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# Drainage Engineering (BTAI3103)

## Module 4

### Subsurface Flow to Drains and Drainage Equation

### (Lecture-09: Steady-State Flow to Drains)

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# Sub-surface Drainage

- Controlling the depth of water table and the level of salinity in the root zone
  - pipe drains (present practice)
  - Open ditches (Earlier practice)
- In a system of drains which acts as ‘sinks’ following drains are used;
  - Field drains or field laterals – to control groundwater depth
  - Collector drains – to collect water from the field drains and to transport it to the main drains
  - Main drains – to transport the water out of the area



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# Sub-surface drainage...contd.

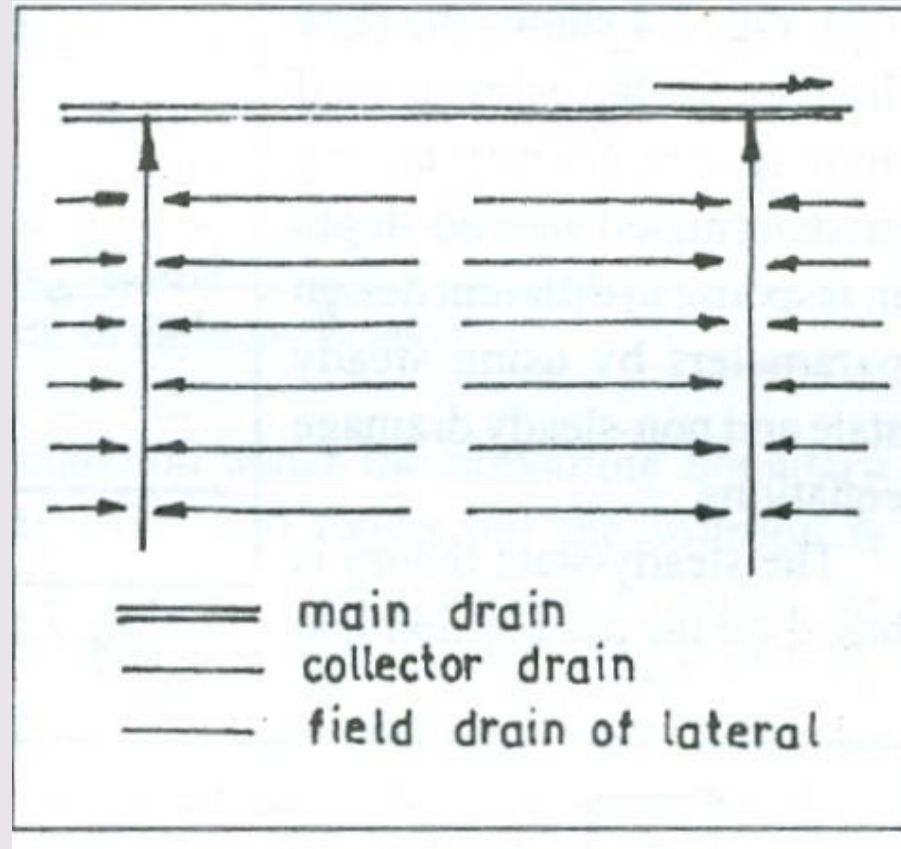


Fig. System of drains



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## Sub-surface drainage...contd.

- Water table is usually curved, its elevation being the highest mid way between the drains
- Symmetrical system – only one half is considered

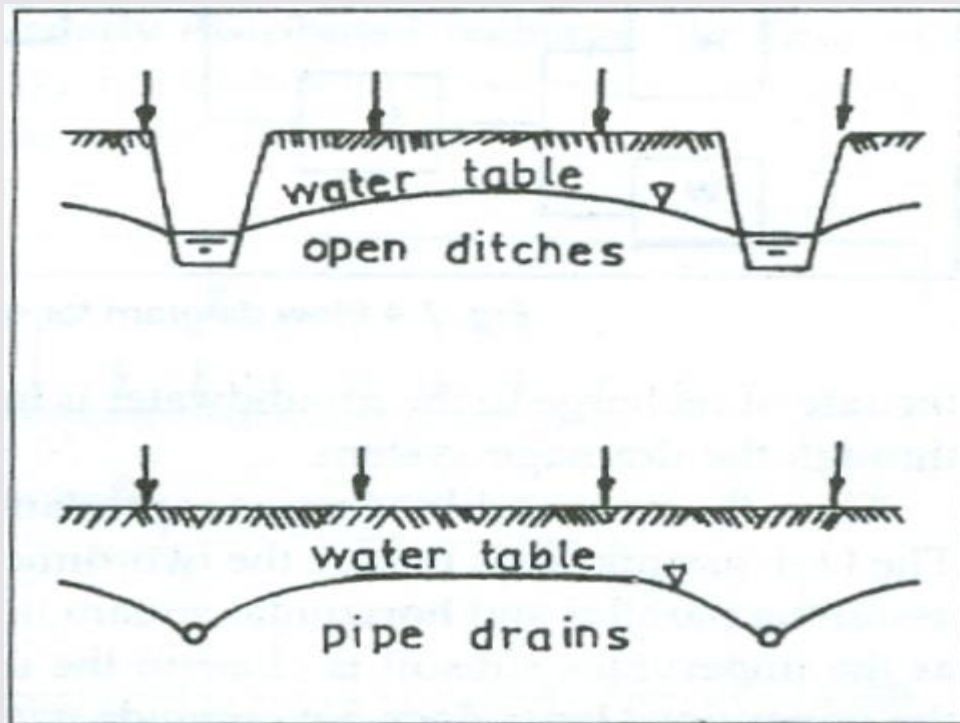


Fig. Laterals showing a curved water table and symmetry of the system



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## **Sub-surface drainage...contd.**

- Factors influencing the height of water table:
  - Precipitation and other source of recharge
  - Evaporation and other source of discharge
  - Depth and spacing of drains
  - Soil properties
  - Cross sectional area of the drains
  - Water level in the drains
  
- Above factors are inter related by drainage equations based on Dupuit – Forchheimer (D-F) theory



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# Dupuit – Forchheimer (D-F) theory

## Assumptions:

- Two dimensional flow, i.e., the flow is identical in any cross-section perpendicular to the drains
- A uniform distribution of the recharge, steady or non-steady over the area between the drains
- Homogenous and isotropic soil ignoring variation in the hydraulic conductivity within a soil layer



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# Ground Water Drainage Design

## Steady State

- Purpose of drainage: to maintain water table at a given depth ( $H$ ) under given design recharge,  $q$  (depth  $W$  preselected)
- Steady state theory: rate of recharge to the groundwater is uniform and steady and equals the discharge through the drainage system
- The D-F assumption reduce the 2d flow to a 1d flow by assuming parallel and horizontal stream lines
- The D-F assumptions are violated if the impervious layer does not coincide with the bottom of the drain as the flow in the vicinity of the drain will no longer be horizontal



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# Components of drainage design

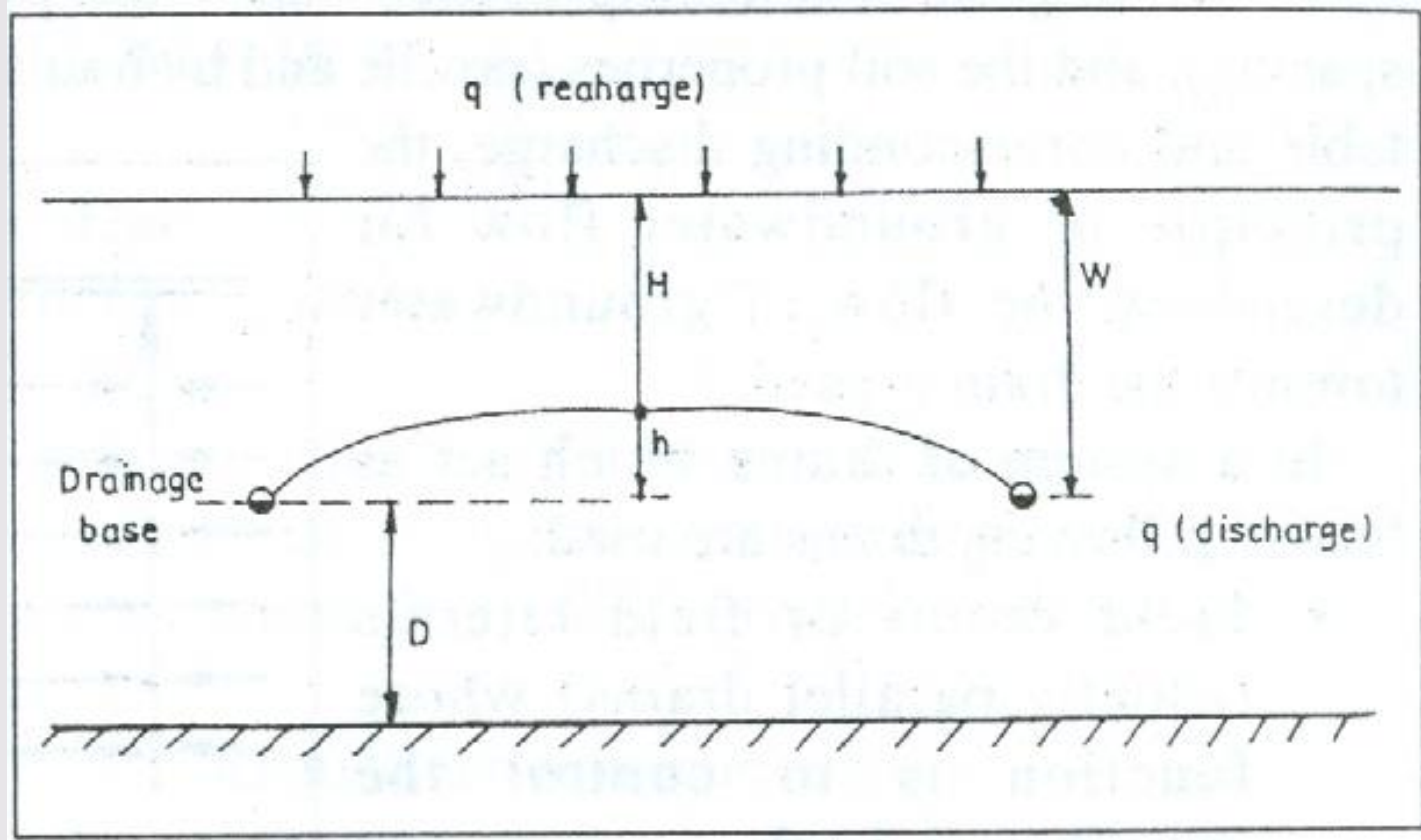


Fig. Different components of drainage design





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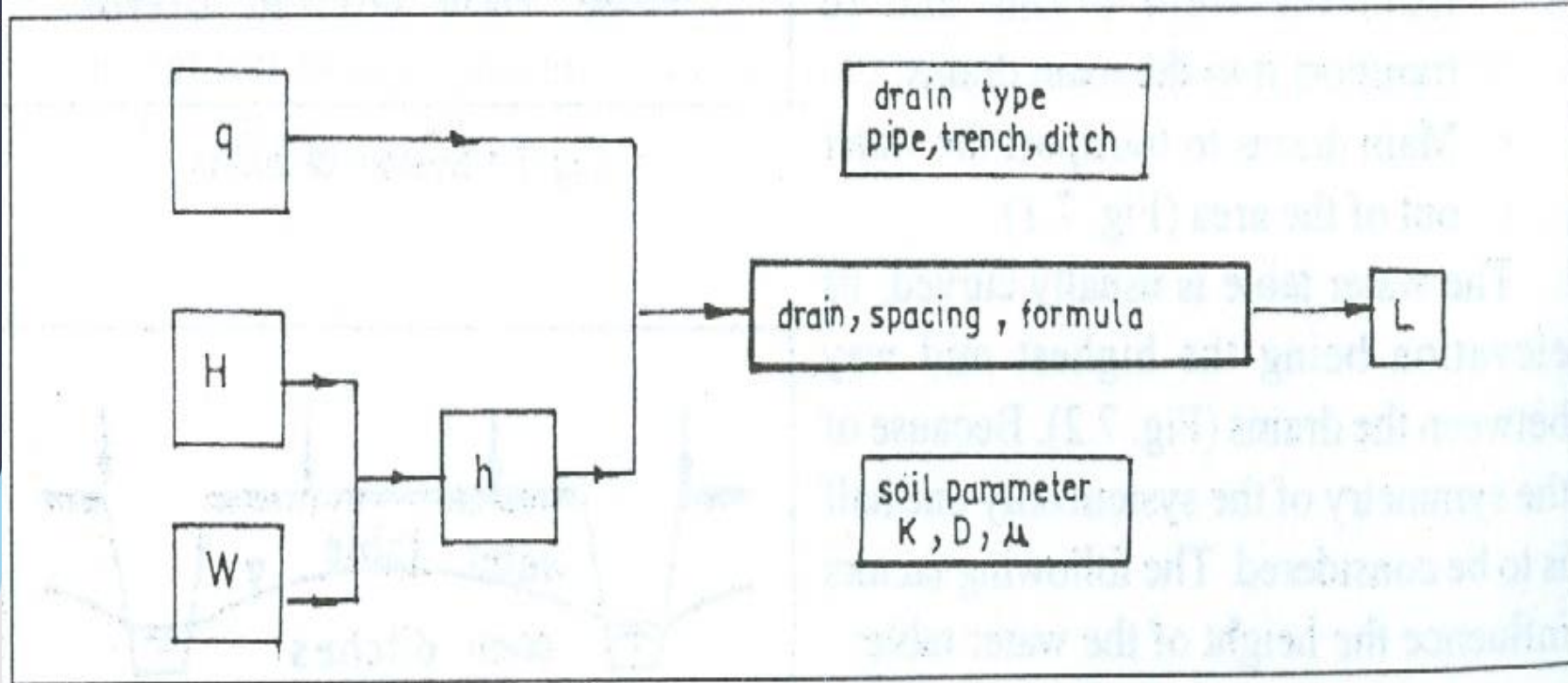


Fig. Flow diagram for estimation of drain spacing



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# Estimation of Drain Spacing

- The analytical solution of the Laplace equation under the prevailing boundary conditions for estimation of drain spacing exists
  - *But the solution is complicated and thus not easily available*
- The partial, semi-analytical solutions for schematized flow patterns give reasonable satisfactory solutions
  - Kirkham (1958), Toksoz and Kirkham (1961), Toksoz and Kirkham (1971) presented analytical solutions and prepared nomographs.
  - Dagan (1964) considered radial flow close to the drain and horizontal flow away from it
  - Ernst presented a solution for layered soil
  - The **Hooghoudt** equation gave the best results



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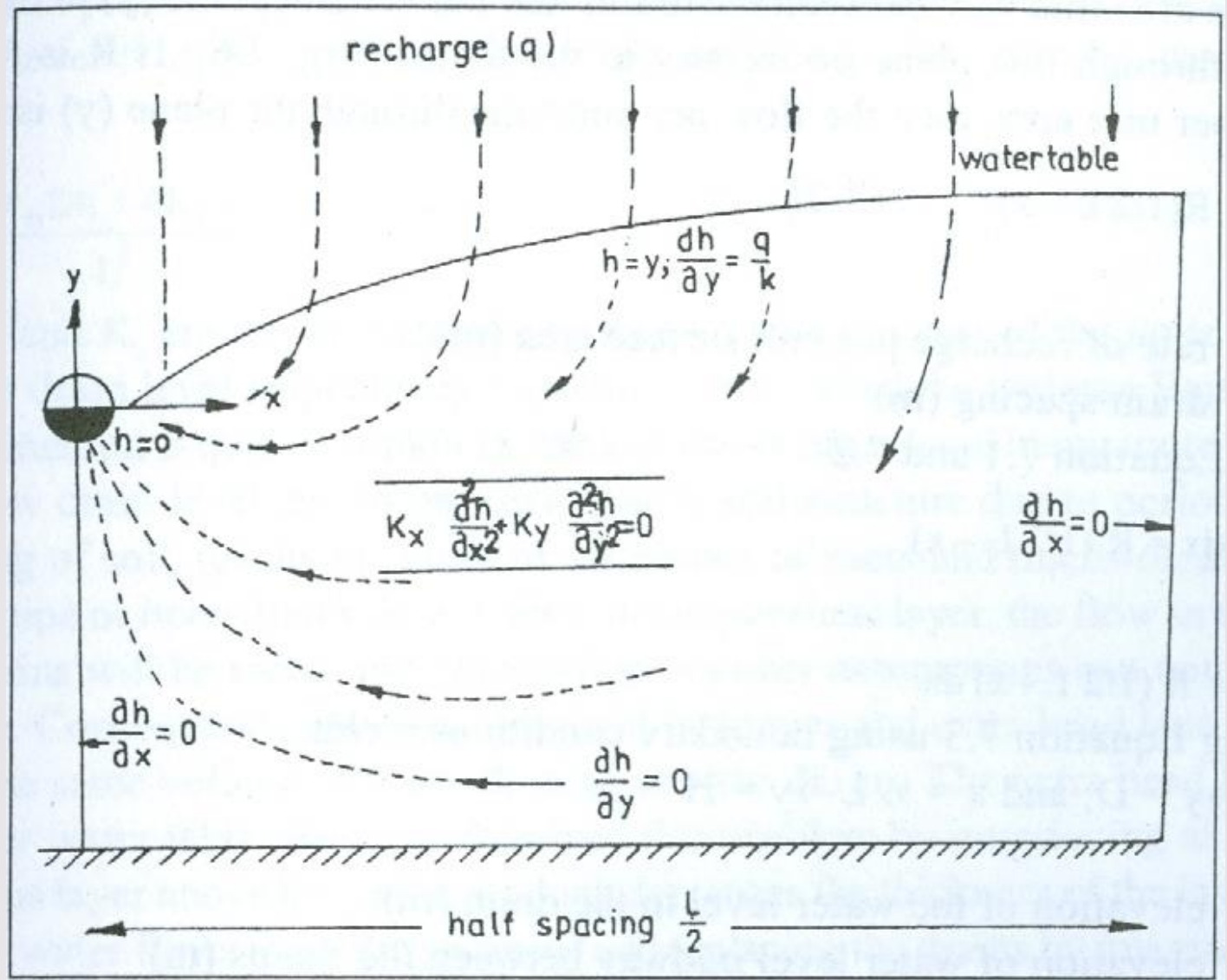


Fig. Boundary conditions for Laplace equation



# Hooghoudt Equation

- Consider, a steady-state flow to vertically walled open drains reaching an impervious layer and linearly distributed recharge
- The flow of groundwater ( $q_x$ ) through a vertical plane ( $y$ ) at a distance ( $x$ ) from the ditch using Darcy's equation and D-F assumptions,

$$q_x = K \cdot y \frac{dy}{dx} \dots\dots\dots(1)$$

where;

$q_x$  = unit flow rate in x direction, ( $m^2/d$ )

$K$  = hydraulic conductivity of the soil, ( $m/d$ )

$Y$  = height of the water table at  $x$ , ( $m$ )

$dy/dx$  = hydraulic gradient at  $x$  (-)



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## Hooghoudt equation...contd.

➤ Using principle of continuity, all the water entering the soil in the surface area mid way between the drains and the vertical plane (y) at distance (x) will pass through this plane on its way to the drains

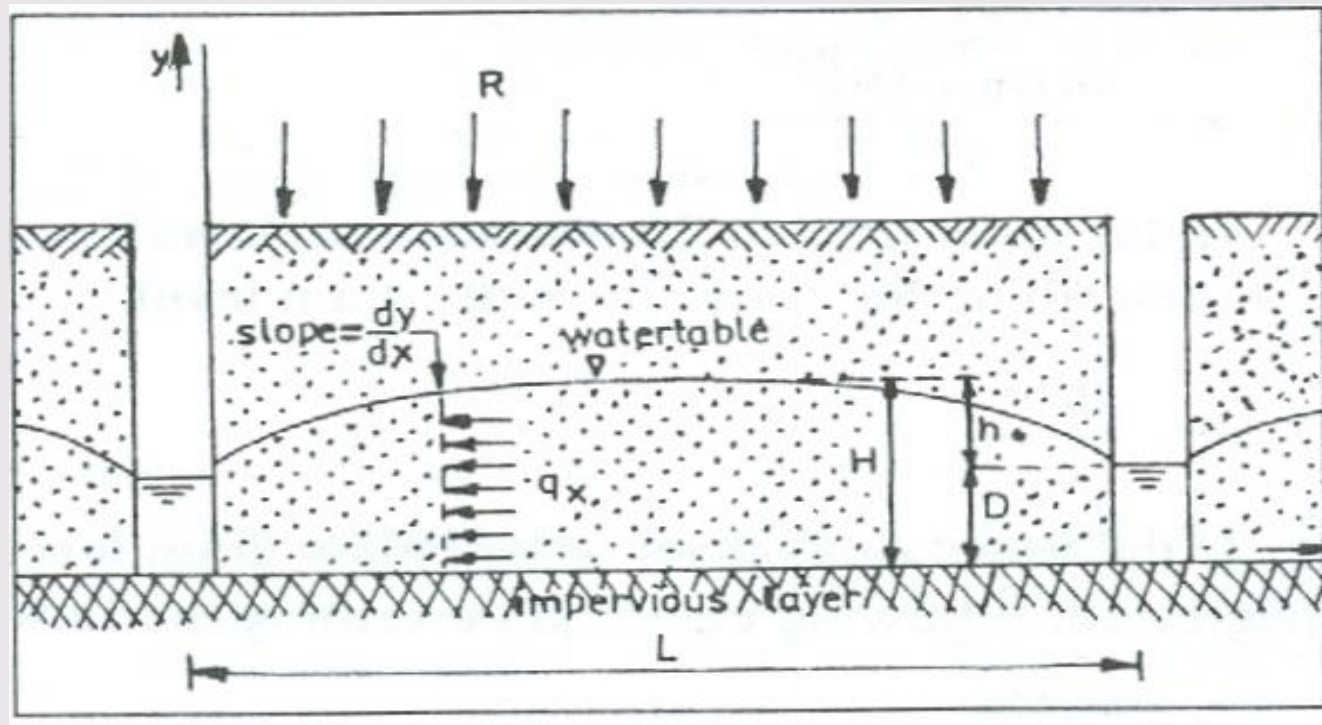


Fig. Diagram for Hooghoudt drain spacing formula



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*Flow per unit time through the plane (y),*

$$q_x = R \left( \frac{1}{2} L - x \right) \dots\dots\dots(2)$$

where

R = rate of recharge per unit surface area (m/d)

L = drain spacing (m)

Equating equation 1 & 2

$$K \cdot y \frac{dy}{dx} = R \left( \frac{1}{2} L - x \right)$$

$$K \cdot y \cdot dy = R \left( \frac{1}{2} L - x \right) dx \dots\dots\dots(3)$$



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*Integrating equation 3 using boundary conditions yields*

$$x=0 \longrightarrow y=D; \text{ and } x=1/2 L \longrightarrow y=H$$

where

D = elevation of the water level in the drain (m)

H = elevation of water level midway between the drains (m)

$$L^2 = \frac{4K(H^2 - D^2)}{R}$$

For steady state condition,  $q=R$

$$q = R = \frac{4K(H^2 - D^2)}{L^2} \dots\dots\dots(4)$$

where

Q = drain discharge (m/d)



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*Equation 4 derived by Hooghoudt in 1936 is known as Donnan Equation (Donnan 1946)*

$$q = R = \frac{4K(H + D)(H - D)}{L^2}$$

H-D= h and H+D=2D+h, where h is the height of water table above the water level in the drain.

*Then equation 4,*

$$q = \frac{8KDh + 4Kh^2}{L^2} \dots\dots\dots(5)$$





*If the water level in the drain is very low ( $D=0$ );*

$$q = \frac{4Kh^2}{L^2} \dots\dots\dots(6)$$

*If the impervious layer is far below drain level ( $D \gg h$ ),*

$$q = \frac{8K Dh}{L^2} \dots\dots\dots(7)$$

If the soil profile consists of two layers with different K values and the drain level is at the interface between the soil layers, equation 5 can be,

$$q = \frac{8K_b Dh + 4K_a h^2}{L^2} \dots\dots\dots(8)$$

Where;  $K_a$  and  $K_b$  are respectively the hydraulic conductivities of the layer above and below the drain level



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- Soil above drain level being more permeable than below drain level due to improvement in soil structure due to periodic wetting and drying of soil results in cracks and presence of roots and micro-organisms
- If the pipe or open drains do not reach the impervious layer , the flow in the vicinity of the drains will be radial and Dupuit-Forchheimer assumptions can not be applied.

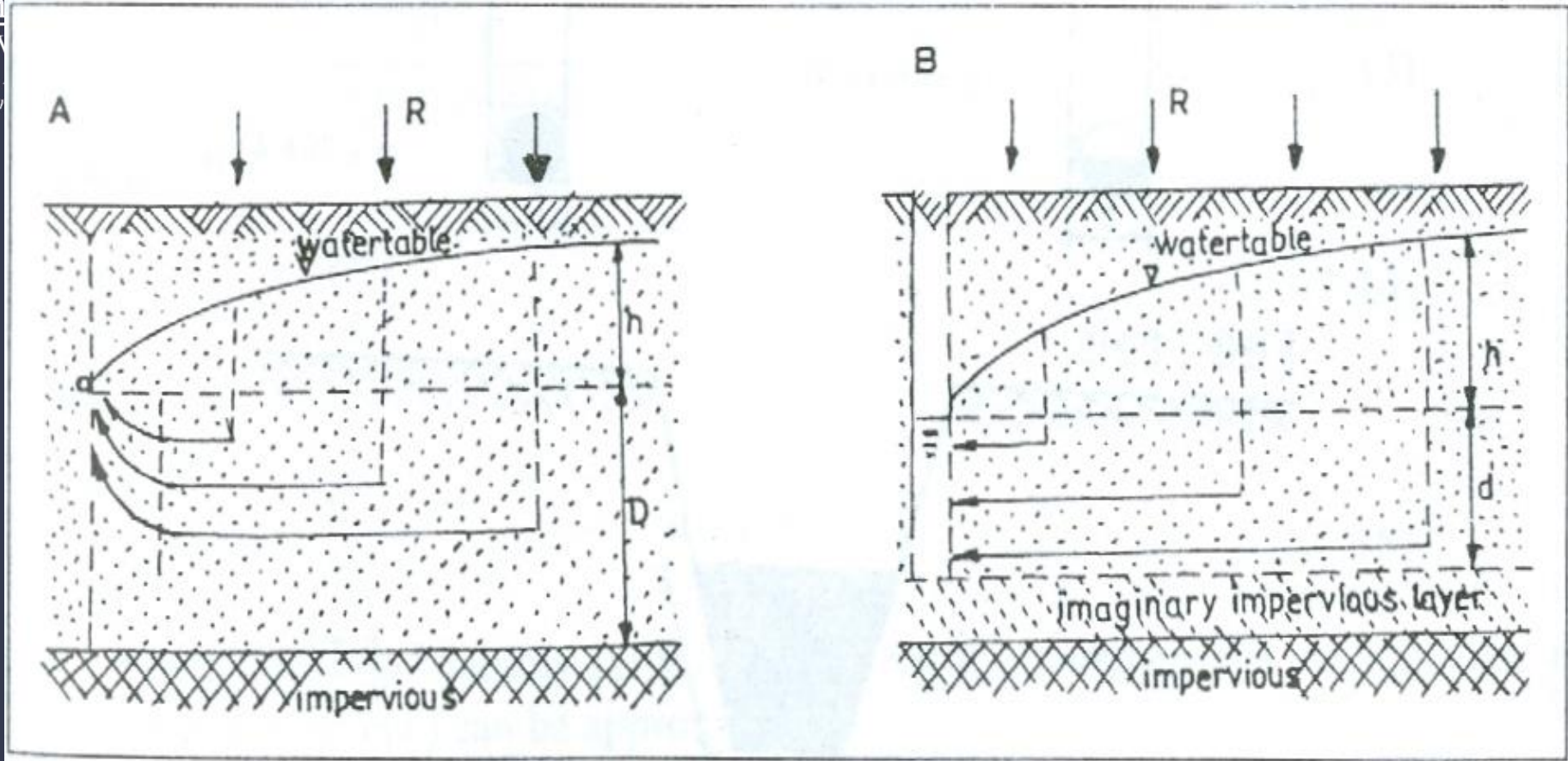


Fig. Equivalent depth to convert horizontal and radial flow (A) into an equivalent horizontal flow (B)



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- Consequently, the flow lines will be longer and extra head loss is required to have the same volume of water flowing into the drains
- The extra head loss results in higher water table
- Hooghoudt solved this problem by introducing an imaginary impervious layer above the real one which decreases the thickness of the layer through which the water flows towards the drains and replacing the drains by imaginary ditches with their bottom on the imaginary impervious layers



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- The equation 5 can be used to describe the flow towards the drains simply by replacing the actual depth to the impervious layer ( $D$ ) with a small equivalent depth ( $d$ ) representing the imaginary thinner soil layer through which the same amount of water will flow per unit time as in the actual situation

*Then we get,*

$$q = \frac{8Kdh + 4Kh^2}{L^2} \dots\dots\dots(9)$$



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- Hooghoudt made following assumptions while deriving the equation
  1. The drains are running half full
  2. The drains have no entrance resistance ( $u = \pi r_0$ )
- In case of no entrance resistance, drain is considered an ideal one. This holds good if the hydraulic conductivity of the drain trench is at least 10 times higher than that of the undisturbed soil outside the trench
- If the hydraulic conductivity is less, an envelope material is used to decrease the entrance resistance, so that a greater part of the total head is available for the flow through the soil



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