 \text{ kW (Ans)}$$

23/09/19 Module - V

* Losses of energy in pipe when a fluid is flowing thr. a pipe the fluid experience some resistance due to which some of the energy of the fluid is lost this loss of energy is classified as energy loss.

Energy loss

Major loss

- this is due to friction and it is calculated by following formula: Darcy Weisbach formula, Chezy's formula.

Minor loss

- this is due to sudden contraction, enlargement, bending of pipe, pipe fitting or any obstruction in the pipe.

* Chezy's formula

$$h_f = \frac{F'}{fg} \times \frac{p}{A} \times L \times v^2$$

where, h_f = loss of head due to friction.
 p = wetted perimeter of the pipe.

A = Area of cross-section of pipe

L = length of pipe

v = Mean velocity of the flow.

f' = coefficient of friction

$\therefore m$ = hydraulic mean depth = $\frac{A}{P}$

A = Area of pipe

P = perimeter of pipe

$$m = \frac{A}{P} = \frac{\frac{\pi d^2}{4}}{\pi d} = \frac{d}{4} \Rightarrow \boxed{m = \frac{d}{4}}$$

$$\boxed{\frac{P}{A} = \frac{1}{m}}$$

v can be calculated by using formula

$v = c\sqrt{mi}$, where c = constant known as Chezy's const.

$$c = \sqrt{\frac{fg}{f'}}$$

$i = \frac{h_f}{L}$ which is known as loss of head per unit length of the pipe.

Q) Find the loss of head due to friction in a pipe of dia 300mm and length 50m through which water is flowing at a velocity of 3m/s using Darcy Formula and Chezy's Formula for which $c = 60$.

Sol: = Given $D = 300\text{mm}$. Take kinematic viscosity of water is 0.01 stoke.

Sol: Given $D = 300 \text{ mm}$, $A = 0.07065 \text{ m}^2$
 $= 0.3 \text{ m}$

$\nu = 0.01 \text{ Stoke}$
 $= 0.01 \times 10^{-4} \text{ m}^2/\text{sec}$

$v = 3 \text{ m/s}$

$L = 50 \text{ m}$

$C = 60$

we know

$$h_f = \frac{4fLv^2}{2gD}$$

$$Re = \frac{fvD}{\mu} = \frac{vD}{\nu} = \frac{3 \times 0.3}{0.01 \times 10^{-4}}$$

$$= \frac{0.9 \times 10^{-2}}{10^{-5}} = \frac{0.9 \times 10^{-2}}{0.01}$$

$= 90 \times 10^4 > 2000$ \Rightarrow turbulent flow

$$f = \frac{0.0791}{(9 \times 10^5)^{1/4}} = \frac{0.0791}{30.8}$$

$$= 2.56 \times 10^{-3}$$

$$h_f = \frac{4 \times 2.56 \times 10^{-3} \times 50 \times 3^2}{2 \times 9.81 \times 0.3}$$

$$= 70 \times 10^{-3}$$

$$= 0.07 \text{ m}$$

Using chezy's formula

Using chezy's formula

$$v = \sqrt{mi} = \sqrt{A/p \times h_f}$$

$$0.07065 = \sqrt{\frac{d}{4} \times \frac{hf}{L}}$$

$$= 0.000$$

$$v = \sqrt{\frac{d}{4} \times \frac{hf}{L}}$$

$$3 = \sqrt{0.0015 hf \times 60}$$

$$9 = 0.0015 hf \times 60$$

$$\left(\frac{3}{60}\right)^2 = 0.0015 hf$$

$$(0.05)^2 = 0.0015 hf$$

$$hf = 1.66 \text{ m (Ans)}$$

Q) Find the dia. of pipe of length 2000 m when the rate of flow of water thr. the pipe is 200 l/s and the head lost due to friction is 4m. Take the value of c = 50 in chezy's formula.

Sol: - Given L = 2000 m

$$Q = 200 \text{ l/s}$$

$$= 0.2 \text{ m}^3/\text{s}$$

$$A = \frac{\pi}{4} d^2$$

$$Q = v \cdot A \cdot v = Q$$

$$v = \frac{Q}{A}$$

$$= 0.2$$

$$hf = 4 \text{ m}$$

$$c = 50$$

$$v = c \sqrt{mi}$$

$$0.785 d^2$$

$$\frac{0.254}{d^2} = 50 \times \sqrt{\frac{d}{4}} \times \frac{4}{2000}$$

$$\frac{0.254}{d^2 \times 50} = \frac{\sqrt{d} \times 4}{4 \times 2000}$$

$$\left(\frac{5.08 \times 10^{-3}}{d^2} \right)^2 = \frac{50 \sqrt{d}}{44.72}$$

$$\frac{5.08 \times 10^{-3}}{d^2} = \frac{50 \sqrt{d}}{44.72}$$

$$\left(\frac{5.08 \times 10^{-3}}{d^2} \right)^2 = d$$

$$\left(5.08 \times 10^{-3} \right)^2 = d \times d^4$$

$$\left(5.08 \times 10^{-3} \right)^2 = d^5$$

$$25.8064 \times 10^{-6} = d^5$$

$$\sqrt[5]{25.8064 \times 10^{-6}} = d$$

$$d = 1.208 \times 10^{-5}$$

$$\frac{0.254}{d^2} = 50 \times \sqrt{\frac{d}{4}} \times \frac{4}{2000}$$

$$\frac{50}{44.72} \times \sqrt{d}$$

$$\frac{0.254}{d^2} = 1.118 \times \sqrt{d}$$

$$\left(\frac{0.2271}{d^2} \right)^2 = d \Rightarrow d^5 = 0.051529$$

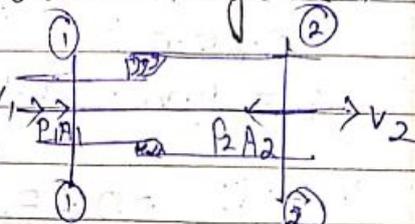
$$d = 0.552 \text{ m}$$

Minor Energy (Head) Loss

Loss of head ~~of~~ or energy due to friction is called in a pipe is known as major loss while loss of energy due to change of velocity ^{of the flowing fluid in mag. of dia.} in pipe is called minor energy loss.

* Loss of head due to sudden enlargement

Consider a fluid flowing v_1 through a pipe which has Sudden enlargement.



Consider the 2 sections 1 and 2 before and after enlargement.

p_1, v_1, A_1 are pressure intensity, velocity of flow Area of pipe at section 1 and similarly p_2, v_2, A_2 are for section 2.

Due to the sudden change of dia of pipe from D_1 to D_2 the flow separates from the boundary and turbulent eddies are formed.

Applying Bernoulli's equation at section 1 and 2

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + (\text{Loss of head of energy}) (h_e)$$

$$z_1 = z_2$$

$$h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g} \right) + \left(\frac{v_1^2}{2g} - \frac{v_2^2}{2g} \right)$$

$$h_e = \frac{(v_1 - v_2)^2}{2g}$$

Loss of head due to sudden enlargement

Q) Find the loss of head when a pipe of dia 200mm is suddenly enlarged to a dia of 400mm the rate of flow of water through the pipe is 250 l/s

Sol:- Given $D_1 = 200 \text{ mm} = 0.2 \text{ m}$ $D_2 = 400 \text{ mm} = 0.4 \text{ m}$
 $A_1 = 0.0314 \text{ m}^2$ $A_2 = 0.125 \text{ m}^2$

$Q = 250 \text{ l/s}$
 $= 0.25 \text{ m}^3$

$Q = A_1 v_1 = A_2 v_2$

$v_1 = \frac{Q}{A_1} = \frac{0.25}{0.0314}$
 $= 7.96 \text{ m/s}$

$v_2 = \frac{Q}{A_2} = \frac{0.25}{0.125}$
 $= 1.98 \text{ m/s}$

We know,

$h_e = \frac{(v_1 - v_2)^2}{2g}$

$= \frac{(7.96 - 1.98)^2}{2 \times 9.81}$

$= \frac{5.98^2}{19.62}$

$h_e = 1.82 \text{ m}$

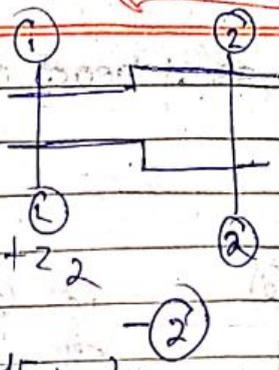
Q) At a certain enlargement of a water main pipe from 240mm to 480mm Diameter the hydraulic gradient rises by 10mm then estimate the rate of flow.

Sol:- Hydraulic Gradient $= \left(\frac{P_2}{\rho g} + z_2 \right) - \left(\frac{P_1}{\rho g} + z_1 \right)$
 $= 10 \text{ mm} = 0.01 \text{ m}$

2007

22/5

Applying Bernoulli's eqⁿ



$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_e + z_2$$

$$\therefore h_e = \frac{(v_1 - v_2)^2}{2g} = 0.45 v_2^2$$

$$Q = A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{Q}{A_1}$$

$$v_2 = \frac{Q}{A_2}$$

$$\Rightarrow v_1 = \frac{A_2 v_2}{A_1}$$

$$= \frac{\frac{\pi}{4} \times D_2^2 \times v_2}{\frac{\pi}{4} \times D_1^2 \times v_1} = \frac{(0.04)^2 \times v_2}{(0.08)^2}$$

$$v_1 = \frac{0.2304 v_2}{0.0576}$$

$$v_1 = 4v_2$$

From (2) $\frac{v_1^2}{2g} - \frac{v_2^2}{2g} = \left(\frac{p_2}{\rho g} - \frac{p_1}{\rho g} \right) + h_e$

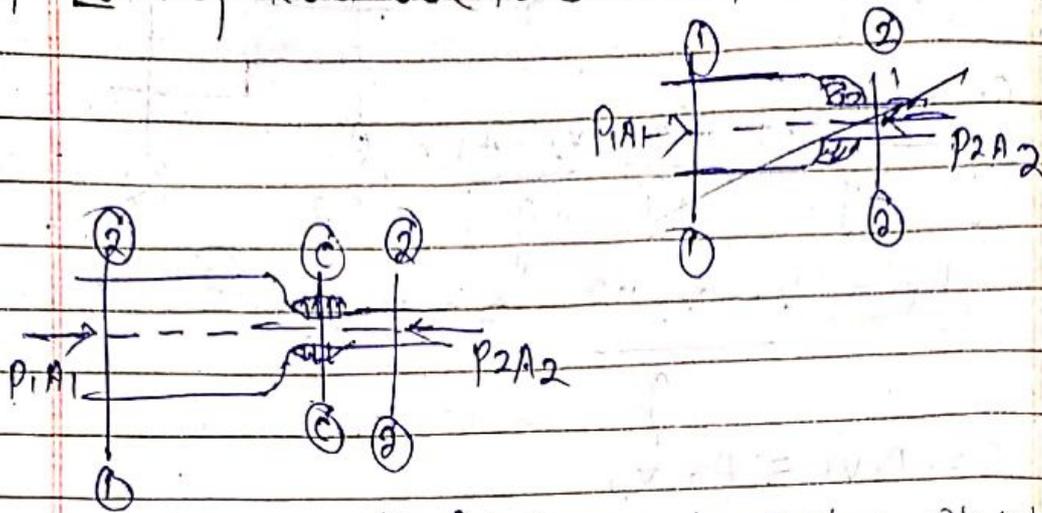
$$\frac{15v_2^2}{2g} = 0.01 + 0.45v_2^2$$

$$0.76v_2^2 - 0.45v_2^2 = 0.01$$

$$v_2 = \sqrt{\frac{0.01}{0.31}} = 0.179 \text{ m/s}$$

$$Q = A_2 v_2 = 0.179 \times 0.180 = 0.032 \text{ m}^3/\text{sec} \text{ (Ans)}$$

Loss of head due to Sudden Contraction



consider a liquid flowing in a pipe which has a sudden contraction in area. Consider 2 sections 1 and 2 before and after. The section C is called vena contracta where the area of flow goes on decreasing.

A_c = Area of flow at section C.

v_c = Velocity of flow at section C.

$A_2 v_2$ = Area and velocity of flow at section 2.

$A_1 v_1$ = Area and velocity of flow at section 1.

h_c = Loss of head due to sudden contraction

h_c is actually loss of head due to enlargement from section C to 2.

$$h_c = \frac{v_2^2}{2g} \left[\frac{1}{C_c} - 1 \right]^2$$

C_c = Coefficient of contraction.

$$h_c = \frac{k v_2^2}{2g}$$

$$k = \left[\frac{1}{C_c} - 1 \right]^2, \quad C_c = 0.62$$

$$k = \left[\frac{1}{0.62} - 1 \right]^2 = 0.375$$

$$h_c = \frac{0.375 v_2^2}{2g}$$

① If the c_c is not given, then head loss due to contraction

$$h_c = \frac{0.5v^2}{2g}$$

Loss of head at the entrance of the pipe

This is the loss of energy which occurs when the liquid enters a pipe which is connected to a large tank or reservoir.

$$h_i = \frac{0.5v^2}{2g}$$

Where, h_i = loss of head at the entrance of the pipe.

Loss of head at the exit of the pipe

$$h_o = \frac{v^2}{2g}$$

Where, v = velocity of liquid at outlet of the pipe.

Loss of head due to an obstruction in the pipe

This occurs due to

$$\text{Loss of head due to obstruction} = \frac{v^2}{2g} \left(\frac{A}{c_c(A-a)} - 1 \right)$$

Where, c_c = Coefficient of contraction

a = ~~or~~ Maximum area of the obstruction

A = Area of the pipe

v = velocity of the liquid in the pipe.

$$c_c = \frac{\text{Area at venacontractor } (a_c)}{(A-a)}$$

Loss of head due to bend in pipe

When there is a bend in the pipe the velocity of the flow changes due to which the separation of the flow from the boundaries and also eddies takes place and loss of head due to bend

$$h_b = \frac{Kv^2}{2g}$$

v is velocity of flow
 K = coefficient of bend.

The value of k depends upon

- ① Angle of bend
- ② Radius of curvature of bend
- ③ Diameter of the pipe.

Loss of head due to pipe fitting

The loss of head due to pipe fitting such as valves, coupling etc. is expressed as $\frac{kv^2}{2g}$

where

k = Co-efficient of pipe due to fitting.

v = velocity of liquid flowing through the pipe.

17/09/19

Q) Ex. A 150mm Dia. pipe reduces in Dia to 100mm dia if the pipe carries water at 30 l/sec. Calculate the pressure loss across the contraction. Take $C_c = 0.6$

Sol: - Given $D_1 = 150\text{mm}$
 $= 0.15\text{m}$

$D_2 = 100\text{mm}$
 $= 0.1\text{m}$

$A_1 = 0.01766\text{m}^2$

$A_2 = 0.00785\text{m}^2$

$Q = 30\text{ l/sec}$
 $= 0.03\text{ m}^3/\text{sec}$

$Q = A_1 v_1 = A_2 v_2$

$v_1 = \frac{Q}{A_1} = \frac{0.03}{0.01766}$

$v_2 = \frac{Q}{A_2} = \frac{0.03}{0.00785}$

$v_1 = 1.698\text{ m/s}$

$v_2 = 3.82\text{ m/s}$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_c \quad (z_1 = z_2)$$

$$h_c = \frac{v_2^2}{2g} \left[\frac{1}{C_c^2} - 1 \right]$$

$$= \frac{(3.82)^2}{2 \times 9.81} \left(\frac{1}{0.6} - 1 \right)^2$$

$$h_c = 0.74 (0.66)^2 = 0.32$$

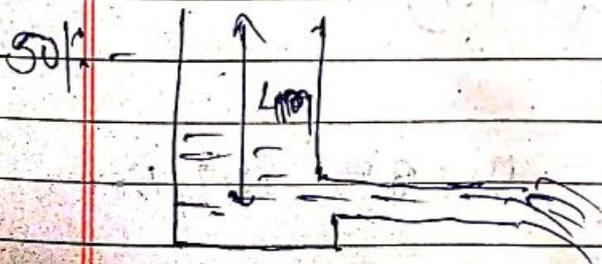
$$\Rightarrow \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + 0.32$$

$$\Rightarrow \frac{(p_1 - p_2)}{\rho g} = \frac{1}{2g} (v_2^2 - v_1^2) + 0.32$$

$$\Rightarrow p_1 - p_2 = \frac{1}{2} [(3.82)^2 - (1.69)^2] \times 1000$$

$$= 0.9 \times 10^4 \text{ N/m}^2 \text{ (Ans)}$$

Q) Determine the rate of flow of water thr. a pipe of dia 20cm and length 50m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the height of water in the tank is 4m above the centre of the pipe. Consider all minor losses and take frictional factor $f = 0.009$ and $h_f = \frac{4fL v^2}{2g}$.



Given Dia = 20cm
= 0.2m

$$A = 3.14 \times 10^{-4}$$

$$L = 50 \text{ m}$$

$$f = 0.009$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \text{all losses}$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f + h_i$$

$$\Rightarrow 0 + 0 + 4 = 0 + \frac{v_2^2}{2g} + 0 + \frac{4fLv^2}{2g} + \frac{0.5v_2^2}{2g}$$

(Here, $\frac{P_1}{\rho g} = 0, \frac{v_1^2}{2g} = 0, z_2 = 0, v_1 = v_2$)

$$\Rightarrow 4 = \frac{v^2}{2g} + \frac{4fLv^2}{2g} + \frac{0.5v^2}{2g}$$

$$4 \times 2g = v^2 (1 + 4fL + 0.5)$$

$$78.48 = v^2 (1 + 90 + 0.5)$$

$$v^2 = 78.48$$

$$\frac{78.48}{91.5} = 10.5$$

$$v = 0.92 \text{ m/s}$$

$$v = 2.733 \text{ m/s}$$

$$Q = A \times v$$

$$= 3.14 \times (0.2)^2 \times 2.733$$

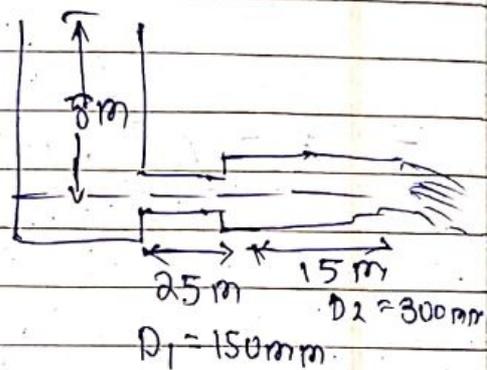
$$= 2.76 \text{ m}^3/\text{s} = 0.085 \text{ m}^3/\text{sec}$$

Q) A horizontal pipeline 40m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25m of its length from the tank the pipe is 150mm dia and its dia is suddenly enlarged to 300mm the height of water level in the tank is 8m above the centre of the pipe considering all losses of head which occur. Determine the rate of flow and take $f = 0.01$ for both sections of the pipe.

Sol:- Given $L = 40\text{m}$
 $f = 0.01$

$D_1 = 150\text{mm}$ $L_1 = 25\text{m}$
 $= 0.15\text{m}$

$D_2 = 300\text{mm}$ $L_2 = 15\text{m}$
 $= 0.3\text{m}$



$$A_1 = 0.0176\text{m}^2, A_2 = 0.07065\text{m}^2$$

$$f = 0.01$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \text{all losses}$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_{f1} + h_{f2} + h_{e1} + h_{e2} + h_{i1} + h_{i2}$$

$$0 + 0 + 8 = 0 + \frac{v_2^2}{2g} + 0 + \frac{4fL_1v^2}{2gD_1} + \frac{0.5v^2}{2g}$$

$$\Rightarrow 8 = \frac{v_2^2}{2g} + \frac{4fL_1v^2}{2gD_1} + \frac{0.5v^2}{2g}$$

$$Q = A_1v_1 = A_2v_2 \Rightarrow v_1 = \frac{A_2v_2}{A_1}$$

$$V_1 = \frac{\pi (D_2)^2 \cdot V_2}{\pi (D_1)^2} = \frac{(0.3)^2 V_2}{(0.15)^2}$$

$$V_1 = 4V_2$$

$$\Rightarrow \delta = \frac{V_2^2}{2g} + \frac{4fL_1 V_1^2}{2gD_1} + \frac{4fL_2 V_2^2}{2gD_2} + \frac{0.5V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g}$$

$$\Rightarrow \delta = \frac{V_2^2}{2g} + \frac{4 \times 0.01 \times 25 \times 16V_2^2}{19.62 \times 0.15} + \frac{0.5 \times 16V_2^2}{19.62} +$$

$$\frac{4 \times 0.01 \times 15 V_2^2}{19.62 \times 0.3} + \frac{(9V_2^2)}{19.62}$$

$$\Rightarrow \delta = \frac{V_2^2}{19.62} (100 + 106.66 + 8 + 2 + 9)$$

$$\Rightarrow V_2 = \sqrt{\frac{8 \times 19.62}{126.66}}$$

$$= 1.113 \text{ m/s}$$

~~$$V_1 = 4 \times$$~~

$$Q = A_2 V_2$$

$$= 0.0706 \times 1.113$$

$$= 0.0786 \text{ m}^3/\text{s} \text{ (Ans)}$$

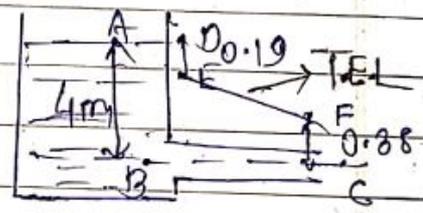
Hydraulic gradient line and total energy line:

(H.G.) Hydraulic gradient line It is defined as the line which gives sum of pressure head and datum head of the flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates showing the pressure head of a flowing fluid in a pipe from the centre of the pipe.

* Total energy line (T.E.L): It is defined as the line which gives the sum of pressure head datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe.

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- Q) Draw the HGL and TEL where take $L = 50m$
 $D = 200mm$



$H = 4m$, $f = 0.009$
Velocity of the flow = $2.734 m/s$.

$$T.E.L \text{ at } A = \frac{P}{\rho g} + \frac{v^2}{2g} + z_1$$

$$= 0 + 0 + 4m$$

$$= 4m$$

$$T.E.L \text{ at } B = \frac{P}{\rho g} + \frac{v^2}{2g} + z_1 - \frac{0.5v^2}{2g} \left[h_f = \frac{0.5v^2}{2g} \right]$$

$$= 4 - 0.19$$

$$= 3.81 \text{ m}$$

$$\text{T.E.L at C} = \frac{p}{\rho g} + \frac{v^2}{2g} + z$$

$$= 0 + \frac{(2.734)^2}{2 \times 9.81} + z = 0.38 \text{ m}$$

$$\text{H.G.L at A} = \frac{p}{\rho g} + z = 0 + 4 = 4 \text{ m}$$

$$\text{H.G.L at B} = \text{T.E.L} - \frac{v^2}{2g} = 0.38 - 0.38 = 0$$

$$\text{H.G.L at D} = \text{T.E.L} - \frac{v^2}{2g}$$

$$= 3.81 - 0.38$$

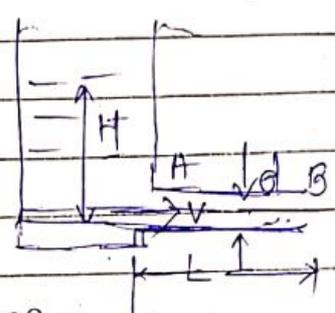
$$= \text{3.43 m}$$

Q) 11.25, 11.26

* Power transmission through pipe: by flowing of water or other liquid depends upon weight of the liquid flowing through the pipe total head available at the end of the pipe.

Consider a pipe AB connected to a tank and the power available at the end of B of the pipe can be calculated by using the following formula

- H = height of water from the centre of the pipe.
- A = inlet of pipe.
- B = outlet of pipe.
- v = velocity of liquid flowing through the pipe.
- d = dia. of the pipe.
- L = length of pipe.



The head available at the outlet of the pipe = total head at the inlet - Loss of head due to friction.

$$= H - h_f$$

$$= H - \frac{4fLv^2}{2gd}$$

Weight of water flowing through the pipe per second = mg

$W = \rho g \times \text{volume of water per second}$

$[mg/sec = \rho v/sec]$

$$= \rho g \times \text{area} \times \text{velocity}$$

$$= \rho g \times \frac{\pi}{4} d^2 \times v$$

$$P = \text{Weight of water flowing per sec} \times \text{head at the outlet}$$

$$= \rho g \times \frac{\pi}{4} d^2 \times v \times [H - h_f]$$

$$P = \rho g \times \frac{\pi}{4} d^2 \times v \left[H - \frac{4fLv^2}{2gd} \right] \text{ in watt}$$

$$P = \frac{\rho g \times \pi}{1000 \times 4} d^2 \times v \left[H - \frac{4fLv^2}{2gd} \right] \text{ in kW}$$

* Efficiency of power transmission

Efficiency = $\frac{\text{power available at the outlet of pipe}}{\text{power supply at the inlet of the pipe}}$

power supply at the inlet of the pipe
= weight of water flowing per sec \times head at the outlet of pipe

weight of water per sec \times head at inlet of pipe

$$= \frac{W \times (H - h_f)}{W \times H} = \frac{H - h_f}{H}$$

* Condition for Maximum transmission of power

The condition for maximum transmission of power is obtained by differentiating equation of power w.r.t v and equating the same to zero.

$$\Rightarrow \frac{d}{dv} \left[\frac{\rho g \times \pi}{1000 \times 4} d^2 \left(H - \frac{4fLv^2}{2gd} \right) \right] = 0$$

$$\Rightarrow \frac{d}{dv} \left[\frac{\rho g \times \pi}{1000 \times 4} d^2 \left(Hv - \frac{4fLv^3}{2gd} \right) \right] = 0$$

$$\Rightarrow \frac{\rho g \times \pi}{1000 \times 4} d^2 \left(H - \frac{6fLv^2}{2gd} \right) = 0$$

$$\frac{\rho g \times \pi d^2 \times H}{1000 \times 4} = \frac{\rho g \times \pi d^2 \times 6FLv^2}{1000 \times 4 \times 9D}$$

$$H - \frac{6FLv^2}{2gd} = 0 \Rightarrow H - \frac{3 \times 4FLv^2}{2gd} = 0$$

$$H = 3h_f$$

* Maximum efficiency of transmission of power:
Efficiency of power transmission through pipe is $\eta = \frac{H - h_f}{H}$ [$h_f = \frac{H}{3}$]

$$\eta = \frac{H - \frac{H}{3}}{H} = \frac{2H}{3H}$$

$$\eta = \frac{2}{3} = 0.66\%$$

Q) A pipe of dia 300mm and length 3500m is used for the transmission of power by water. The total head at the inlet of pipe is 500m. Find the maximum power available at the outlet of pipe, if $f = 0.006$.

So: - Given $D = 300 \text{ mm}$

$$= 0.3 \text{ m}$$

$$A = 0.07065 \text{ m}^2$$

$$L = 3500 \text{ m}$$

$$H = 500 \text{ m}$$

$$f = 0.006$$

$$\text{Now, } h_f = \frac{4fLv^2}{2gd} = \frac{4 \times 0.006 \times 3500 \times v^2}{2 \times 9.81 \times 0.3}$$

$$\text{and } H = 3h_f$$

$$\text{or, } h_f = H/3 = \frac{500}{3} = 166.66 \text{ m} \quad \text{--- (2)}$$

From eqn (1) & (2)

$$14.271 V^2 = 166.66 \text{ m}$$

$$V = \sqrt{\frac{166.66}{14.27}}$$

$$V = \sqrt{11.618}$$

$$= 3.417 \text{ m/s}$$

$$\therefore Q = A \times V$$

$$= (0.07065 \times 3.417)$$

$$= 0.241564 \text{ m}^3/\text{s}$$

Head available at the outlet of the pipe

$$= H - h_f$$

$$= H - H/3$$

$$= 500 - 166.66$$

$$= 333.34 \text{ m}$$

$$\therefore P_{\text{max}} = \rho g \times Q \times \text{head at outlet}$$

$$1000$$

$$= 1000 \times 9.81 \times 0.241564 \times 333.34$$

$$1000$$

$$P_{\text{max}} = 789.93 \text{ kW (Ans)}$$

Q1) A pipe line of length 2000m is used for power transmission if 110.8625 kW power is to be transmitted for the pipe in which water having a pressure of 490.5 N/cm² at inlet is flowing. Find the dia. of the pipe and efficiency of transmission if the

pressure drop over the length of the pipe is
 98.1 N/cm^2 . $f = 0.0065$.

Sol: Given $L = 2000 \text{ m}$

$$P = 110.3625 \text{ kW}$$

$$P_1 = 490.5 \text{ N/cm}^2$$

$$= 490.5 \times 10^4 \text{ N/m}^2$$

$$P_1 - P_2 = 98.1 \text{ N/cm}^2$$

$$= 98.1 \times 10^4 \text{ N/m}^2$$

$$f = 0.0065$$

find $D = ?$

$\Rightarrow n = 9$

$$H = \frac{P}{\rho g} = \frac{490.5 \times 10^4 \text{ N/m}^2}{10^3 \times 9.81} = 500 \text{ m}$$

$$\frac{P}{\rho g}$$

$$h_f = \frac{\Delta P}{\rho g}$$

$$h_f = \frac{H}{3} = \frac{500}{3} = 166.66 \text{ m}$$

$$h_f = \frac{98.1 \times 10^4}{10^3 \times 9.81} = 100 \text{ m}$$

$$P = \rho g Q (H - h_f)$$

$$1000$$

$$110.3625 = \frac{10^3 \times 9.81 \times Q (500 - 100)}{1000}$$

$$1000$$

$$110.3625 = \frac{400}{3} \times 9.81 \times Q$$

$$Q = \frac{110.3625}{3270.06} = 0.0337 \text{ m}^3/\text{s}$$

$$Q = 0.0281 \text{ m}^3/\text{sec}$$

$$Q = AV$$

$$0.0281 = \frac{\pi}{4} D^2 V$$

$$D^2 V = 0.0429 \quad 0.035$$

$$V = 0.0429 / D^2 \quad \text{--- (1)}$$

$$V = 0.035 / D^2 \quad \text{--- (2)}$$

$$hf = \frac{4\pi Lv^2}{2gD}$$

$$100466.66 = \frac{4 \times 0.0065 \times 2000 \times v^2}{2 \times 9.81 \times D}$$

$$100466.66 = 2.65v^2$$

$$\frac{v^2}{D} = \frac{37.73}{62.89} \quad \text{--- (ii)}$$

Substituting the value of v in (ii)

$$\frac{(0.035D^2)^2}{D} = \frac{37.73}{62.89}$$

$$\frac{1.840 \times 10^{-3}}{D} = 62.89$$

$$D = \frac{1.840 \times 10^{-3}}{62.89} = 0.0000294 \text{ m}$$

$$\frac{1.840 \times 10^{-3}}{D^5} = 62.89 \quad D = 0.1277 \text{ m}$$

$$D^5 = \frac{0.001840}{62.89}$$

$$D^5 = 2.925 \times 10^{-5}$$

$$D = 0.123 \text{ m}$$

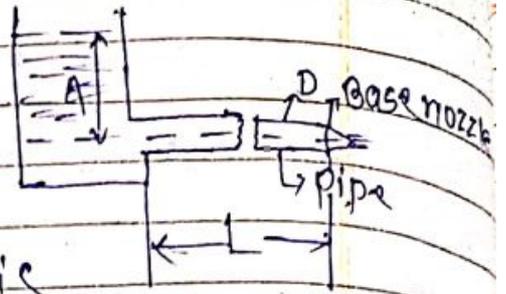
$$n = \frac{H - hf}{H} = \frac{500 - 100}{500} = 0.8$$

$$H = 0.8 = 80\%$$

$$500 = 66\%$$

Flow through nozzle:

The Fig. shows the nozzle fitted at the end of a long pipe. The total energy at the end of the pipe consist of pressure energy and kinetic



energy. Then by fitting the nozzle at the end of the pipe the total energy will be converted into kinetic energy. Thus, nozzles are used for higher velocity of flow as required.

D = dia. of the pipe

A = Area of the pipe

L = Length of the pipe

v = velocity of the flow in pipe.

H = Total head at the inlet of the pipe.

d = dia. of nozzle at outlet.

v = Velocity of flow at outlet of nozzle.

a = area of nozzle at outlet = $\frac{\pi}{4} d^2$.

f = coefficient of friction of pipe.

$$\text{Loss of head due to friction } (h_f) = \frac{4fLv^2}{2gD}$$

Head available at the end of pipe or at the base of the nozzle = head at inlet of the pipe - head lost due to friction. = $H - h_f$

$$= H - \frac{4fLv^2}{2gD}$$

Neglecting the minor losses and also assuming losses in nozzle negligible we have total head at the inlet of the pipe = total head at the outlet of the nozzle + losses.

Total head available at the outlet of the pipe is Kinetic head = $\frac{v^2}{2g}$

$$H = \frac{v^2}{2g} + h_f = \frac{v^2}{2g} + \frac{4fLv^2}{2gD}$$
$$= \frac{v^2}{2g} + \frac{4fLv^2}{2gD} \quad \text{--- (1)}$$

From continuity eqⁿ in the pipe and outlet of nozzle.

$$Av = av$$

$$v = \frac{av}{A} \quad \text{--- (2)}$$

Substituting value of v in (1)

$$H = \frac{v^2}{2g} + \frac{4fL}{2gD} \left(\frac{av}{A} \right)^2$$

$$= \frac{v^2}{2g} \left(1 + \frac{4fLa^2}{DA^2} \right)$$

$$\Rightarrow v^2 = \frac{2gH}{\left(1 + \frac{4fLa^2}{DA^2} \right)}$$

$$\therefore v = \sqrt{\frac{2gH}{\left(1 + \frac{4fLa^2}{DA^2} \right)}}$$

Power Transmitted through Nozzle :-

The kinetic energy of the jet at the outlet of the nozzle

$$= \frac{1}{2} m v^2$$

mass of liquid at the outlet of the nozzle per sec

$$\dot{m} = \rho a v$$

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$$K.E = \frac{1}{2} \rho a v \cdot v^2$$

$$= \frac{1}{2} \rho a v^3$$

Power in kW at the outlet of the nozzle

$$= K.E / \text{sec} \times \frac{1}{1000}$$

1000

$$= \frac{1}{2} \frac{\rho a v^3}{1000} \text{ kW}$$

Efficiency of power transmission through nozzle

(η) = $\frac{\text{power available at the outlet of the nozzle}}{\text{power available at the inlet of the pipe}}$

power available at the inlet of the pipe

$$= \frac{1}{2} \frac{\rho a v^3}{1000} / \frac{\rho g a H}{1000}$$

$$= \frac{1}{2} \frac{v^2}{gH}$$

$$\eta = \frac{v^2}{2gH} = \frac{1}{\left(1 + \frac{4fL a^2}{DA^2}\right)}$$

Condition for maximum power transmission through Nozzle: $h_f = H/3$

* Diameter of the nozzle for maximum power transmission through nozzle $\frac{A}{a} \sqrt{\frac{8fL}{D^5}}$

Q) Find the Maximum power Transmitted by a jet of water discharging freely out of nozzle fitted to a pipe = 300m long and 100mm dia. With coefficient of friction as 0.01. The available head at the nozzle is 90m.

Sol:- Given $L = 300\text{m}$
 $D = 100\text{mm} = 0.1\text{m}$
 $f = 0.01$, $H = 90\text{m}$

$$H = \frac{v^2}{2g} = 90\text{m}$$

$$v = \sqrt{90 \times 2 \times 9.81}$$

$$= 42.02\text{m/s}$$

$$Q = A \times v$$

$$= \frac{\pi}{4} d^2 \times v$$

$$d = \left(\frac{D^5}{8fL} \right)^{1/4}$$

$$= \left(\frac{(0.1)^5}{8 \times 0.01 \times 300} \right)^{1/4} = 0.0254\text{m}$$

$$Q = a \times v$$

$$= \frac{\pi}{4} (0.0254)^2 \times 42.02$$

$$= 0.02129\text{m}^3/\text{sec}$$

∴ Maximum power transmitted
 $= \rho g Q \times \text{head available at outlet of nozzle}$

$$= 1000 \times 9.81 \times 0.02129 \times 90$$

$$= 18.795\text{KW (Ans)}$$

Method of Dimensional Analysis

1) Rayleigh's method

2) Buckingham's Π Theorem

1) Rayleigh's method

Let x_1 is a variable which depends on x_1, x_2, x_3 variables then according to Rayleigh's method x is a function of $x = (x_1, x_2, x_3)$

$$x = k x_1^a x_2^b x_3^c$$

Q) The time period 't' of a pendulum depends upon the length 'l' of the pendulum and accn due to gravity 'g'. Derive an expression for the time period.

$$t = k (l)^a (g)^b$$

$$\Rightarrow M^0 L^0 T^1 = k (L)^a (M L^{-1} T^{-2})^b$$

$$\Rightarrow M^0 L^0 T^1 = k L^a L^{-2b} T^{-2b}$$

$$\Rightarrow M^0 L^0 T^1 = k L^{a+b} T^{-2b}$$

$$a+b=0 \quad -2b=1$$

$$a=-b \quad b=-1/2$$

$$a=1/2$$

$$\Rightarrow t = k (l)^{1/2} (g)^{-1/2}$$

$$= k \sqrt{\frac{l}{g}}$$

$$\Rightarrow \boxed{t = 2\pi \sqrt{l/g}}$$

2) Buckingham's Π Theorem - If there are small 'n' variables (independent & dependent variables) in a physical phenomenon and if these variables contain 'm' fundamental dimension (MLT) then the variables are arranged into (n-m) dimensionless term. Each term is called a Π term.

Let $x_1, x_2, x_3, \dots, x_n$ are the variables involved in a physical problem.

Let x_1 be the dependent variable x_2, x_3, \dots, x_n are the independent variables on which x_1 depends than $x_1 = f(x_2, x_3, \dots, x_n)$.

and Mathem

$$F(x_1, x_2, x_3, \dots, x_n) = 0$$

It is a dimensionally homogeneous equations

It contains 'n' variable. If there are 'm' no. of fundamental dimensions than a/c to the Buckingham's Π theorem can be written in terms of no. of dimensionless groups in which Π term = (n-m).

Hence, equation becomes $F(\Pi_1, \Pi_2, \dots, \Pi_{n-m}) = 0$

Each Π term contains (m+1) variables where m is the no. of fundamental dimension and is also called repeating variables.

Let, in the above case x_2, x_3 and x_4 are repeating variables if the fundamental dimension $m(M, L, T) = 3$. Then each Π -term is written as

$$\Pi_1 = x_2^{a_1} \cdot x_3^{b_1} \cdot x_4^{c_1} \cdot x_1$$

$$\Pi_2 = x_2^{a_2} \cdot x_3^{b_2} \cdot x_4^{c_2} \cdot x_5$$

$$\vdots$$

$$\Pi_{n-m} = x_2^{a_{n-m}} \cdot x_3^{b_{n-m}} \cdot x_4^{c_{n-m}} \cdot x_n$$

Q) The resisting force 'R' of a supersonic plane during flight can be considered as ~~and~~ dependent upon the length of the air craft, 'L', velocity 'V', air viscosity μ , air density ρ and bulk modulus of air 'K'. Express the functional relationship b/w these variables and the resisting force.

Sol: $n = 6$

$m = 3$

$(n - m) = 3$ no. of π terms.

The resisting force 'R' depends on L, V, μ , ρ , K

$$R = f(L, V, \mu, \rho, K)$$

$$= f(R, L, V, \mu, \rho, K) = 0$$

Total no. of variables (n) = 6

no. of fundamental dimensions (m) = 3

$$R = [M^1 L^1 T^{-2}]$$

$$L = [M^0 L^1 T^0]$$

$$V = [M^0 L^1 T^{-1}]$$

$$\mu = [M^1 L^{-1} T^{-1}]$$

$$\rho = [M^1 L^{-3} T^0]$$

$$K = [M^1 L^{-1} T^{-2}]$$

Where $(n - m)$ is 3 and is repeating variables. Out of 6 no. of variables there are 3 no. of variables are to be selected as repeating variables. R is dependent variable and should be selected as repeating variables. Out of three variables one variable should have geometric property, second is flow property and another is fluid. So, L, V and ρ are selected as repeating.

$$\hat{\Pi}_1 = [l^{a_1} v^{b_1} \rho^{c_1}] R \Rightarrow [M^0 L^0 T^0] = [L^{a_1}] [L^{-1}]^{b_1} [M L^{-3}]^{c_1} [M L^{-2}]$$

$$\hat{\Pi}_2 = [l^{a_2} v^{b_2} \rho^{c_2}] \mu$$

$$\hat{\Pi}_3 = [l^{a_3} v^{b_3} \rho^{c_3}] k$$

$$\Rightarrow [M^0 L^0 T^0] = [L]^{a_1} [L T^{-1}]^{b_1} [M L^{-3}]^{c_1} [M L^{-2}]$$

$$\Rightarrow M^{c_1+1} = M^0$$

$$a_1 + b_1 - 3c_1 + 1 = 0$$

$$\Rightarrow a_1 - 2 - 3(-1) + 1 = 0$$

$$\Rightarrow c_1 + 1 = 0$$

$$-b_1 - 2 = 0 \Rightarrow b_1 = -2$$

$$\Rightarrow a_1 = -2$$

$$\Rightarrow c_1 = -1$$

$$\hat{\Pi}_1 = l^{-2} v^{-2} \rho^{-1} R = \frac{R}{l^2 v^2 \rho}$$

$$\Rightarrow [M^0 L^0 T^0] = [L]^{a_2} [L T^{-1}]^{b_2} [M L^{-3}]^{c_2} [M L^{-2} T^{-2}]$$

$$M^0 = M^{c_2+1}$$

$$L^0 = [a_2 + b_2 - 3c_2 - 2]$$

$$\Rightarrow c_2 + 1 = 0$$

$$\Rightarrow a_2 + b_2 - 1 = 0$$

$$\Rightarrow c_2 = -1$$

$$\Rightarrow a_2 - 1 - 1 = 0$$

$$\Rightarrow a_2 = 2$$

$$L^0 = [a_2 - 3c_2 - 2]$$

$$T^0 = T^{-b_2 - 2}$$

$$\Rightarrow a_2 - 3c_2 - 2 = 0$$

$$\Rightarrow -b_2 - 1 = 0$$

$$\Rightarrow a_2 + 3 = 2 = 0$$

$$b_2 = -1$$

$$\Rightarrow a_2 = -2$$

$$\hat{\Pi}_2 = [l]^{-2} [v]^{-1} [\rho]^{-1} \mu$$

$$= \frac{\mu}{l^2 v \rho}$$

$$\Rightarrow [M^0 L^0 T^0] = [L]^{a_3} [L T^{-1}]^{b_3} [M L^{-3}]^{c_3} [M L^{-2} T^{-2}]$$

$$\Rightarrow 0 = c_3 + 1$$

$$0 = a_3 + b_3 - 3c_3 - 1$$

$$c_3 = -1$$

$$a_3 + b_3 + 1 + 1 = 0 \quad a_3 - 2 + 3$$

$$a_3 - 2 + 1 = 0$$

$$= 0$$

$$a_3 = 2$$

$$a_3 = 0$$

$$0 = -b_3 - 2$$

$$b_3 = -2$$

$$\Pi_3 = [L]^0 [V]^{-2} [\rho]^{-2} R = \frac{R}{V^2 \rho}$$

$$\frac{R}{L^2 V^2 \rho} = \phi \left[\frac{u}{L V \rho}, \frac{K}{V^2 \rho} \right]$$

$$\rightarrow R = f L^2 V^2 \rho \left[\frac{u}{L V \rho}, \frac{K}{V^2 \rho} \right]$$

4. Similarity or

Similitude is defined as the similarity b/w the model and its prototype in every aspects which means that the model and prototype have similar property or model and prototype are completely similar.

Three types of similarity exists b/w prototype and model.

- i) Geometric Similarity
- ii) Kinematic Similarity
- iii) Dynamic Similarity

i) The geometric similarity is said to exist b/w the model and prototype if the ratio of all corresponding linear dimension in model and prototype are equal.

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r$$

Where, L_r = scale ratio

L_p = length of prototype

L_m = " " model

b_p = breadth of prototype

b_m = " " model

D_p = Dia. of prototype

D_m = " " model

$$\frac{A_p}{A_m} = \frac{l_p \times b_p}{l_m \times b_m} = L_r \times L_r = L_r^2$$

A_p = Area of prototype.
 A_m = " " model.

ii) Kinematic Similarity means the similarity of motion b/w the model and prototype. Kinematic Similarity is said to exist if

$$\frac{V_p}{V_m} = \frac{V_p}{V_m} = V_r \text{ (velocity ratio)}$$

$$\frac{a_p}{a_m} = \frac{a_p}{a_m} = a_r \text{ (accn ratio)}$$

iii) Dynamic Similarity means the similarity of force b/w the model and prototype.

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m}$$

F_i = inertia force
 F_v = viscous force
 F_g = gravitational force.

F_r = force ratio

* Model laws or similarity law:

i) Reynold's model law is based on which models are based on Reynold's no.

$$(Re)_m = (Re)_p$$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\Rightarrow \frac{\rho_p v_p L_p}{\rho_m v_m L_m} \times \frac{1}{\mu_p / \mu_m} = 1$$

$$\Rightarrow \frac{\rho_r v_r L_r}{\mu_r} = 1 \quad \text{where } \rho_r, v_r, L_r, \mu_r \text{ are the scale ratio of density, velocity, length and viscosity}$$

$$\text{Time scale ratio } (t_r) = \frac{L_r}{v_r}$$

$$Ar = \frac{v_r}{f_r}$$

$$\begin{aligned} F_r &= \text{mass} \times \text{acc}^n \\ &= M_r \times a_r \\ &= \rho_r A_r v_r \times a_r \\ &= \rho_r L_r^2 v_r \times a_r \end{aligned}$$

$$\begin{aligned} \text{Discharge scale ratio } (Q_r) &= (\rho A v)_r \\ &= \rho_r A_r v_r \\ &= \rho_r L_r^2 v_r \end{aligned}$$

31/10/19 Froude Model: It is the difference b/w inertial to gravitational force.

$$\text{Froude no.} = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho A v^2}{\rho A L g}} = \frac{v}{\sqrt{Lg}}$$

Froude Model Law:-

$$(F_e)_{\text{model}} = (F_e)_{\text{prototype}}$$

$$\Rightarrow \frac{v_m}{\sqrt{L_m g_m}} = \frac{v_p}{\sqrt{L_p g_p}}$$

$$\text{As } g_m = g_p$$

$$\Rightarrow \frac{v_m}{\sqrt{L_m}} = \frac{v_p}{\sqrt{L_p}}$$

$$\Rightarrow \frac{v_m}{v_p} \times \frac{1}{\sqrt{L_m/L_p}} = 1$$

$$\Rightarrow \frac{V_p}{V_{om}} = \sqrt{\frac{L_p}{L_m}} = \sqrt{L_r}$$

$$\boxed{\frac{V_p}{V_{om}} = \sqrt{L_r}}$$

$$\Phi = L^3/T$$

* Euler's Model law

$$\text{Euler's no.} = \frac{F_i}{F_p} = \frac{\rho A v^2}{\rho A} = \frac{v^2}{p/\rho} = \frac{v}{\sqrt{p/\rho}}$$

Euler Model = Euler prototype

$$\frac{V_{om}}{\sqrt{\rho_m/\rho_m}} = \frac{V_p}{\sqrt{\rho_p/\rho_p}}$$

$$\frac{V_{om}}{\sqrt{\rho_m}} = \frac{V_p}{\sqrt{\rho_p}}$$

† Weber's number (We):

$$We = \frac{F_i}{F_s} \quad F_s = \text{surface tension force}$$

$$= \frac{\rho A v^2}{\sigma L} = \frac{v}{\sqrt{\sigma/\rho L}}$$

(We) Model = (We) prototype

$$\frac{V_{om}}{\sqrt{\sigma_m/\rho_m L_m}} = \frac{V_p}{\sqrt{\sigma_p/\rho_p L_p}}$$

If $\sigma_m = \sigma_p$

$$\frac{V_{om}}{V_p} =$$

$$\text{Mach no.} \sqrt{F_i/F_e} = \sqrt{\frac{\rho A v^2}{K L^2}} = \frac{v}{\sqrt{K/\rho}} = \frac{v}{\sqrt{K/\rho}}$$

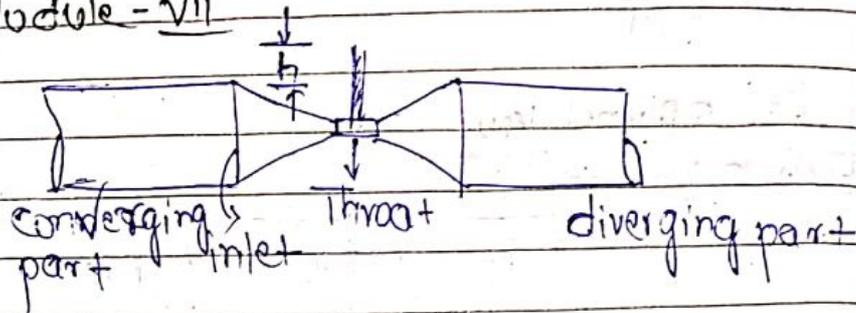
$$M = v/L$$

Mach model Law

(Mach no.) Model = (Mach no.) Prototype

$$\frac{v_m}{\sqrt{k_m/\rho_m}} = \frac{v_p}{\sqrt{k_p/\rho_p}}$$

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$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

$$p_1 - p_2 = \rho \frac{v_2^2 - v_1^2}{2}$$

$$h = \frac{v_2^2 - v_1^2}{2g}$$

$$a_1 v_1 = a_2 v_2 \Rightarrow v_1 = \frac{a_2 v_2}{a_1}$$

$$h = \frac{v_2^2}{2g} - \frac{(a_2 v_2 / a_1)^2}{2g}$$

$$= \frac{v_2^2}{2g} \left[1 - \left(\frac{a_2}{a_1} \right)^2 \right]$$

$$= \frac{v_2^2}{2g} \left[\frac{a_1^2 - a_2^2}{a_1^2} \right]$$

$$V_2^2 = 2gh \times \frac{a_1^2}{(a_1^2 - a_2^2)}$$

$$V_2 = \frac{\sqrt{2gh} \cdot a_1}{\sqrt{a_1^2 - a_2^2}}$$

$$Q = a_2 v_2$$

$$= a_2 \cdot \frac{a_1}{\sqrt{a_1^2 - a_2^2}}$$

$$Q_H = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}$$

$$Q_{net} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

value of 'h' given by differential U-tube manometer.

Case-1 $h = x \left[\frac{s_H}{s_0} - 1 \right]$ $s_H = \text{sp. gravity heavier liquid.}$
 $s_0 = \text{sp. gravity of the liquid flowing thro pipe.}$
 $x = \text{Diff. of the heavier liquid column in U-tube}$

$$s_H = 13.6$$

$$s_0 = 1$$

Case-2 $h = x \left[1 - \frac{s_L}{s_0} \right]$ $s_L = \text{sp. gravity lighter liquid.}$

* Notches and weirs

A notch is a device which is used for measuring rate of discharge through a small channel or tank.

→ A weir is a concrete or machinery structure based in an open channel over which flow occurs.

* Classification of notch and weir

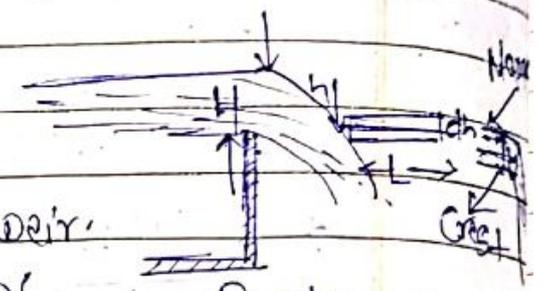
→ Notch are classified as

- a) Rectangular
- b) Triangular notch
- c) Stepped notch
- d) Trapezoidal notch

Discharge over a rectangular Notch or Weirs

H = head of water over the crest.

L = length of the notch or weir.



For finding the discharge of water flowing over the weir or notch consider an elemental strip having a thickness of element length L and depth h from the free surface of water.

Area of strip = $L \times dh$

theoretical velocity = $\sqrt{2gh}$

$dQ = dx \times \text{Area of strip} \times \text{theo. velocity}$
 $= Cdx \times L \times dh \times \sqrt{2gh}$

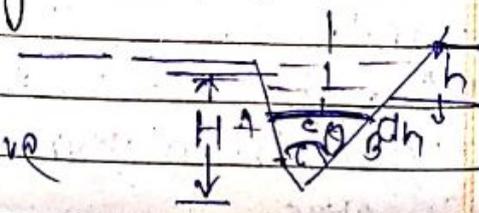
$\therefore Q = \int_0^H dQ = \int_0^H Cdx \times L \times dh \times \sqrt{2gh}$
 $= Cdx \times L \times \sqrt{2g} \int_0^H h^{1/2} dh$

$\Rightarrow Q = Cdx \times L \times \sqrt{2g} \left[\frac{2}{3} h^{3/2} \right]_0^H$
 $= \frac{2}{3} Cdx \times L \times \sqrt{2g} H^{3/2}$

Discharge over a triangular notch or Weirs.

θ = angle of Notch

H = head of water above the notch



* Advantage of triangular notch or weir over rectangular notch :-

→ A triangular notch or weir is preferred to a rectangular notch due to the following reasons

- The expression for discharge for a right angle V notch or weir is very simple.
- For measuring low discharge a triangular notch gives more accurate results than a rectangular notch. In case of triangular notch only one reading i.e. H is required for the computation of discharge.
- Ventilation of triangular notch is not necessary.

Q) Determine the height of a rectangular weir or notch of length 6m to be built across a rectangular channel. The max^m depth of water on the upstream side of weir is 1.8m and discharge is 2000 l/sec. Take $C_d = 0.6$ and neglect end contraction.

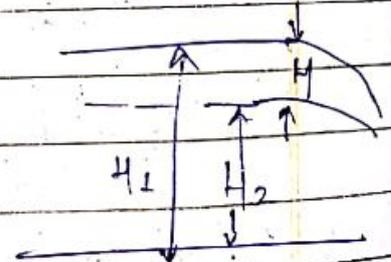
Sol: - Given

$$H_1 = 1.8 \text{ m}$$

$$L = 6 \text{ m}$$

$$Q = 2000 \text{ l/s}$$

$$= 2 \text{ m}^3/\text{s}$$



$$Q = C_d \times L \times \sqrt{2g} H^{3/2} \Rightarrow H = 0.044$$

$$2 = 0.6 \times 6 \times \sqrt{19.62} H^{3/2}$$

$$H_2 = H_1 - H$$

$$= 1.8 - 0.044$$

$$= 1.756$$

$$= 1.756$$