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Drainage Engineering

(BTAI3103)

Module 4

Subsurface Flow to Drains and Drainage Equation

(Lecture-11: Ernst Equation)

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Ernst Equation

- It is applicable to any type of two layered soil profile; the interface between the two layers, either above or below the drain
- Ernst divided the flow to the drains into
 - Horizontal flow (h_h)
 - Vertical flow (h_v)
 - Radial flow (h_r)

$$H = h_h + h_v + h_r$$



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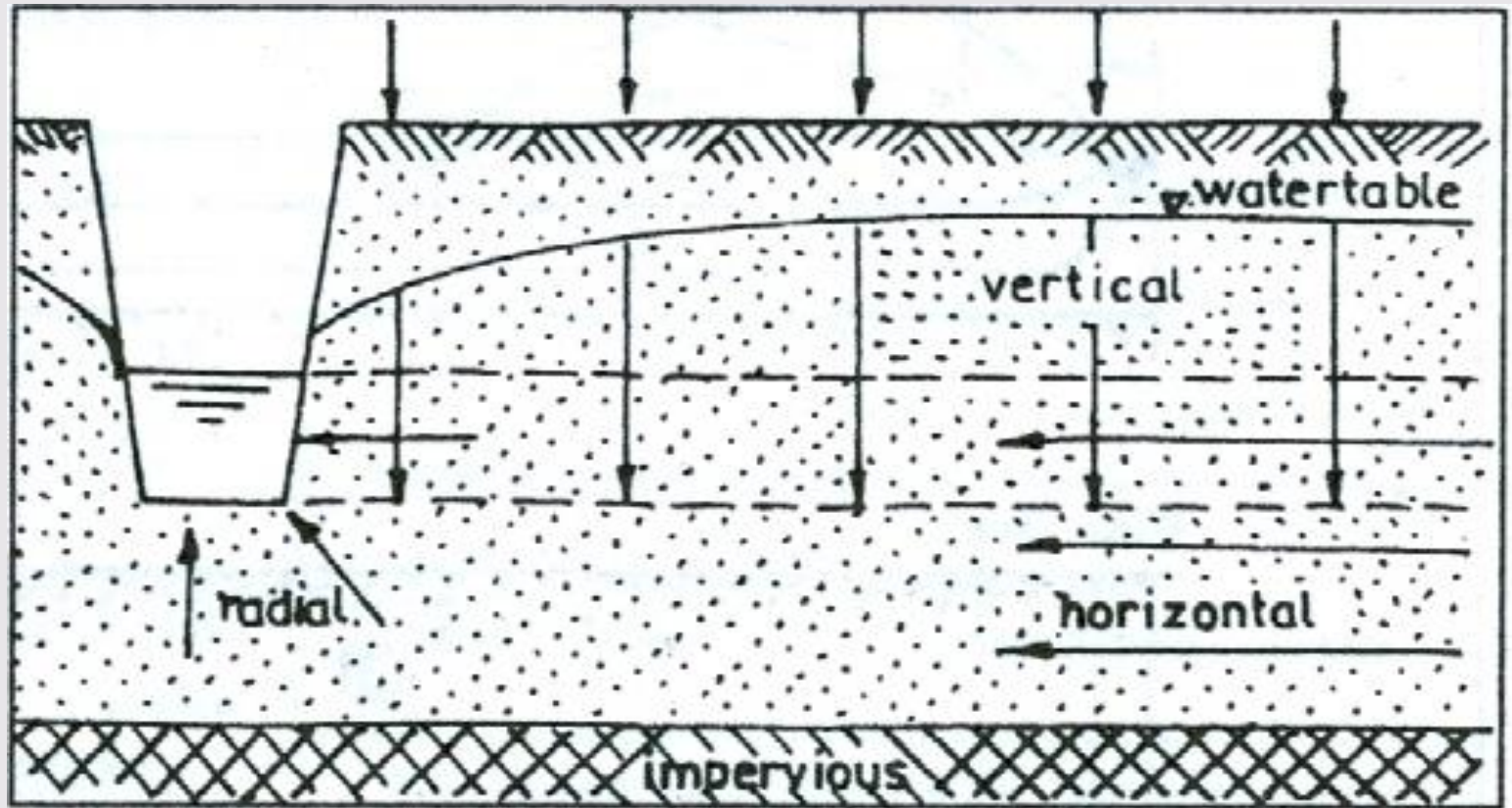


Fig. Different components of flow towards drain



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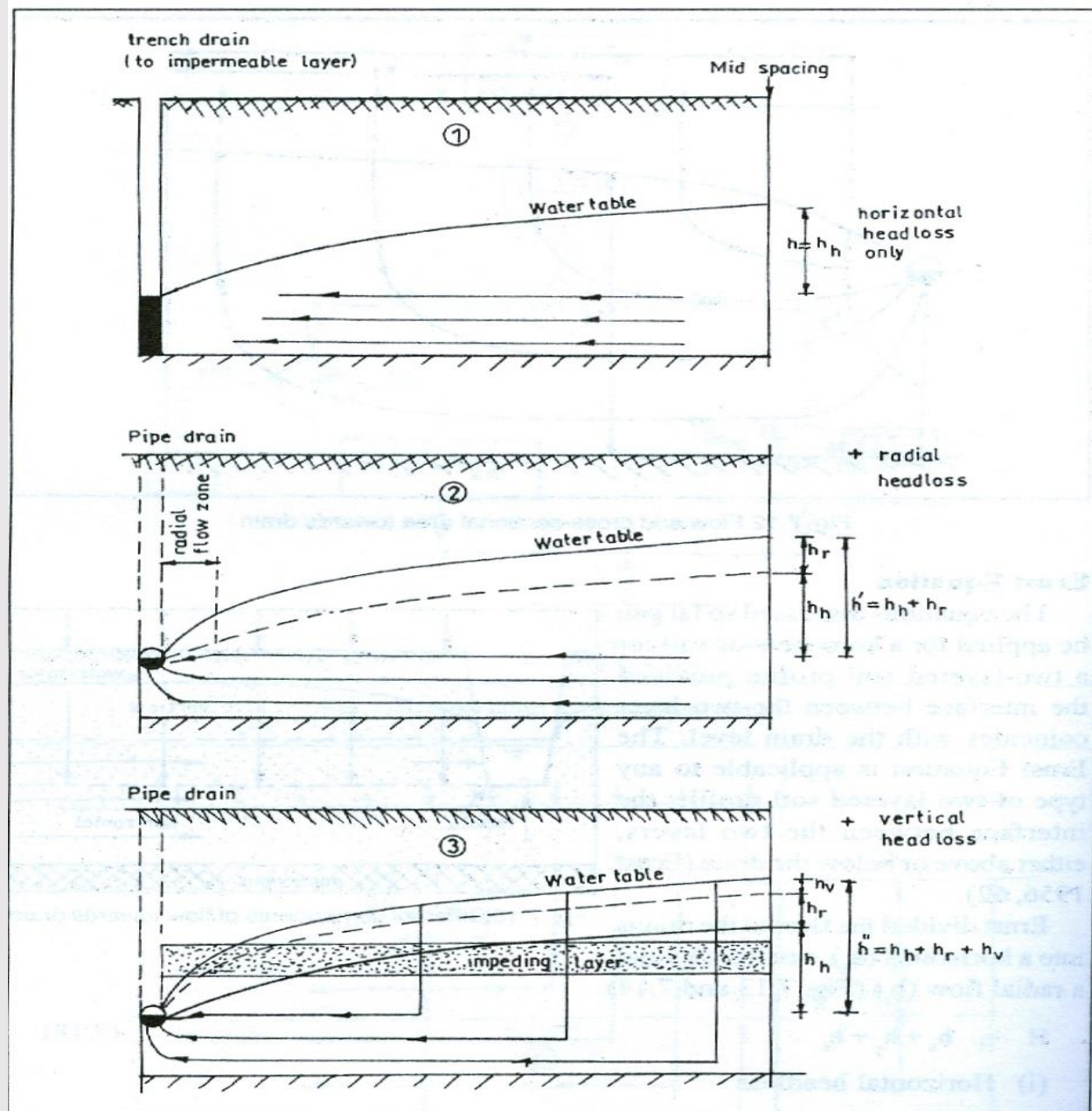


Fig. Horizontal, radial and vertical head loss



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Horizontal Head Loss

$$H = q \frac{L^2}{8 \sum (KD)_h}$$

Where

$\Sigma(KD)_h$ = transmissivity of the soil layers through which the water flows, (m²/d)

If the impervious layer is very deep, the value of $\Sigma(KD)_h$ increases to infinity and consequently, the horizontal head loss decreases to zero

The maximum thickness of the soil layer below drain level through which the flow is considered $\Sigma(D)_h$ is restricted to $\frac{1}{4} L$ to prevent the above situation



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Head loss in Vertical Flow (h_v)

- Vertical flow is assumed to take place in the layer between the water table and the drain level

$$q = K_v \frac{h_v}{D_v}$$

or

$$h_v = q \frac{D_v}{K_v}$$

Where;

D_v = thickness of the layer in which vertical flows is considered (m)

K_v = vertical hydraulic conductivity (m/d)



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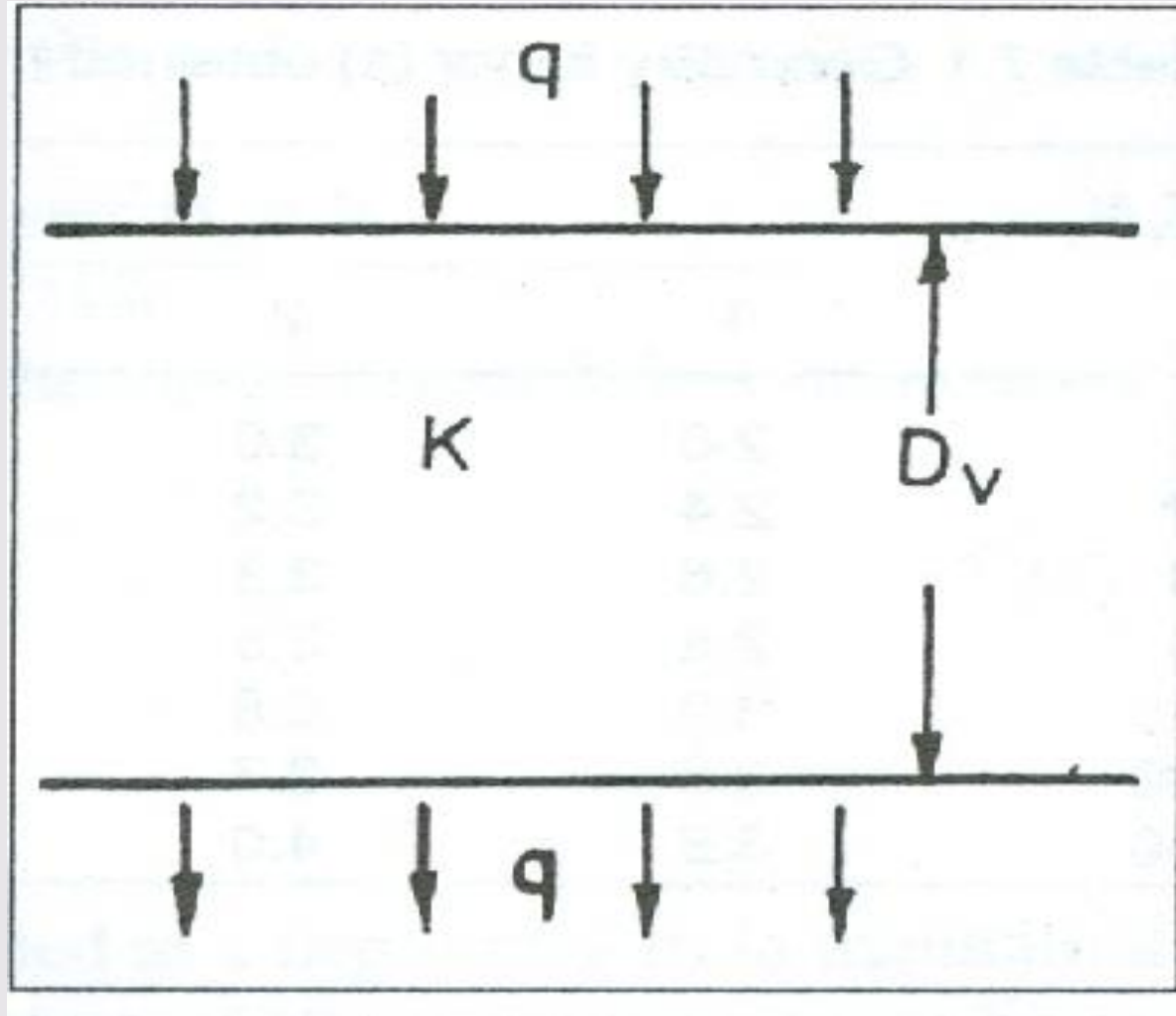


Fig. Head loss in vertical flow (h_v)



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Head loss in Radial Flow (h_r)

The radial flow is assumed to take place below the drain level and can be expressed as

$$h_r = K_v q \frac{L}{\pi K_r} \ln \frac{a D_r}{u}$$

Where,

K_r = radial hydraulic conductivity, (m/d)

a = geometry factor for the radial resistance, (-)

D_r = thickness of the layer in which the radial flow is considered $\leq \frac{1}{4} L$, (m)

u = wetted perimeter of the drain, (m)



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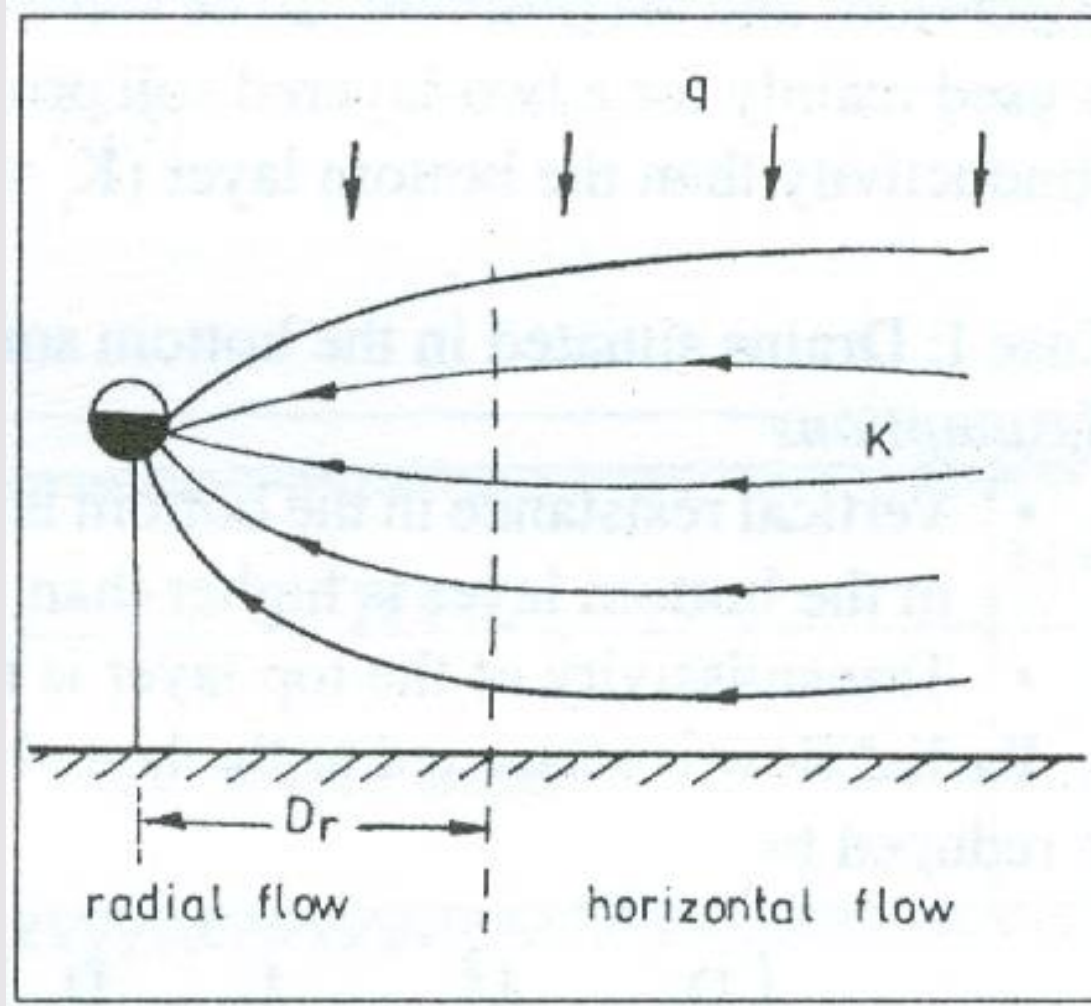


Fig. Head loss in radial flow (h_r)



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- The geometry factor ‘a’ depends on the soil profile and the position of the drain
- In a homogenous soil $a = 1$
- In a layered soil ‘a’ depends on whether the drains are in the top or bottom soil layer
 - For the drains located in the bottom of soil layer, $a=1$, as the radial flow is assumed to be restricted to this layer
 - For the drain in the top layer, values of ‘a’ are given below depending upon the ration of hydraulic conductivity of bottom (K_b) and top (K_t) layer



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$$K_b/K_t < 0.1; a=1$$

(bottom layer can be considered impervious, hence homogenous profile)

$$0.1 \leq K_b/K_t \leq 50; \text{ depending upon } K_b/K_t \text{ and } D_b/D_t$$

$$K_b/K_t \geq 50; a=4$$

$$h = q \left(\frac{D_v}{K_v} + \frac{L^2}{8 \sum (KD)_h} + \frac{L}{\pi K r} \ln \frac{a D_r}{u} \right)$$



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- If the drains are located at the interface of the two layers, Hooghoudt Equation can be used
- If the drains are located either above or below the interface of the two soil layers, the Ernst Equation can be used
- If the hydraulic conductivity of the bottom layer is significantly lower than the top layer, the former can be considered as impervious and the problem can be solved as one layered problem
- The Ernst equation is used mainly for a two-layered soil profile when the top layer has a lower hydraulic conductivity than the bottom layer



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Case I: Drains situated in the bottom soil layer

Assumptions

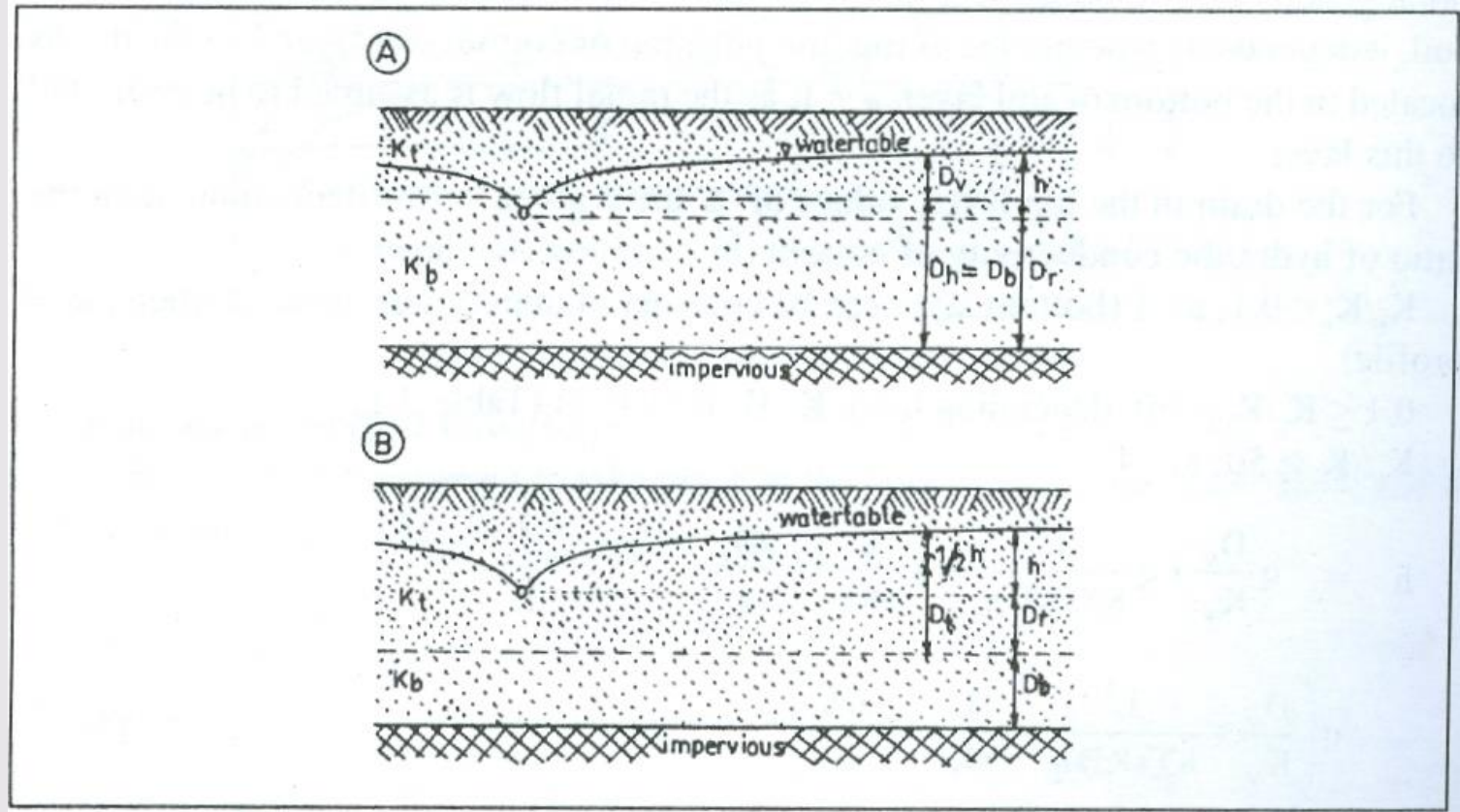
- Vertical resistance in the bottom layer is neglected as the hydraulic conductivity in the bottom layer is higher than the top layer
- Transmissivity of the top layer is neglected as $K_t < K_b$ and also $D_t < D_b$
- Radial flow is restricted to the layer below drain level (D_r), $a=1$, hence equation is reduced to

$$h = q \left(\frac{D_v}{K_t} + \frac{L^2}{8K_b D_b} + \frac{L}{\pi K_b} \ln \frac{D_r}{u} \right)$$



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Two-layered soil profile with drain in the bottom layer (A) and in the top layer (B)



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Case II: Drains located in the top layer

Assumptions

There is no vertical flow in the bottom layer; $D_v = h$

$(KD)_h = K_b D_b + K_t D_t$ in which $D_t = D_r + 1/2 h$

Radial flow is restricted to the region in the top soil layer below drain level

$$h = q \left(\frac{D_v}{K_t} + \frac{L^2}{8(K_b D_b + K_t D_t)} + \frac{L}{\pi K_t} \ln \frac{a D_r}{u} \right)$$



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Drain spacing by NOMOGRAPHS

Drain spacing can be determined by using nomography also

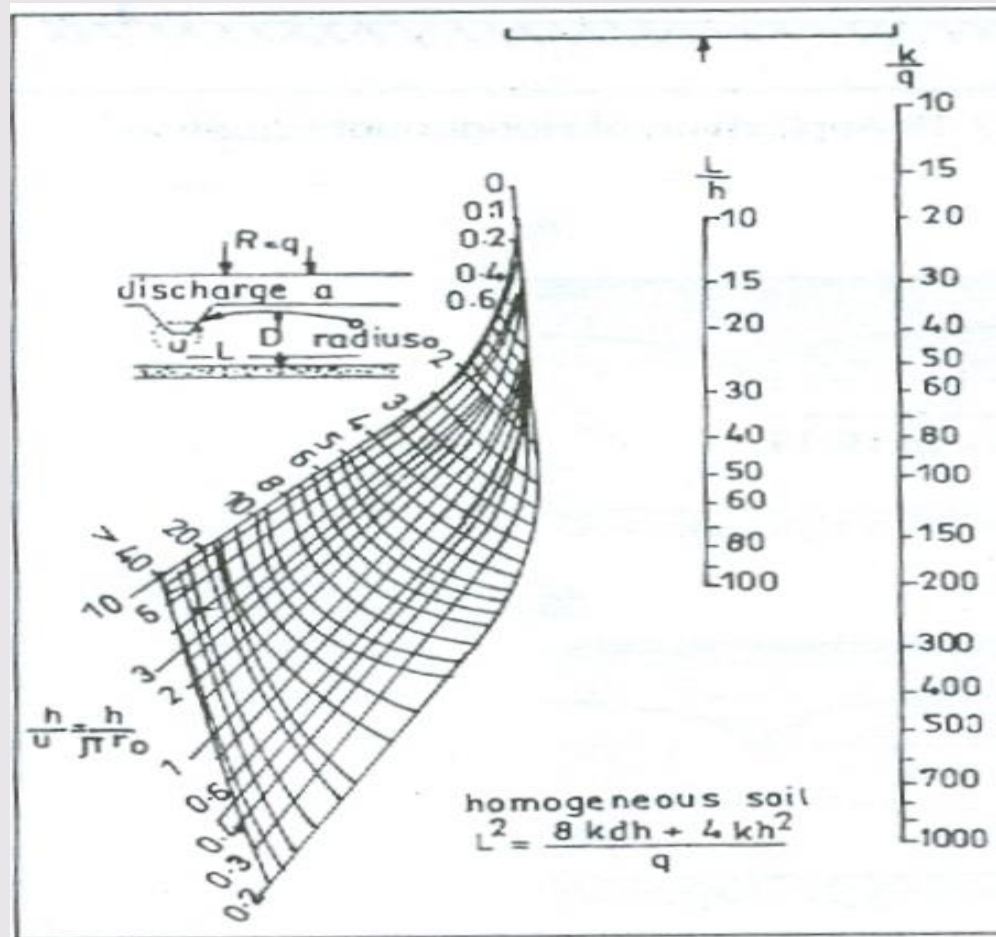
Procedure

1. Find the intersection point of D/h and $h/\pi r_0$
2. Find the value of K/q on the right hand scale and connect it with the above intersection point by a straight line at the intersection
3. Read the value of L/h in the middle scale



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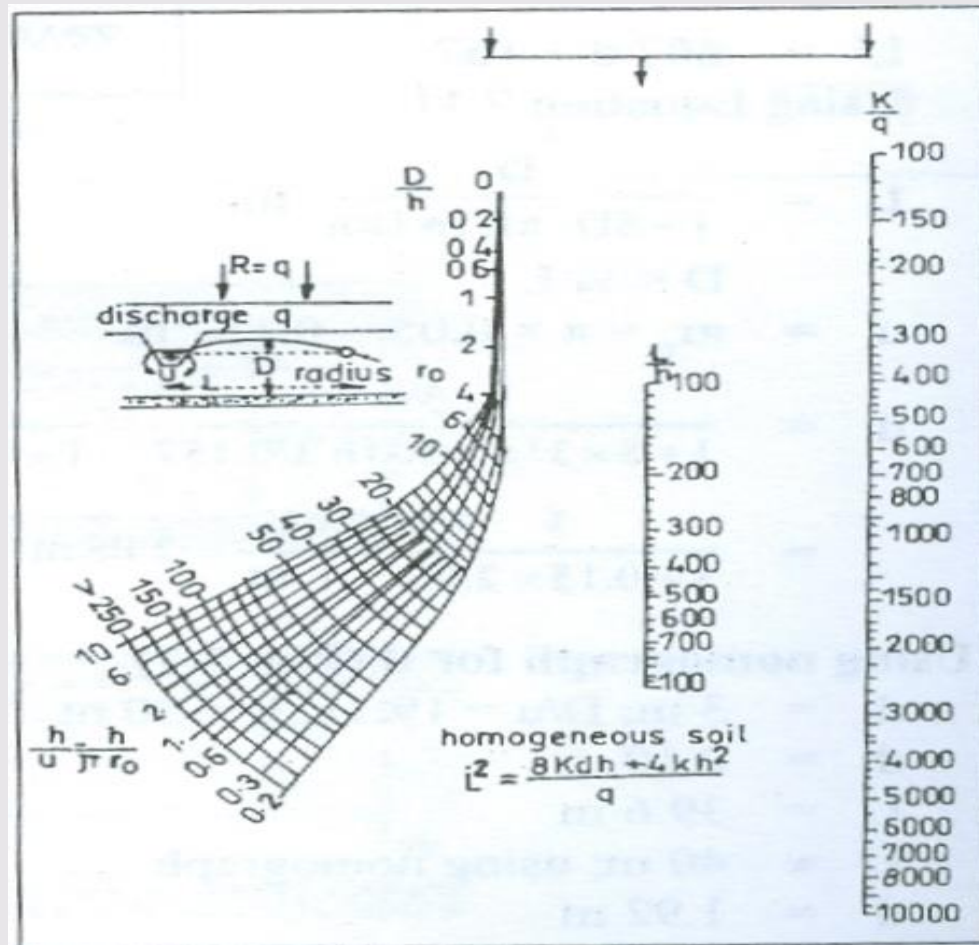
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Nomograph for drain spacing for $L/h < 100$



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Nomograph for drain spacing for $L/h > 100$



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