

VISCOELASTIC MODELS

ELECTRICAL EQUIVALENCE

Viscoelasticity

The word 'viscoelastic' means that the material **simultaneously exhibits** some of the **elastic properties of an ideal solid** and some of the **flow properties of an ideal liquid**. A combined solid-like and liquid-like behavior in which the stress-strain relationship is time dependent.

Examples of viscoelastic foods

Almost all solid foods and fluid foods containing long chain biopolymers

- ✓ Food starch, gums, gels
- ✓ Grains
- ✓ Most solid foods (fruits, vegetables, tubers)
- ✓ Cheese
- ✓ Pasta, cookies, breakfast cereals

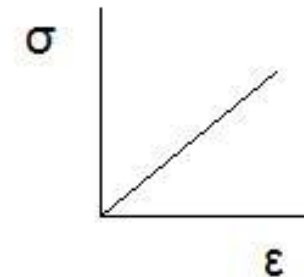
Stress-Strain Curves for a purely elastic material and a viscoelastic material. The red area is a **hysteresis** loop and shows the amount of energy lost (as heat) in a loading and unloading cycle.

Elastic Hysterisis

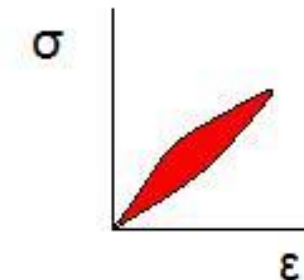
(Behavior of rubber under stress).

- ✓ stress is not proportional to strain.
- ✓ strain lags behind the stress.

On decreasing the load, the stress-strain curve is not retraced but follows a different path. **This lag of strain behind stress is called elastic hysteresis.**

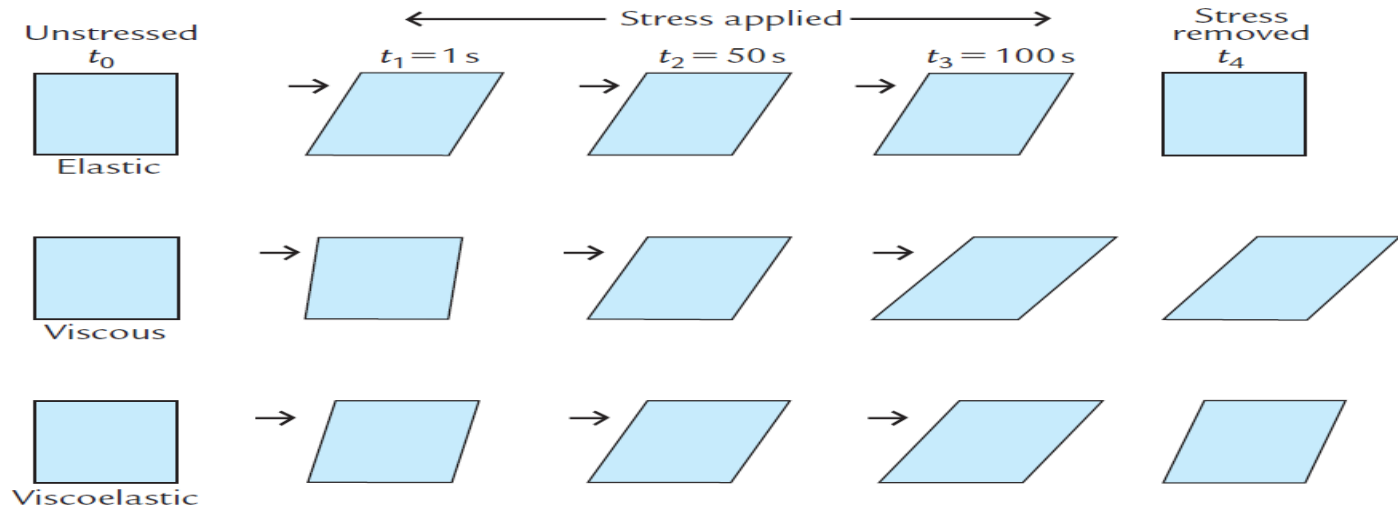


(a)



(b)

- a) Elastic
- b) Viscoelastic material



Elastic solid:

There is an instantaneous deformation when the deforming force is applied and no further deformation with time. There is complete recovery of the original shape when the force is removed.

Newtonian (Viscous) liquid:

The material begins to flow as soon as the deforming force is applied and it continues to flow as long as the force is being applied. There is no recovery of shape when the force is removed.

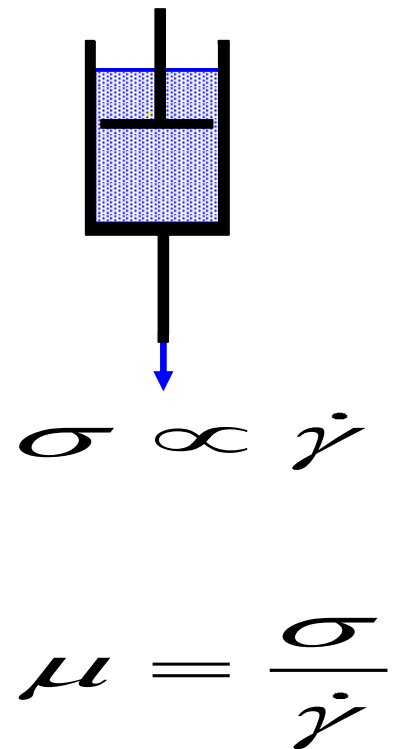
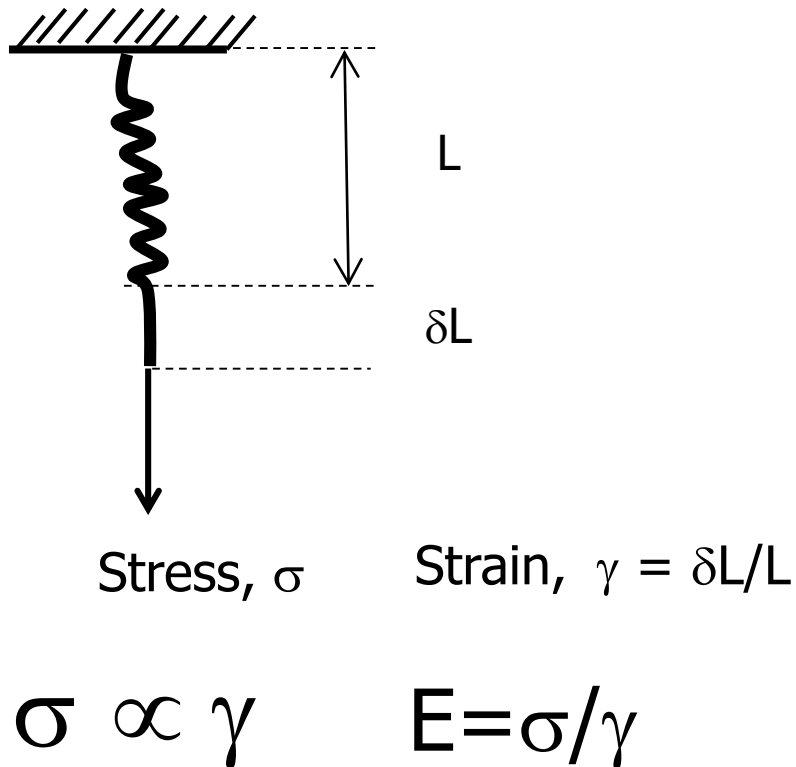
Viscoelastic solid:

There is an instantaneous deformation when the deforming force is first applied, and then the material continues to deform so long as the force is applied. When the force is removed there is some recovery of the original shape (elastic component) but not a full recovery (viscous component).

BASIC MECHANICAL ELEMENTS

The **two basic mechanical elements** used in mechanical models

1. Spring (ideal elastic body) obeys Hooke's law: Stress \propto Strain
2. Dashpot (Newtonian liquid) obeys law of Newton. Stress \propto Strain Rate



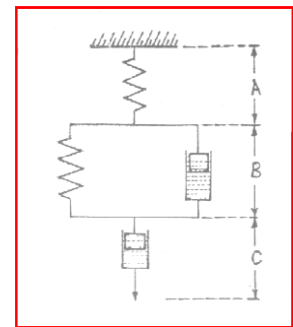
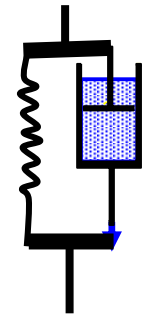
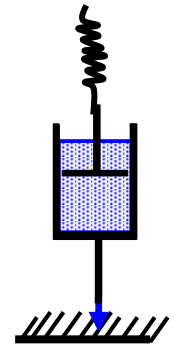
MECHANICAL MODELS

The behaviour of agricultural materials under stress can be represented by any of these three models

Maxwell model - States that the behaviour of agricultural materials under stress can be represented by a spring (representing elastic behaviour) and a dashpot (representing viscous behaviour) in **series**.

Kelvin model - States that the behaviour of agricultural materials under stress can be represented by a spring (representing elastic behaviour) and a dashpot (representing viscous behaviour) in **parallel**.

Burger's model - States that the behaviour of agricultural materials under stress can be represented by a spring (representing elastic behaviour) in series with a dashpot and a combined spring and a dashpot (representing viscous behaviour) in parallel.



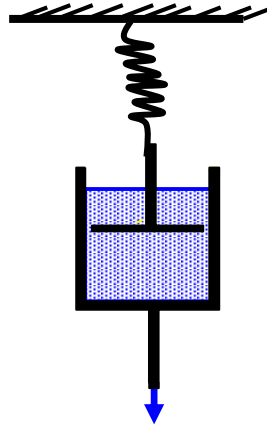
VISCOELASTIC MODELS AND EQUATIONS

Maxwell Model

$$\varepsilon = \varepsilon_s + \varepsilon_v$$

Kelvin Model

$$\sigma = \sigma_s = \sigma_v$$



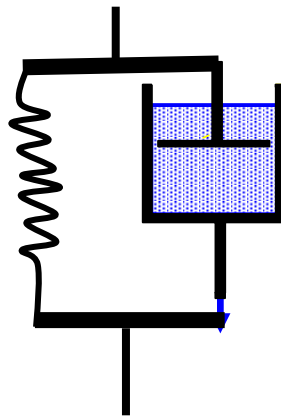
Used for stress relaxation tests

$$\frac{\sigma_s}{\varepsilon_s} = E$$

$$\frac{\sigma_v}{\varepsilon_v} = \eta$$

$$\varepsilon = \varepsilon_s = \varepsilon_v$$

$$\sigma = \sigma_s + \sigma_v$$

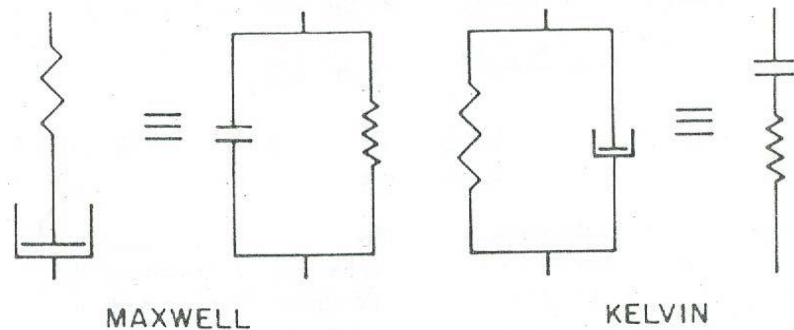
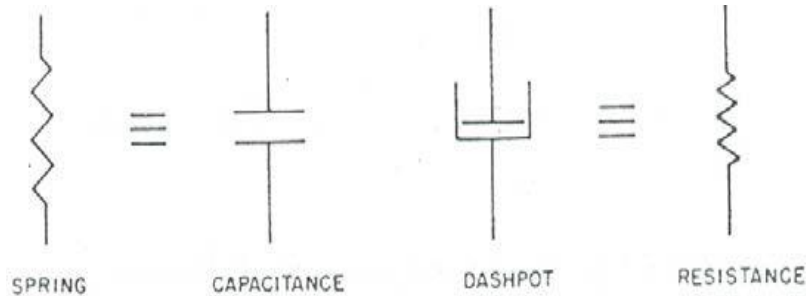


Used for creep tests

$$\frac{\sigma_s}{\varepsilon_s} = E$$

$$\frac{\sigma_v}{\varepsilon_v} = \eta$$

Electrical equivalence of mechanical models



Spring → elasticity → capacitance
dashpot → viscosity → resistance

The **tension and compression** of the spring corresponds to **charging and discharging** of the capacitor.

when elements are coupled in **parallel** in the mechanical model, its equivalent in the electrical network is a **series** coupling. Similarly, a series coupling in the mechanical system has a parallel equivalence in the electrical system.

Electrical analogs of mechanical models

In the electrical network, the **stress** is represented by the **electric potential (voltage)** and the **strain** by **electric current**.

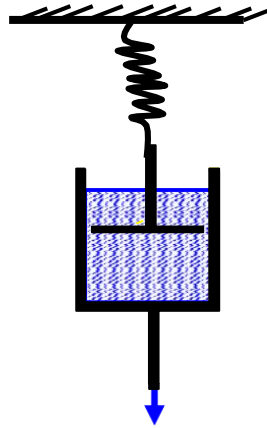
**RHEOLOGICAL
MODELS
EQUATIONS**

Viscoelastic Models

- Maxwell

$$\varepsilon = \varepsilon_s + \varepsilon_v$$

$$\sigma = \sigma_s = \sigma_v$$



Used for stress relaxation tests

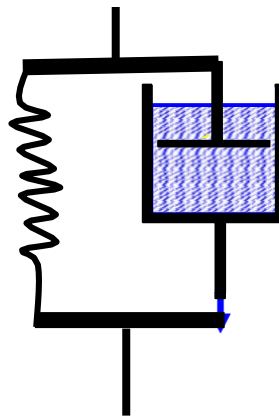
$$\frac{\sigma_s}{\varepsilon_s} = E$$

$$\frac{\sigma_v}{\varepsilon_v} = \eta$$

- Kelvin Model

$$\varepsilon = \varepsilon_s = \varepsilon_v$$

$$\sigma = \sigma_s + \sigma_v$$



Used for creep tests

$$\frac{\sigma_s}{\varepsilon_s} = E$$

$$\frac{\sigma_v}{\varepsilon_v} = \eta$$

MAXWELL MODEL

For
Dashpot

$$\frac{\sigma_v}{\varepsilon_v} = \eta$$

For
Spring

$$\frac{\sigma_s}{\varepsilon_s} = E$$

The strain being additive in Maxwell model,
the total strain

$$\varepsilon = \varepsilon_s + \varepsilon_v$$

by differentiating the above equation,

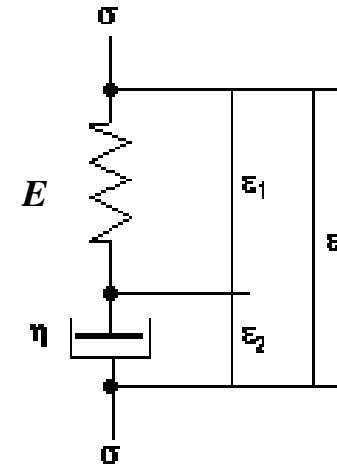
$$\dot{\varepsilon} = \dot{\varepsilon}_s + \dot{\varepsilon}_v$$

substituting

$$\dot{\varepsilon}_s \quad \dot{\varepsilon}_v$$

we
get

$$\dot{\varepsilon} = \frac{\dot{\sigma}_s}{E} + \frac{\dot{\sigma}_v}{\eta}$$



A. Maxwell model

$$\frac{\sigma_s}{\varepsilon_s} = E$$

$$\frac{\dot{\sigma}_s}{\dot{\varepsilon}_s} = E$$

Since stress is constant,
i.e.,

$$\sigma = \sigma_s = \sigma_v$$

the equation
yields

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$

If the model is subjected to constant strain ($\dot{\varepsilon}=0$) and the
term
is replaced by the symbol T_{rel} called

$$\frac{\eta}{E}$$

the Time of relaxation, the above equation
becomes

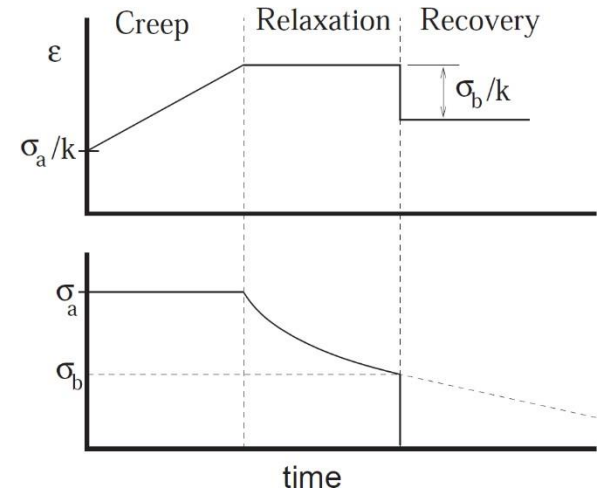
$$0 = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} = \dot{\sigma} + \frac{\sigma}{\frac{\eta}{E}} = \dot{\sigma} + \frac{\sigma}{T_{rel}}$$

By differentiation we get the final solution as

$$\sigma = A e^{-t / T_{rel}} + C$$

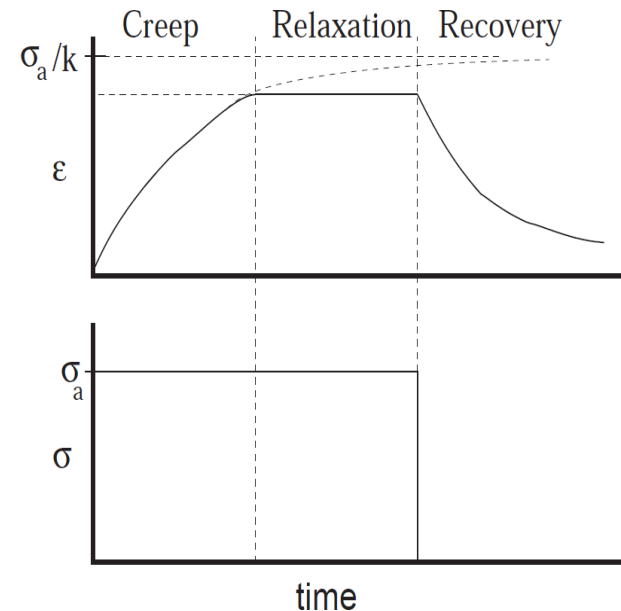
MAXWELL MODEL

- Handles Creep badly
- Handles Recovery badly (model only recovers elastic deformation, and does so instantly)
- Accounts fairly well for Relaxation

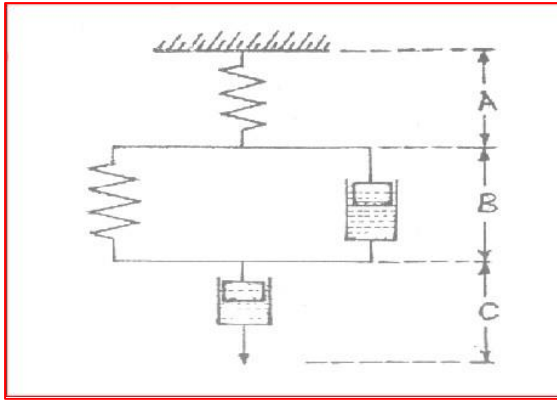


KELVIN MODEL

- Handles Creep fairly well
- Handles Recovery fairly well
- Does not account for relaxation



4-element model (Burgers model)



4-element model (Burgers model)

Burger's model - States that the behaviour of agricultural materials under stress can be represented by a spring (representing elastic behaviour) in series with a dashpot and a combined spring and a dashpot (representing viscous behaviour) in parallel.

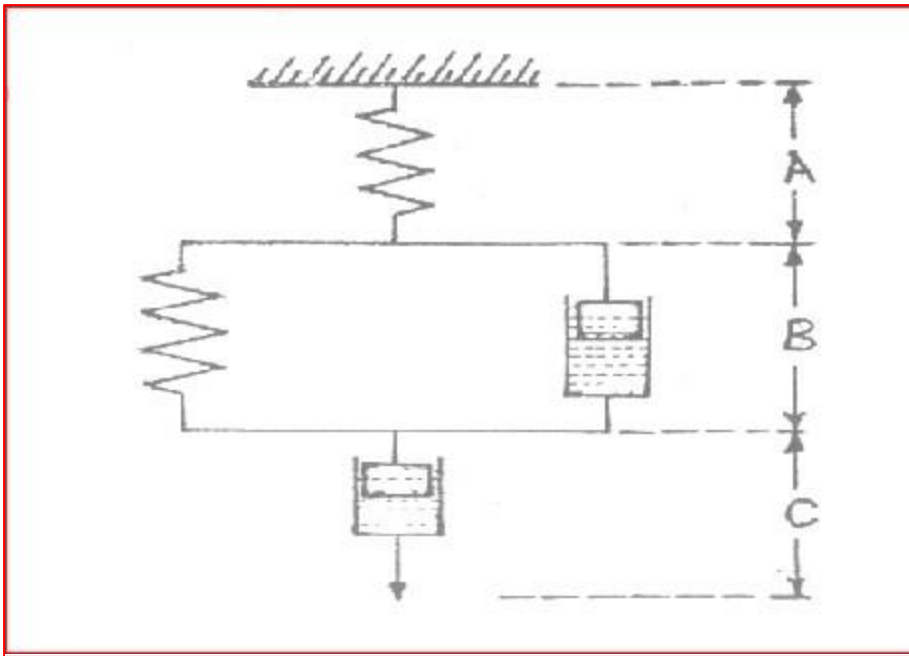


Figure (1): 4-element model (Burgers model)

ϵ = Strain, t = time, min.

σ_0 = stress, MPa

E_0 = instantaneous modulus or modulus at zero time

η = viscosity coefficient of the liquid in the dashpot, Mpa.min; and $T_{ret} = \eta / E_r$ The time of relaxation.

This equation is based on the model consists of a Kelvin model connected in series to a spring and a dashpot element. The model may thus be divided into three parts A, B and C.

$$\epsilon(t) = \sigma_0 \left[\frac{1}{E_0} + \frac{1}{E_r} (1 - e^{-t/T_{ret}}) + \frac{t}{\eta_v} \right]$$

$$\text{at } t=0, \quad \epsilon(0) = \left(\frac{\sigma_0}{E_0} \right)$$

$$\text{at } t=\infty, \quad \epsilon(\infty) = \frac{\sigma_0}{\eta_v}$$

THANK YOU