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# **Drainage Engineering**

## **(BTAI3103)**

### **Module 4**

## **Subsurface Flow to Drains and Drainage Equation**

### **(Lecture-12: Unsteady State flow to drains)**

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# Unsteady State

- Unsteady state approach: recharge is a function of time and flow is not constant
- Steady state assumption is not justified for the areas with periodic irrigation or high intensity rainfall because it changes with time as water is stored or released from the soil
- For the above conditions, unsteady state solutions of the flow problem should be applied
- Advantage: Evaluation of the drainage system can be based on either water table height or drain outflow rate



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- The differential equation for unsteady state flow based on Dupuit's Forchheimer assumption,

$$KD \frac{\partial^2 h}{\partial x^2} = \mu \frac{\partial h}{\partial t} - R$$

or when recharge rate R equals zero

$$KD \frac{\partial^2 h}{\partial x^2} = \mu \frac{\partial h}{\partial t}$$

Where

KD = transmissivity of the aquifer (m<sup>3</sup>/day)

R = recharge rate per unit surface area (m/day)

h = hydraulic head as a function of x and t (m)

t = time (days)

μ = drainable pore space (m/m)



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Different unsteady state approaches have been used to describe the rise or fall of water table :

### **Glover Dumm Equation:**

- It is used to describe a falling water table after a sudden rise due to instantaneous recharge
- This situation is encountered in irrigated areas, where the water table rises sharply during the application of irrigation water and then recedes

### **De Zeeuw-Hellings Equation:**

- It is used to describe the fluctuating water tables wherein a non-uniform recharge is divided into shorter time periods during which recharge to ground water is assumed to be constant
- This type of situation is encountered in humid areas with high intensity rainfall



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## **Kraijenhoff van de Leur – Maasland Equation**

- It is used to describe unsteady flow to drain with steady recharge instead of an instantaneous recharge
- Mainly used for research purposes

## **Glover Dumm Equation**

- The change in storage of soil profile with no recharge to groundwater is given by,

$$\Delta W = \mu \Delta h dx dy$$

Where;

$\Delta W$  = change in water storage per unit surface area over the time considered (m)

$\mu$  = drainable pore space (-)

$\Delta h$  = change in the level of the water table over the time considered (m)



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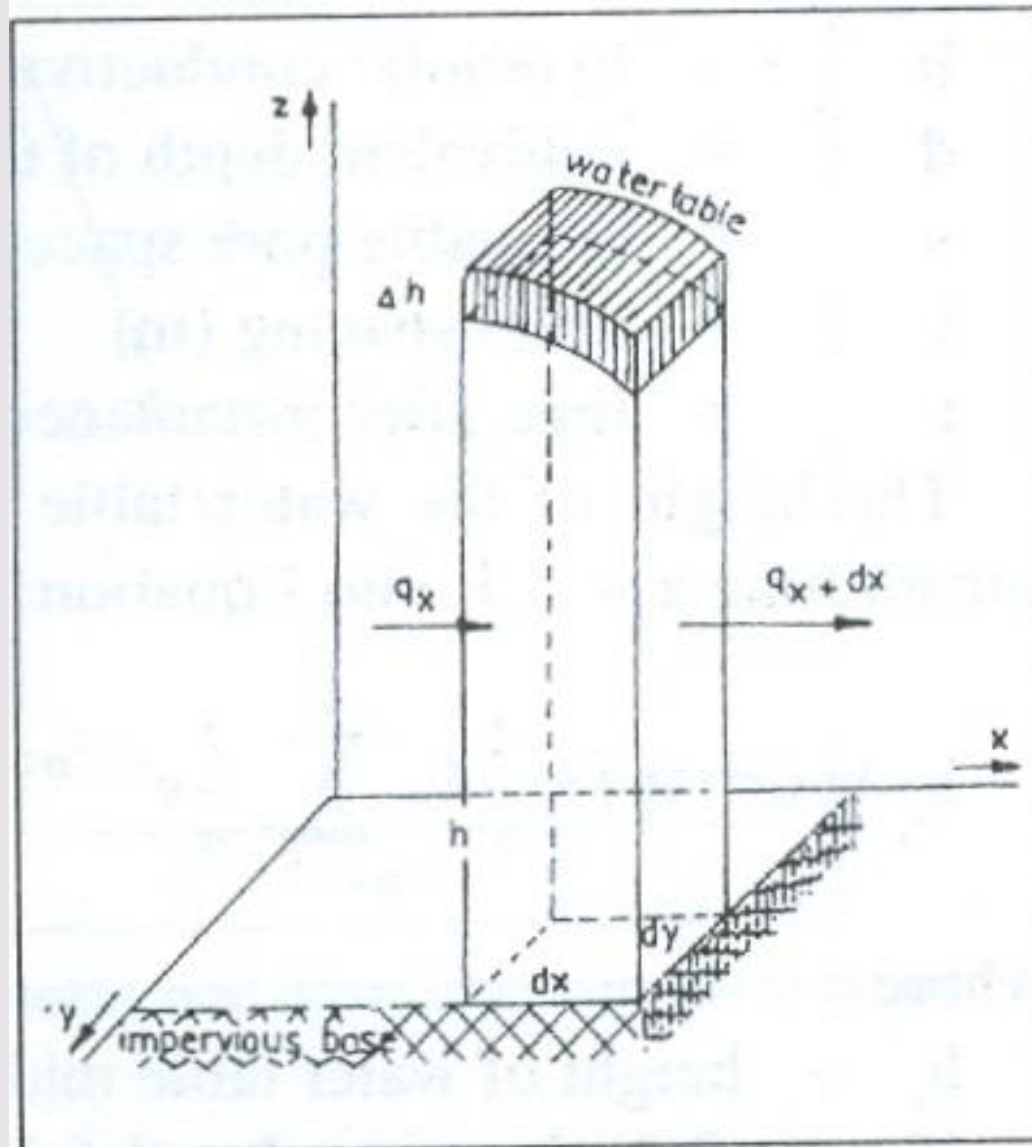


Fig. Change in storage under a falling water table



The change in storage considered over an infinitely small period of time , dt, is

$$dw = \mu \frac{\partial h}{\partial t} dx.dy \dots\dots\dots(4)$$

*Continuity principle:*

Total difference of outgoing minus incoming flow in x and y directions equals the change in storage

Hence the continuity equation can be written as

$$- K \left[ \frac{\partial}{\partial x} \left( h \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( h \frac{\partial h}{\partial y} \right) \right] dx.dy = \mu \frac{\partial h}{\partial t} dx.dy \dots\dots\dots(5)$$



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- The differential equation can be used for unsteady state condition if the flow is in one duration and  $h = D$  ( $h \gg \Delta h$ )
- Dumm (1954) used this differential equation to describe the fall of water table after it has risen to a height to above the drain level

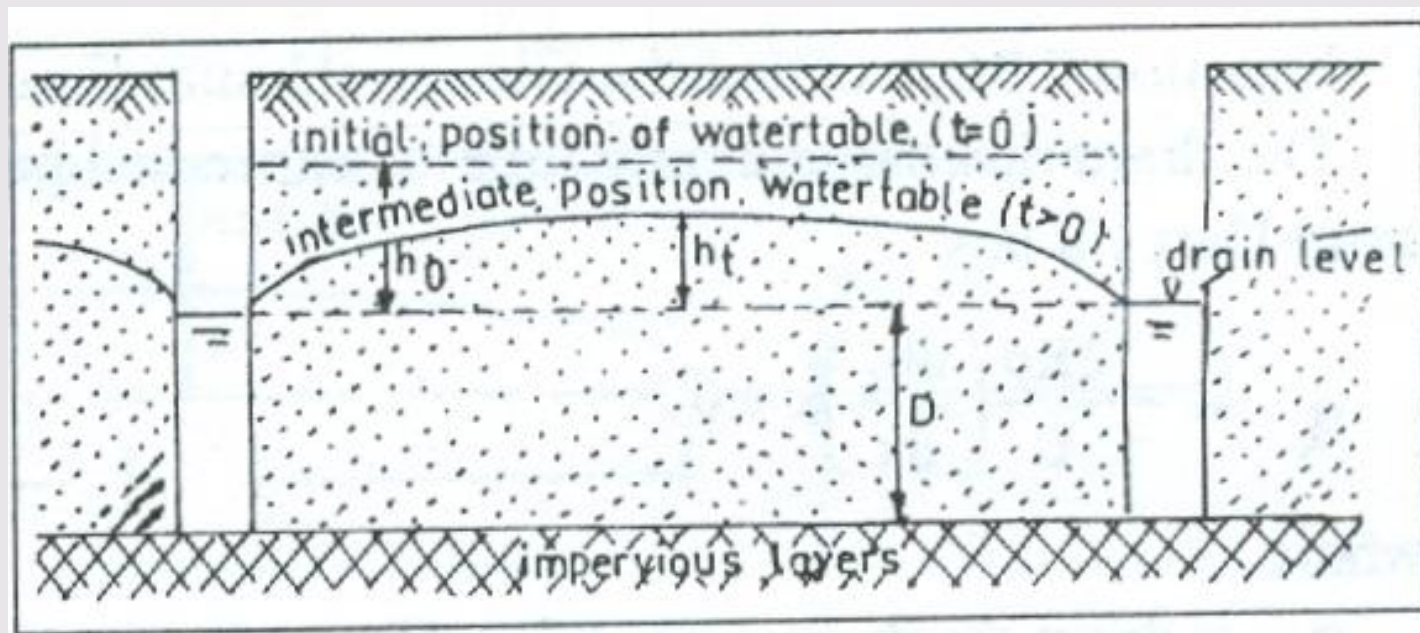


Fig. Boundary conditions for the Glover-Dumm equation with an initial water table





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- The solution is based on Glover formula describing the lowering of initially horizontal water table as a function of time, place, drain spacing and soil properties

$$h(x, t) = \frac{4h_0}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} e^{-n^2 \alpha t} \sin \frac{n \pi x}{L} \dots\dots\dots(6)$$

$$\alpha = \frac{\pi^2 K d}{\mu L^2} \dots\dots\dots(7)$$



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Where;

$h(x, t)$  = height of the water table at distance  $x$  at time  $t$  (m)

$h_0$  = initial height of the water table at  $t=0$  (m)

$\alpha$  = reaction factor ( $d^{-1}$ )

$h$  = hydraulic conductivity (m/d)

$d$  = equivalent depth of the soil layer below drain level  
(m)

$\mu$  = drainable pore space (-)

$L$  = drain spacing (m)

$t$  = time after instantaneous rise of the water table (d)



The height of the water table mid way between the drains can be found by substituting  $x = \frac{1}{4} L$  into equation 6,

$$h_t = h(x = \frac{1}{2} L) = \frac{4}{\pi} h_0 \sum_{n=1,3,5}^{\infty} \frac{1}{n} e^{-n^2 \alpha t} \dots\dots\dots(8)$$

where;

$h_t$  = height of water table mid way between the drain at  $t > 0$   
(m)

If  $\alpha t > 0.2$ , the 2<sup>nd</sup> and following terms of equation are small and can be neglected, so this equation reduces to,

$$h_t = \frac{4}{\pi} h_0 e^{-\alpha t} = 1.27 h_0 e^{-\alpha t} \dots\dots\dots(9)$$



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If instead of being horizontal, the initial water table has the shape of a fourth degree parabola, equation 9 becomes,

$$h_t = 1.16h_0 e^{-\alpha t} \dots\dots\dots(10)$$

By substituting equation 7 into equation 9, the expression for the drain spacing can be found,

$$L = \pi \left( \frac{Kdt}{\mu} \right)^{1/2} \left( \ln 1.6 \frac{h_0}{h_t} \right)^{-1/2} \dots\dots\dots(11)$$

The above equation is called the *Glover-Dumm Equation*



The drain discharge at any time 't' expressed per unit surface area, can be found from Darcy's law,

$$q_t = -\frac{2Kd}{L} \left( \frac{dh_t}{dx} \right)_{x=0} \dots\dots\dots(12)$$

where;

$q_t$  = drain discharge per unit surface area at  $t > 70$  (m/day)

Differentiating equation 6 with respect to x, neglecting all terms

$n > 1$ , substituting  $x = 0$  and combining with equation 12 gives,

$$q_t = \frac{8Kd}{L^2} h_0 e^{-\alpha t} \dots\dots\dots(13)$$



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Substituting equation 9 into equation 13 gives,

$$q_t = \frac{2\pi Kd}{L^2} h_t \dots\dots\dots(14)$$

This is similar to the Hooghoudt equation describing the flow below drain level except that the factor 8 is now replaced by  $2\pi$

According to equation 10 the water level falls with time exponentially

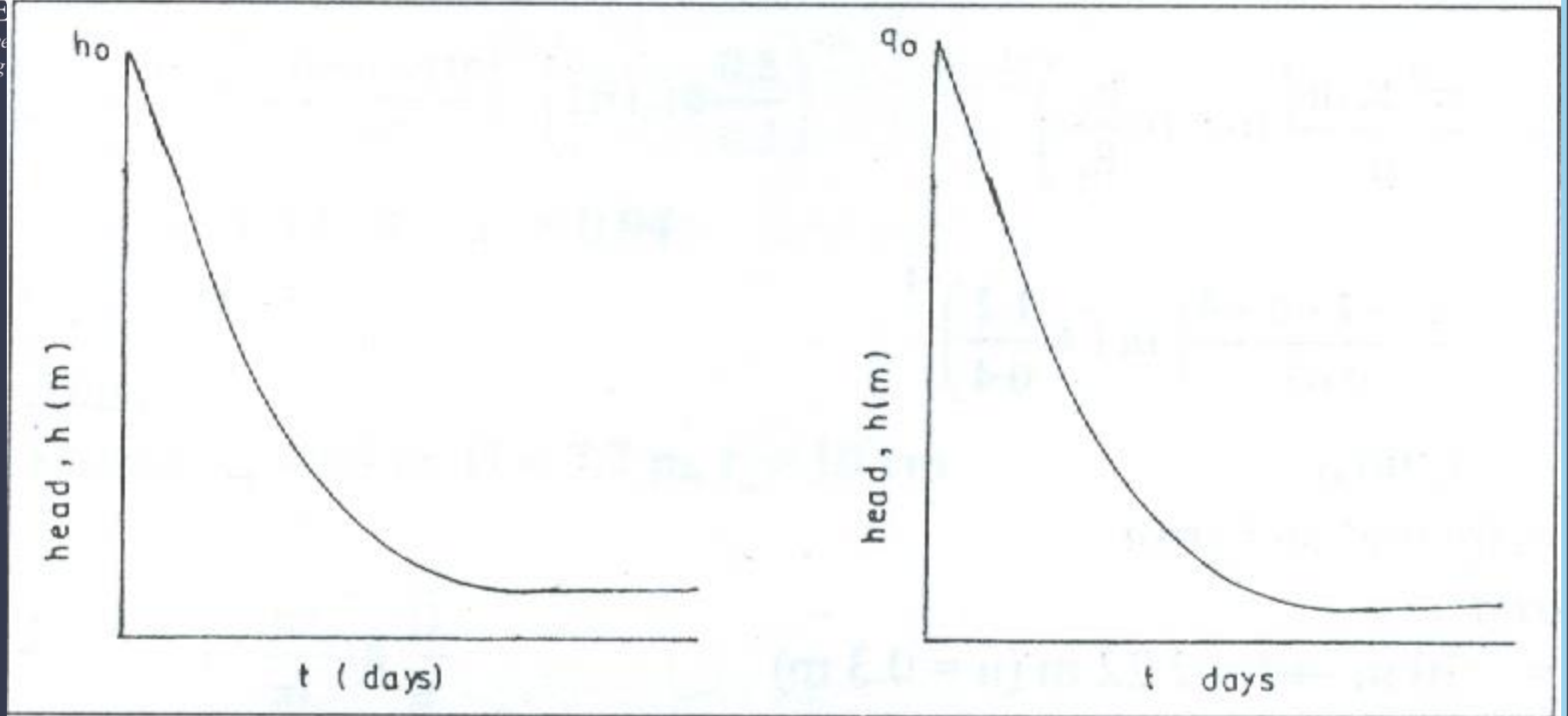


Fig. Exponential relationship between water table and drain discharge with time



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$$h_{t+1} = 1.16h_t e^{-\alpha\Delta t} \dots\dots\dots(15)$$

$$\Delta t = (t + 1) - t$$

Since q and h are linearly related

$$q_t + 1 = 1.16q_t e^{-\alpha\Delta t} \dots\dots\dots(16)$$

Equations 15 and 16 lead to

$$\frac{q_{t+1}}{q_t} = \frac{h_{t+1}}{h_t} = e^{-\alpha\Delta t} \dots\dots\dots(17)$$





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# De Zeeuw-Hellinga Equation

- To simulate the drain discharge over a period with a non-uniform distribution of recharge, if the period is divided into time intervals of equal length and the recharge in each period is assumed to be constant, the change in drain discharge is proportional to the excess recharge ( $R - q$ )

$$\frac{dq}{dt} = \alpha(R - q) \dots\dots\dots(18)$$

- where;
- $\alpha$  = reaction factor



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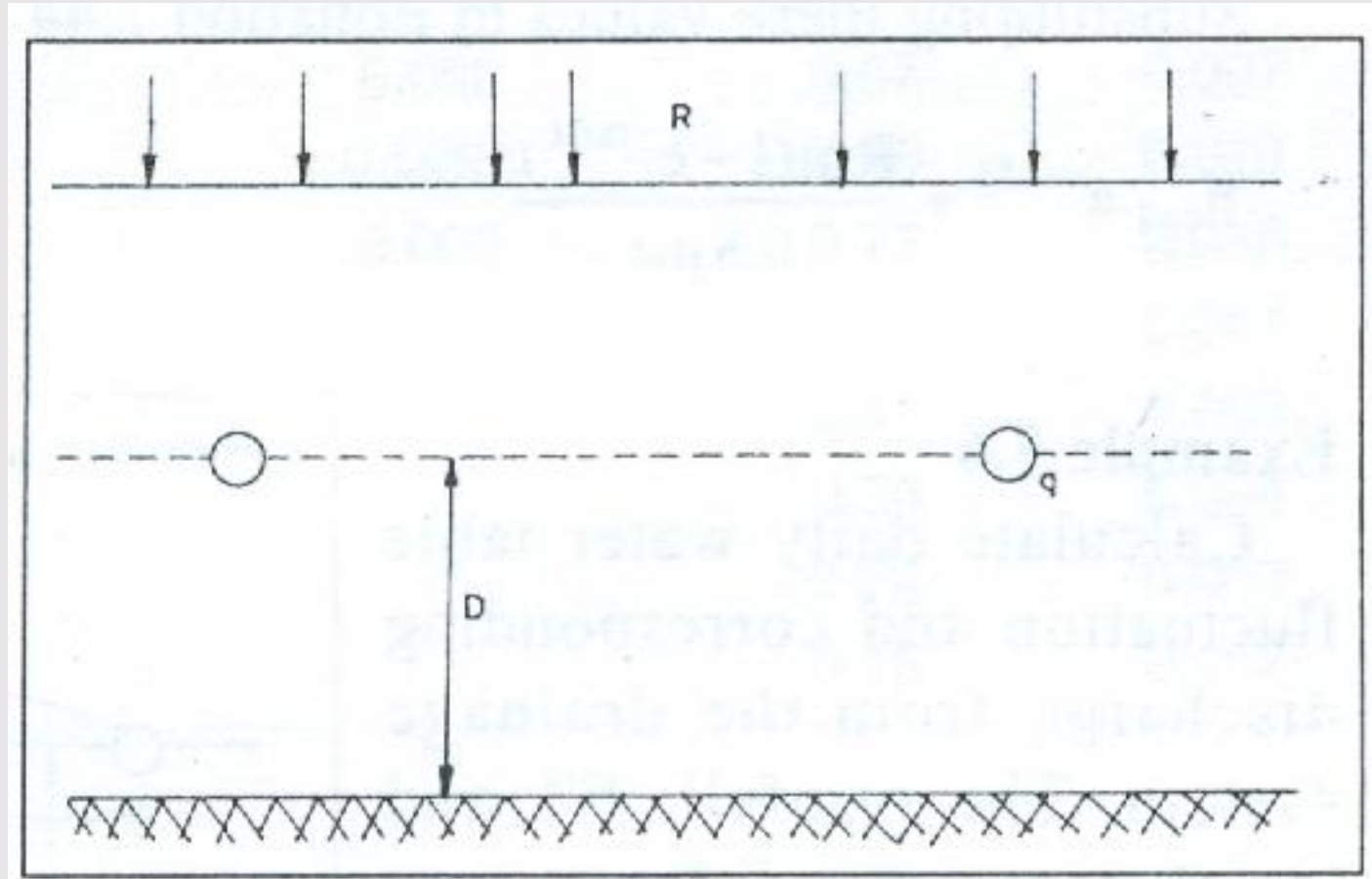


Fig. Drain discharge with non-uniform recharge



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Integration of equation 18 between the limits  $t = t$ ,  $q = q_t$ ,  
 $t = t - 1$ ,  $q = q_{t-1}$  results in,

$$q_t = q_{t-1} e^{-\alpha \Delta t} + R \Delta t (1 - e^{-\alpha \Delta t}) \dots\dots\dots(19)$$

where;

$$\Delta t = t - (t - 1),$$

It is the time interval over which the recharge is assumed to be constant



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The depth of water table can be simulated by simplified Hooghoudt Equation neglecting the flow above drain level

$$q = \frac{8Kd}{L^2} h = \frac{8\mu\alpha}{\pi^2} h \approx 0.8\mu\alpha h \quad \dots\dots\dots(20)$$

$$(i) \quad \frac{dq}{dt} = \frac{8Kd}{L^2} \frac{dh}{dt}$$

water table rises when  $(R-q) > 0$  and,

water table falls when  $(R-q) < 0$ , assuming evapotranspiration to be zero



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$$\frac{dh}{dt} = \frac{dq}{dt} \frac{L^2}{8Kd}$$

$$\alpha = \frac{\pi^2 Kd}{\mu L^2}$$

$$= \frac{\alpha(R - q)\pi^2}{\alpha 8\mu}$$

$$= \frac{(R - q)}{0.8\mu}$$

.....(21)

$$\frac{dq}{dt} = \frac{8Kd}{L^2} \frac{(R - q)}{0.8\mu} \approx \frac{10Kd(R - q)}{\mu L^2}$$



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$$q_t = 0.8\mu\alpha h_t \dots\dots\dots(22 \text{ a})$$

$$q_{t-1} = 0.8\mu\alpha h_t \dots\dots\dots(22 \text{ b})$$

Substituting these values in equation 19,

$$h_t = h_{t-1}e^{-\alpha\Delta t} + \frac{R\Delta t(1 - e^{-\alpha\Delta t})}{0.8\mu\alpha} \dots\dots\dots(23)$$



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## Interpretation of Reaction Factor

$$\alpha = \frac{\pi^2 Kd}{\mu L^2} \approx \frac{10Kd}{\mu L^2}$$

- (i)  $\left. \begin{array}{l} \text{low } k \text{ d} \\ \text{high } \mu \\ \text{large } L \end{array} \right\} \rightarrow \alpha \text{ small} \rightarrow \text{h and q respond slowly to change in recharge}$
- (ii)  $\left. \begin{array}{l} \text{high } k \text{ d} \\ \text{low } \mu \\ \text{small } L \end{array} \right\} \rightarrow \alpha \text{ large} \rightarrow \text{h and q respond rapidly to change in recharge}$



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## Determination of $\alpha$ :

Using equation 23,

$$h_t = h_{t-1}e^{-\alpha\Delta t} + \frac{R\Delta t(1 - e^{-\alpha\Delta t})}{0.8\mu\alpha}$$

where;  $R\Delta t = 0$

$$h_t = h_{t-1}e^{-\alpha\Delta t}$$

$$\alpha = \frac{\ln(h_{t-1}) - \ln(h_t)}{\Delta t}$$

$$q_t = q_{t-1}e^{-\alpha\Delta t}$$

$$\alpha = \frac{\ln(q_{t-1}) - \ln(q_t)}{\Delta t}$$

Fig. 5





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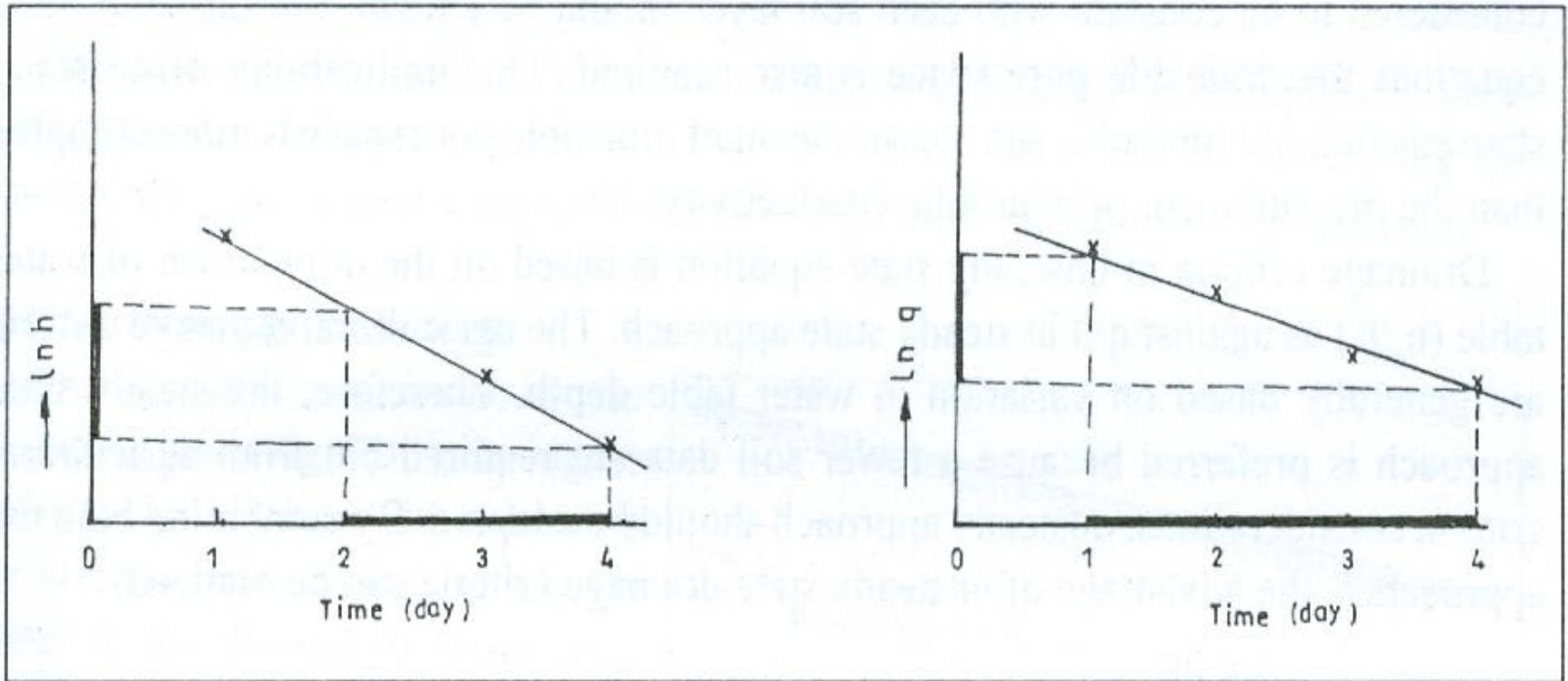


Fig. Relationship of  $h$  and  $q$  (log scale) with time



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# Application of Steady State And Unsteady State Equations

Input requirements for steady and unsteady equations,

	Steady state equation	Unsteady state equation
Soil data		
- Profile description	Yes	Yes
- Hydraulic conductivity	Yes	Yes
- Drainable pore space	Yes	Yes
Agricultural criteria:		
- $q/h$ ratio	Yes	No
- $h_o/h_i$ ratio	No	Yes
Technical criteria:		
- Drain depth, pipe size etc.	Yes	Yes



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- The agricultural design criteria are the required depth of water table ( $h$ ) and the corresponding design discharge ( $q$ ) depending upon type of crop and climate
- The ratio  $q/h$  is called the drainage criterion or drainage intensity
- Higher the  $q/h$  ratio, more is the possibility to prevent high water table
- In unsteady state equation instead of  $q/h$  ratio, water table drawdown  $h_0/h_t$  is required



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- The technical criteria include the drain depth depending upon available machinery, selected construction method, soil conditions and the drain specification ( $r_0$  and  $h$ )
- Unsteady state equations can be applied if a homogenous soil is assumed
- Steady state equations have to be applied for layered soil profile
- For both steady and unsteady equations, hydraulic conductivity considered to be constant with each soil layer should be known



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$$q = \frac{8Kdh}{L^2} \quad (\text{Steady state approach assuming flow drain level})$$

$$\alpha = \frac{\pi^2 Kd}{\mu L^2} \quad (\text{Design criteria in unsteady state in terms of reaction factor})$$

Combining the above two equations,

$$\frac{h}{q} = \frac{\pi^2}{8\mu\alpha}$$

- Using this equation, the unsteady criteria (required drawdown of the water table in certain period of time) can be established in experiment on pilot scale
- The above unsteady design criteria can be converted into steady state criteria which can be applied on a project area



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# Drainage of Heavy Clay Soils

- Low hydraulic conductivity – subsurface drainage with narrowest drain spacing is not justifiable and economically viable
- In such cases, surface drainage system of furrows and small ditches possibly combined with bedding of soil needs to be adopted
- For moderate hydraulic conductivity, subsurface drainage system can be installed in a soil if the infiltration rate allows the rainfall expected in 2 or 3 subsequent days to infiltrate during that time



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- Heavy clay soils of low hydraulic conductivity often have a top layer with high hydraulic conductivity due to activity of plant roots or the presence of a filled layer
- In such cases a perched water table is formed just below the top layer (Fig. 6)
- Under such conditions, the subsurface drainage system can be effective due to the interflow in the permeable top layer
- It will work as long as back filled trend remains more permeable than the original soil



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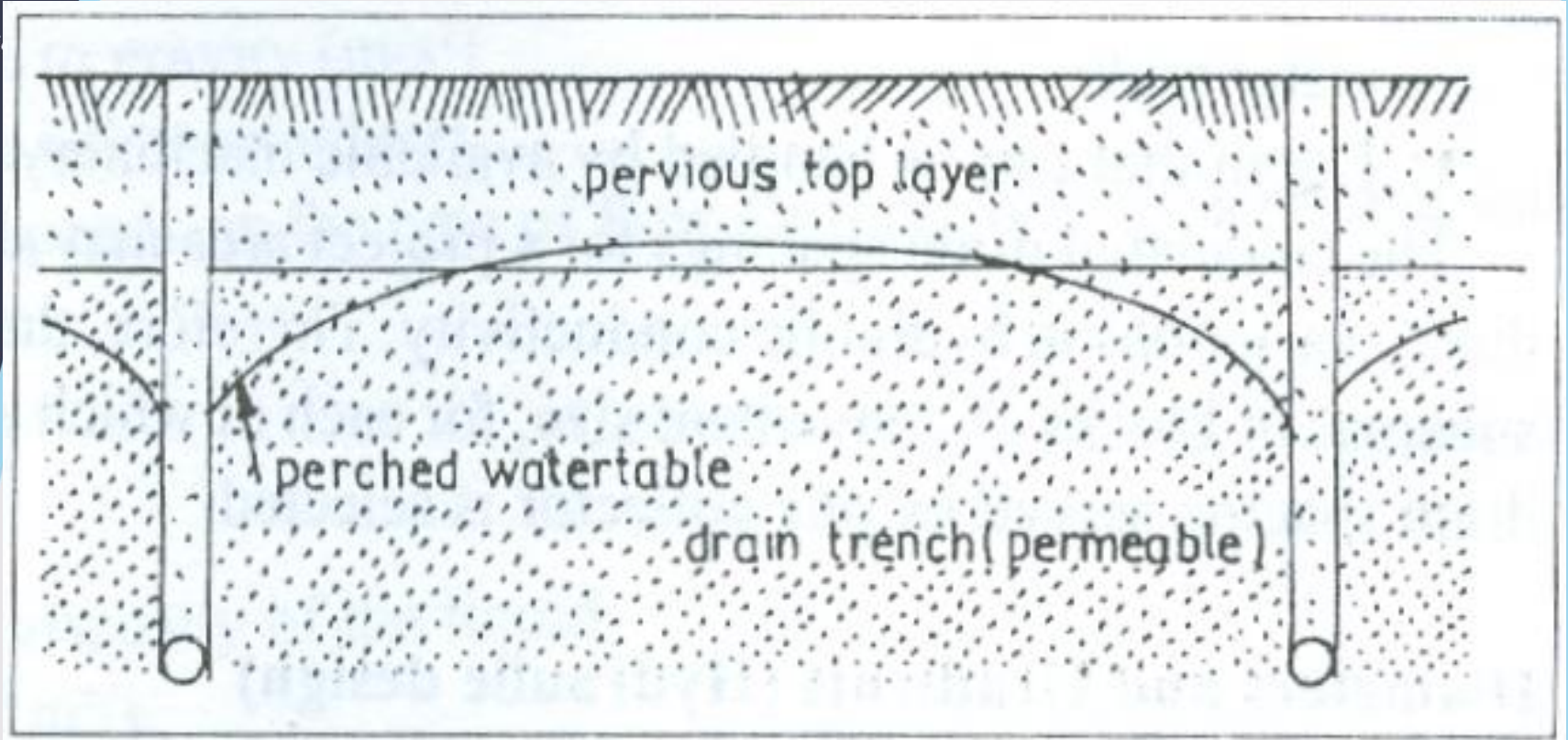


Fig. Perched water table built up in heavy clay soil below the top layer having high hydraulic conductivity





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- Heavy clay soils in basin clay areas often have relatively high permeable layers in the subsoil
- The soils can be drained well by placing subsurface drainage system in such a layer

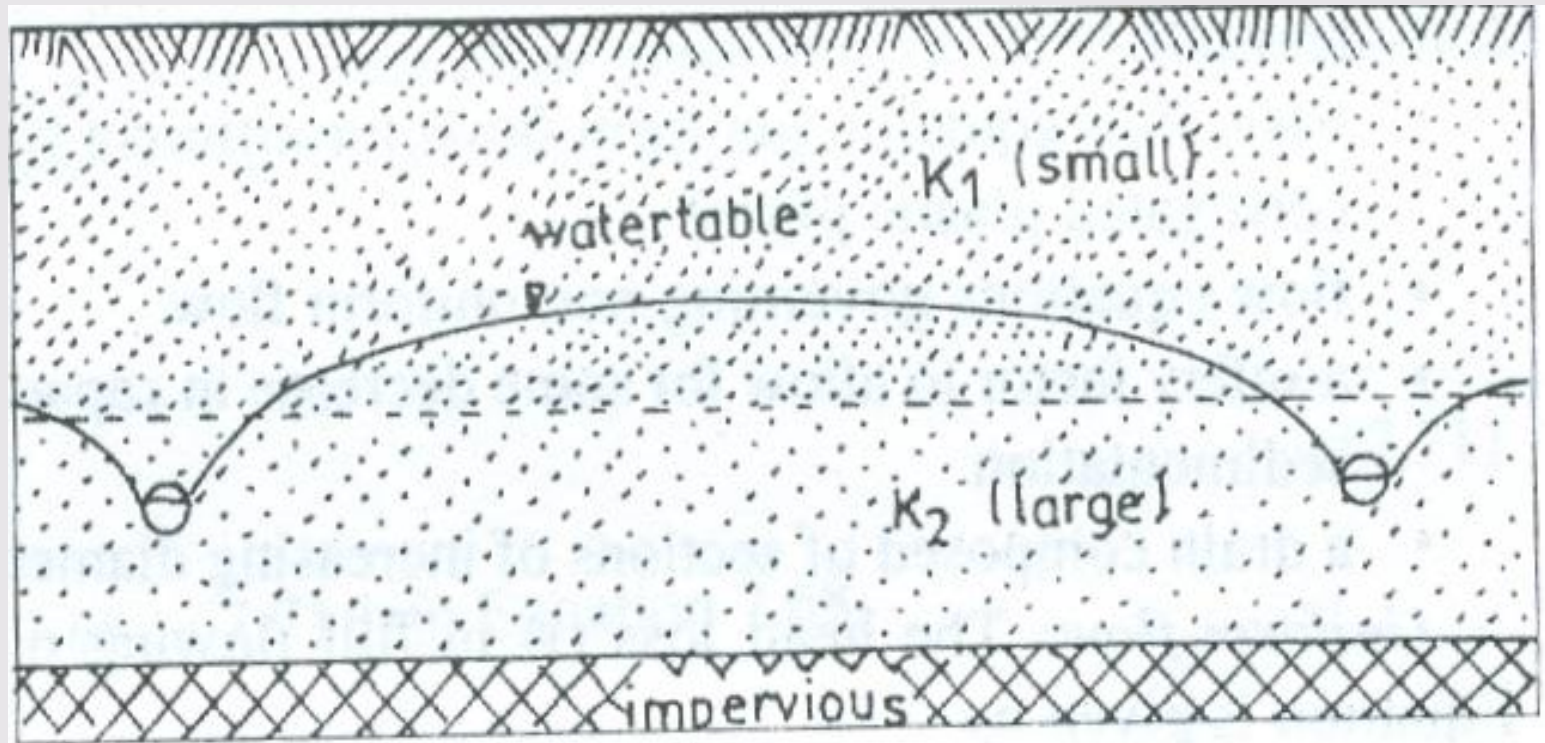


Fig. Drains in layer with high hydraulic conductivity under less permeable heavy clay top soil



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