MECHANICS OF MATERIALS

Stress and Strain – Axial Loading

(we study deformation of a member under axial loading. Also need deformations to solve statically indeterminate structures)

MECHANICS OF MATERIALS

Contents

Normal Strain

(a)   (b)

Fig. 2.1

Fig. 2.3
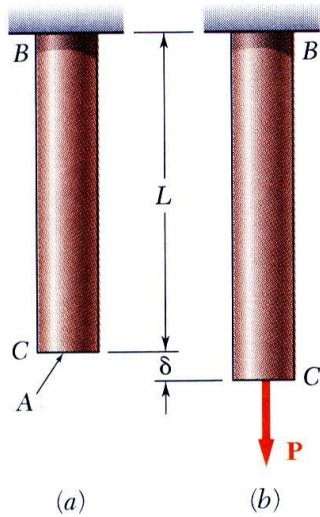
Fig. 2.4

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$

$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$

If Nonuniform strain :

$$\varepsilon = \lim_{\Delta x \to 0} \frac{\Delta \delta}{\Delta x} = \frac{d\delta}{dx}$$

$$\sigma = \frac{P}{A}$$

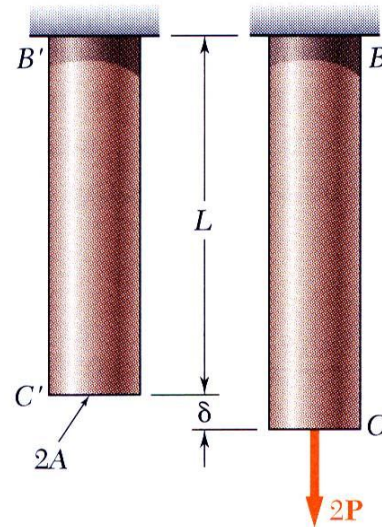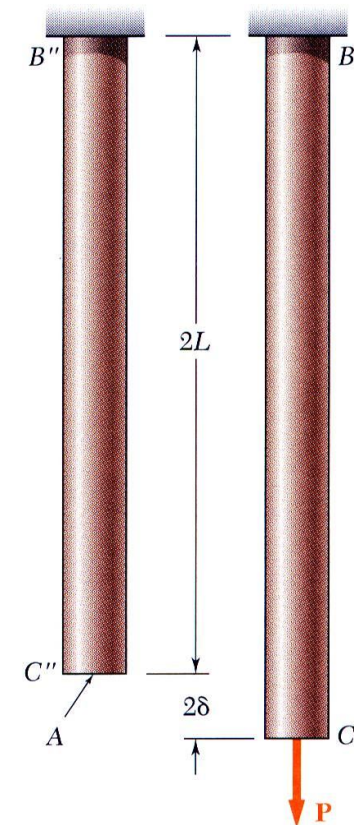$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

MECHANICS OF MATERIALS

Usual procedure to determine how materials behave when they are subjected to loads is to conduct tensile or compressive testing

Tensile Testing of metals:

-Circular specimen with **enlarged ends** where they fit in the grips so that failure doesn't happen near the grips

-A gage length is fixed as per the IS standards.

-Extensometer arms are attached to the specimen at gage marks

- **Load is slowly increased and the elongation is measured.**

- **Displacement controlled test**
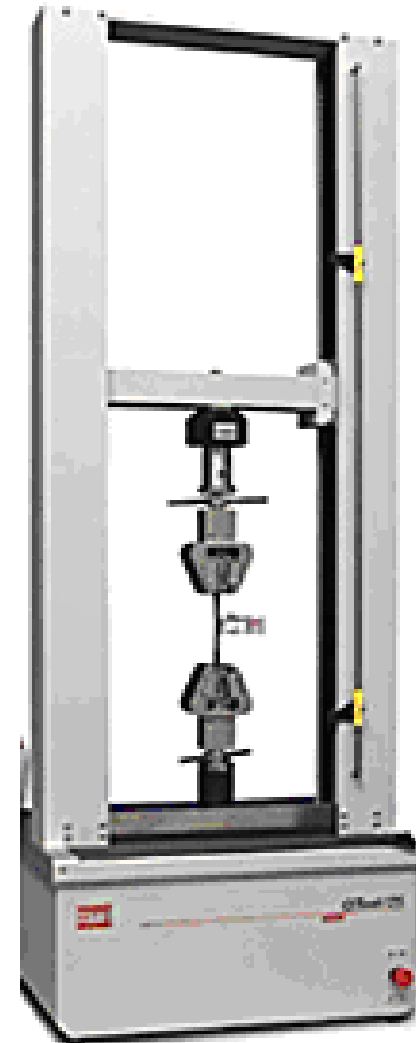
Universal Tensile Testing machine
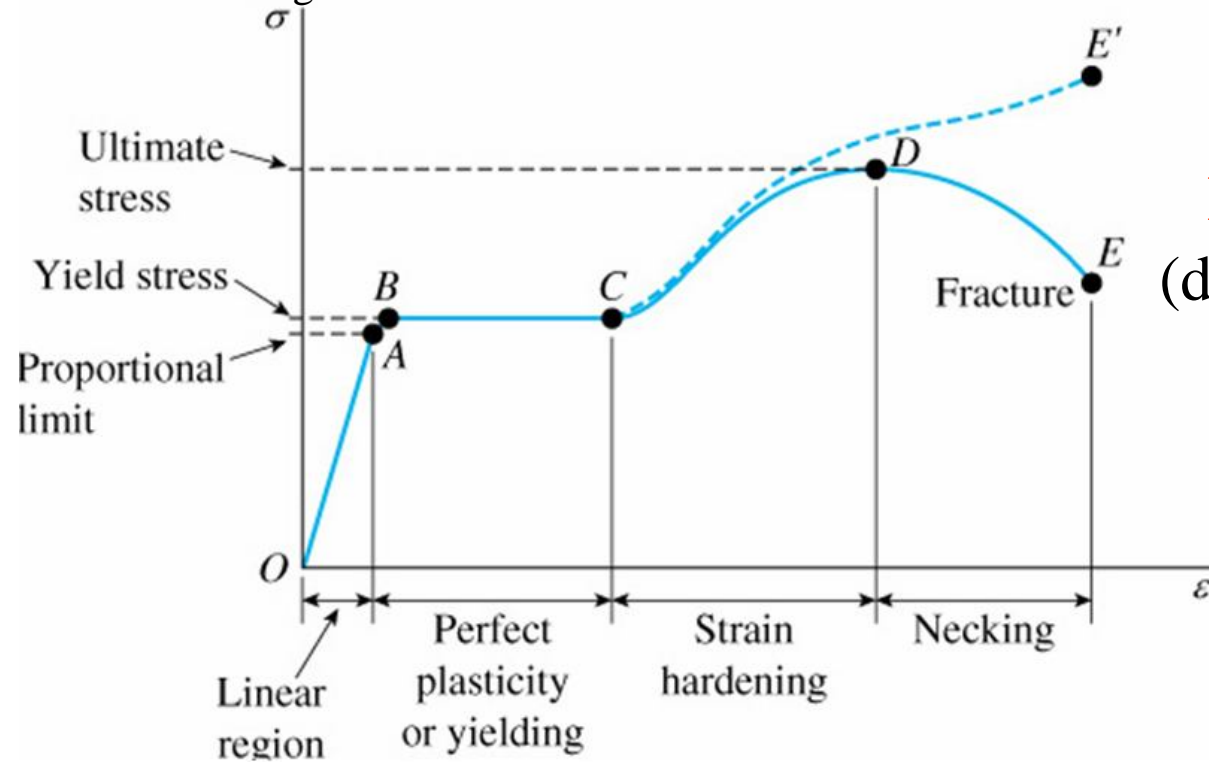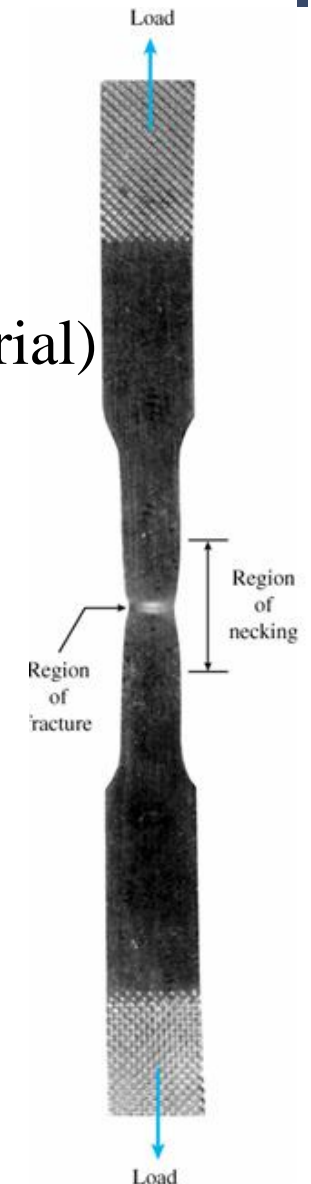
MECHANICS OF MATERIALS

Stress-Strain Diagram for Structural Steel (Mild Steel):

Convert load-elongation curve to stress-strain curve to avoid dimension problem

Mild Steel
(ductile material)

Nominal Stress (aka conventional / engineering stress) – initial area of the specimen is used

True Stress – actual area of specimen used at the cross-section where failure occurs

Stress-Strain Diagram for Aluminum Alloy

-No obvious yield point.

-Yield stress may be determined by offset method

Offset method:

At 0.2% strain (0.002) draw line parallel to linear part. It cuts stress-strain diagram at A, which is defined as yield stress

- Note Aluminum alloy is also ductile because it exhibits plasticity (large permanent deformation) before failure

- Other ductile materials include copper, nickel, bronze, etc.

(b) Aluminum alloy

0.002 offset

Stress-Strain Diagram for Brittle materials (eg. Cast iron)

- Materials that fail in tension at relatively low strains are classified as brittle. Failure is sudden.
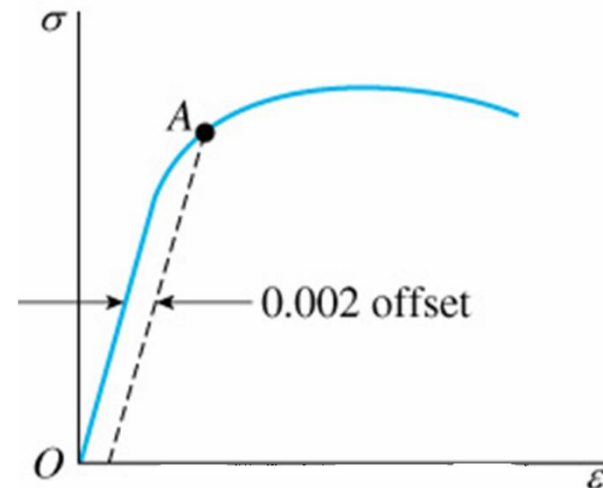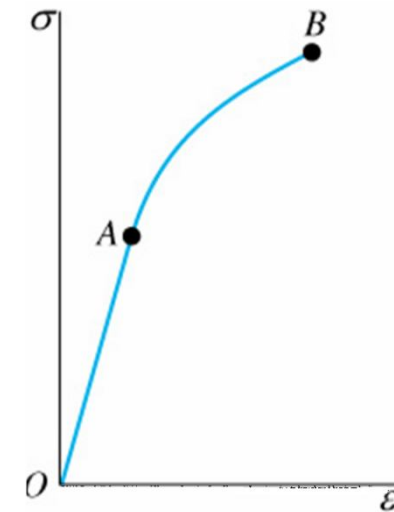
- Brittle materials fail only after a little elongation after the proportional limit (point A) is exceeded and doesn't exhibit significant plasticity as ductile materials

-No Necking

Concrete, glass, cast iron, stone, ceramics

Stress-Strain Diagram for Rubber (elastic material)

- Linear relationship between stress and strain upto relatively large strain (as high as 0.1 or 0.2). Beyond that behavior is non-linear and depends on type of rubber

- Rubber is not ductile because it doesn't give permanent deformation and returns to original configuration upon release of load.

Stress-strain diagram in compression

- For ductile material linear regime remains same as in tension

- With increasing load, specimen takes barrel shape, finally flattens out to provide great resistance to further shortening

- For brittle materials bulging doesn't occur. Material actually breaks. However, ultimate compressive stress is much higher than ultimate tensile stress
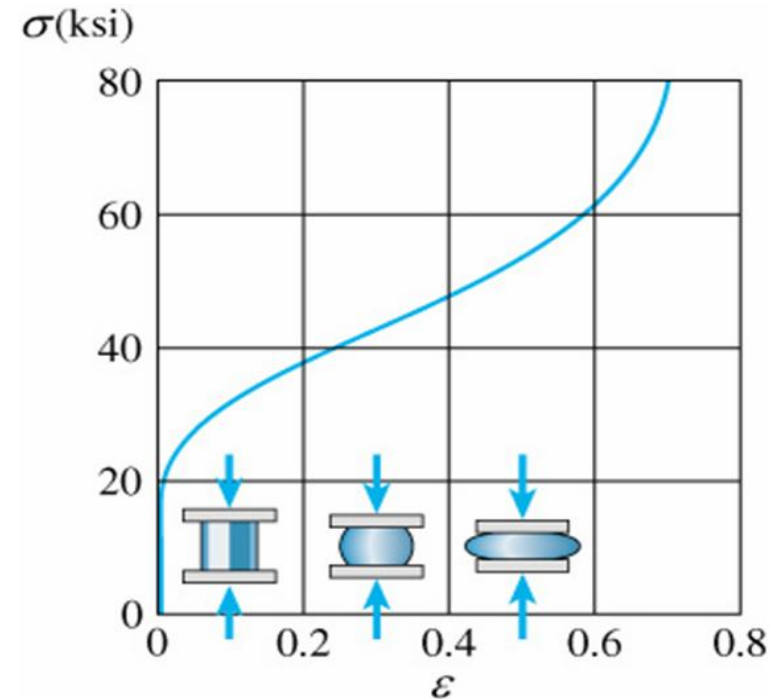
For ductile materials

Elasticity and Plasticity

-Property of material returning to original dimension after unloading, called **elasticity**

- Property of material undergoing inelastic strains beyond the strain at the elastic limit (point *E*) is **plasticity**

- For steel, aluminium, **Elastic limit** or yield point is close to proportional limit, i.e., most of curve *OE* is straight line.

- When material loaded beyond elastic limit, upon unloading it doesn't come back to its original dimension but follows path *BC* which is parallel to tangent at *O* or straight part of loading curve. Material has **permanently yielded** and a **residual strain (or permanent strain)** remains in the material

- Residual elongation called **permanent set**

Elasticity and Plasticity

Aluminium

Reloading of Material

Materials reloaded from point C.

1. Linear elastic behavior from *C* to (almost) *B,* with slope being parallel to tangent to original loading curve at *O*

2. Proportional limit now at (almost) *B* which is higher than the original elastic limit (point *E*),

3. Thus, the material properties have been changed - the proportional limit and elastic limit are raised

4. Ductility reduced because in 'new stress strain diagram' (path *CBF*) the amount of yielding beyond elastic limit (*B* to *F*) is less than in the original stress strain diagram (*E* to *F*)
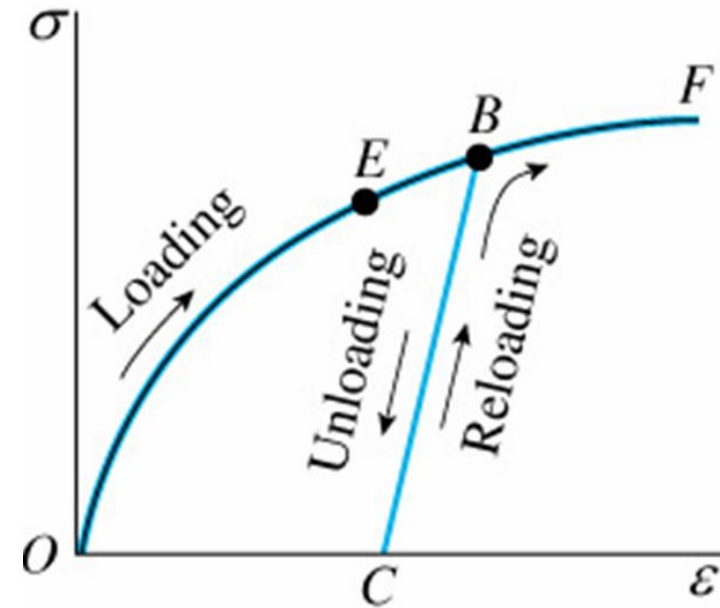
Stress-Strain Relationship: Hooke's Law

The linear relationship between stress and strain for a bar in simple tension or compression is expressed by the equation

$$\sigma = E\varepsilon$$
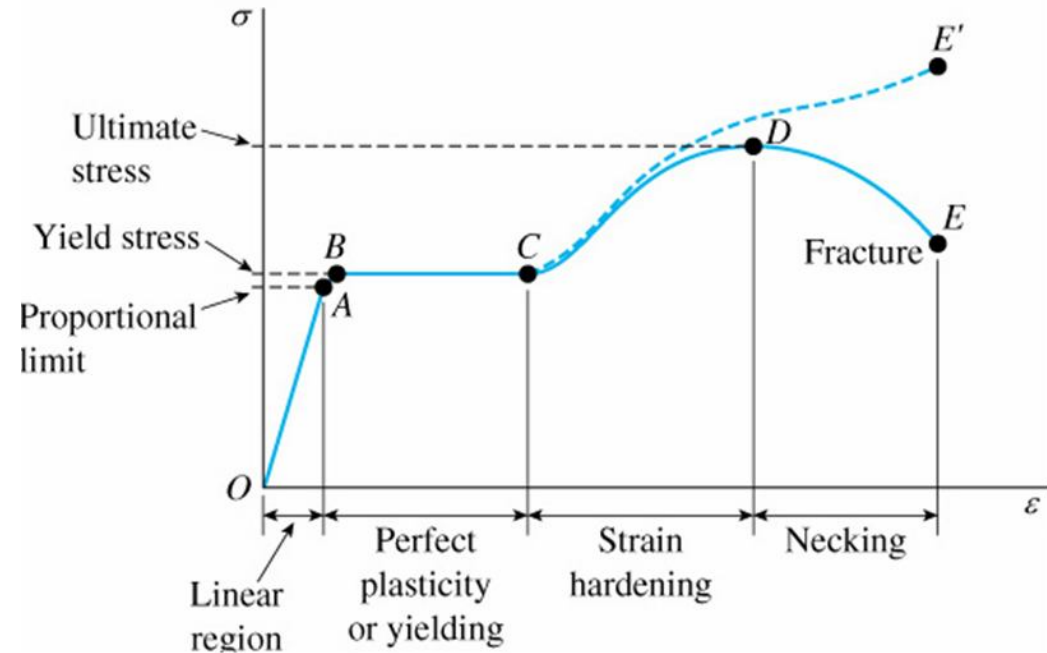
➤ **Modulus of Elasticity (E)** (aka Young's Modulus). It is the ratio of normal stress to normal strain (i.e., measure of resistance to elastic deformation), evaluated below the proportional limit, i.e., **slope** of the straight-line portion of the stress-strain curve.

Structural Steel E=210 GPa (30, 000 ksi)
Aluminum   E=73 GPa (10,600 ksi)

Elongation of prismatic bar in tension
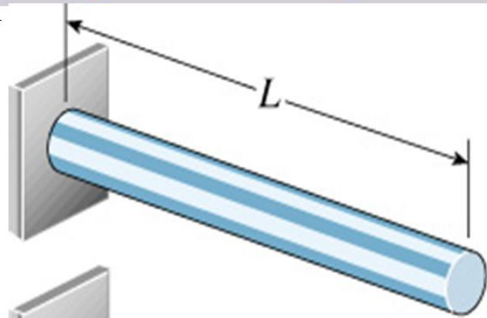
Stress: $\sigma = \dfrac{P}{A}$

Strain: $\varepsilon = \dfrac{\delta}{L}$

Stress-strain relationship:

$$\sigma = E\varepsilon$$

$$\frac{P}{A} = E\frac{\delta}{L}$$

$$\delta = \frac{PL}{AE} \quad \text{or} \quad \delta = \frac{P_i L_i}{A_i E_i}$$
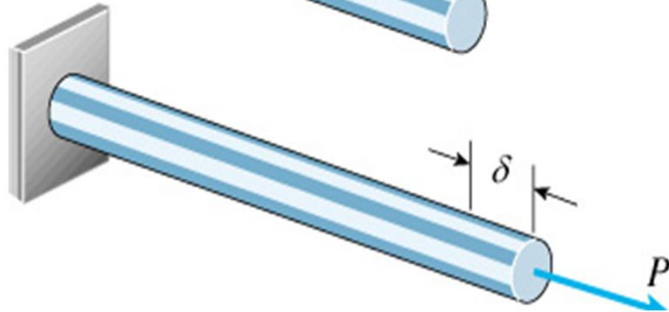
$$Stiffness: k = \frac{EA}{L} \qquad flexibilty: f = \frac{L}{EA}$$

$$EA \Rightarrow Axial\ Rigidity$$

Solid cross sections

Hollow or tubular cross sections

Thin-walled open cross sections

Example 2.1

(a) Bar with external loads acting at intermediate points; (b) (c), and (d) free-body diagrams showing the internal axial forces N_1, N_2, and N_3.

Calculate elongation of bar AD

Step 1: use **equilibrium** to obtain internal forces in each segment

From F.B.D (b),

$$N_1 = P_D + P_C - P_B \quad \text{on segment } AB$$

From F.B.D (c),

$$N_2 = P_C + P_D \quad \text{on segment BC}$$

From F.B.D (b),

$$N_3 = P_D \quad \text{on segment CD}$$
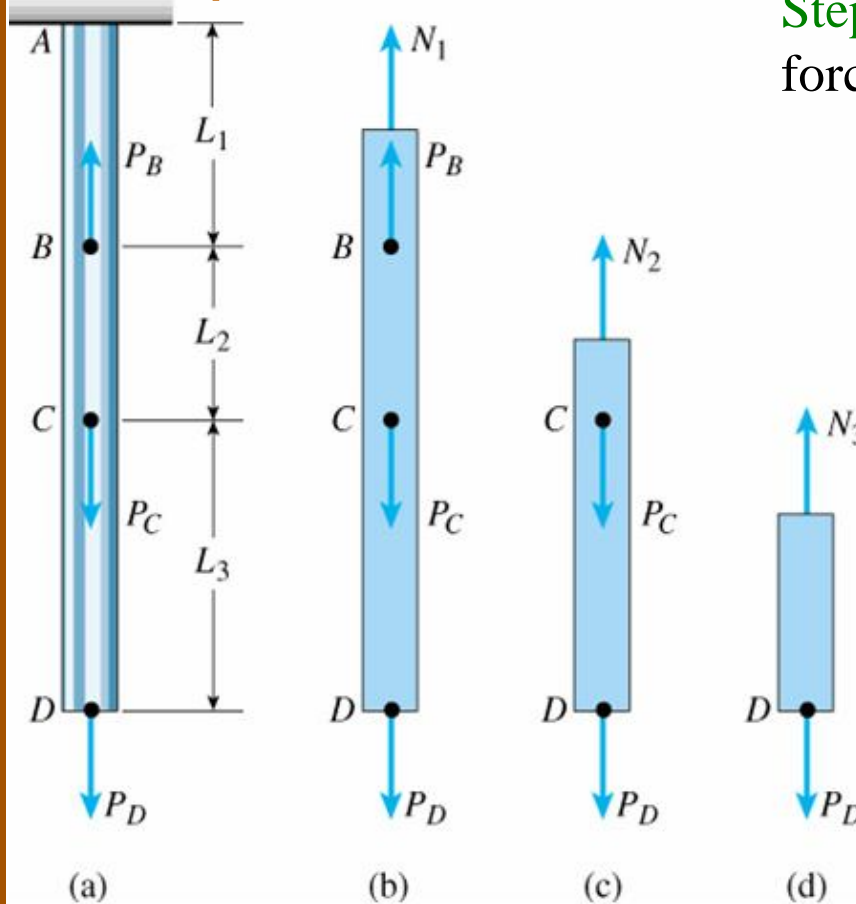
Step 2: calculate **elongation** of each segment and add them

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$\delta_{AD} = \frac{N_1 L_1}{AE} + \frac{N_2 L_2}{AE} + \frac{N_3 L_3}{AE}$$

Variant: non-uniform (stepped) bar

Example 2.2

(a)

(b)

(c)
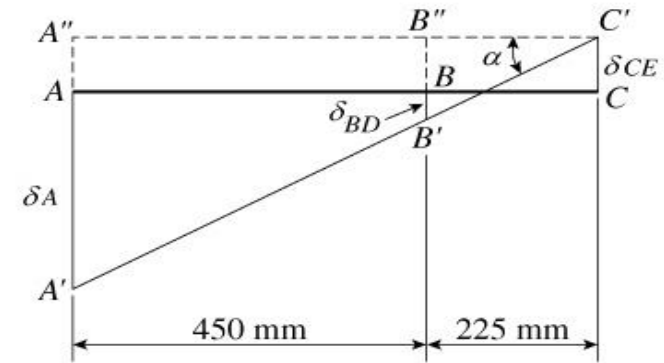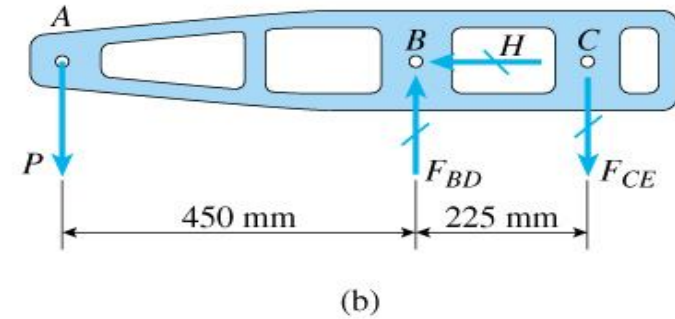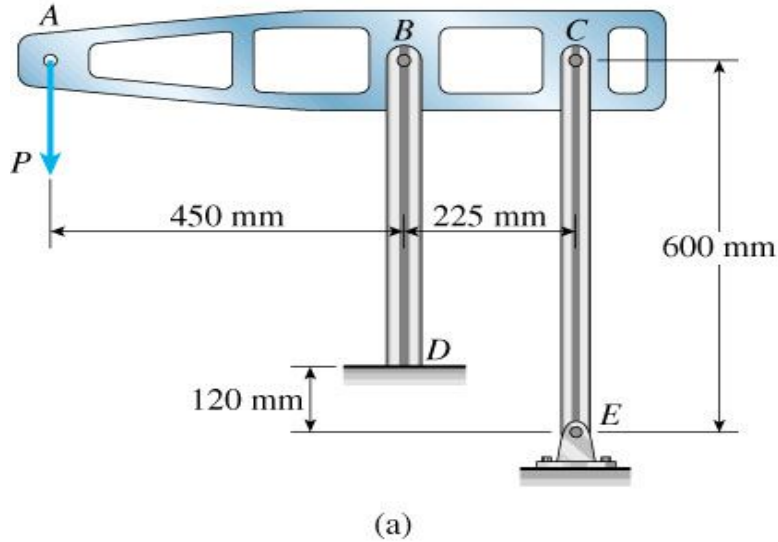
Horizontal rigid beam **ABC** supported by two vertical non-rigid bars.

Calculate deflection at A

Step 1: Equilibrium: $\sum F_x = 0; \quad \sum F_y = 0; \quad \sum M_B = 0$ $H=0; F_{CE}=2P; F_{BD}=3P$

Step 2: Member elongation/shortening

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{EA_{BD}} \qquad \delta_{CE} = \frac{F_{CE} L_{CE}}{EA_{CE}}$$

Step 3: Displacement at A? (construct displacement diagram)

$$\frac{A'A''}{A''C'} = \frac{B'B''}{B''C'} \qquad \frac{\delta_A + \delta_{CE}}{450 + 225} = \frac{\delta_{BD} + \delta_{CE}}{225}$$

MECHANICS OF MATERIALS

Non-uniform Bar with Distributed Axial Loading

(a)   (b)   (c)
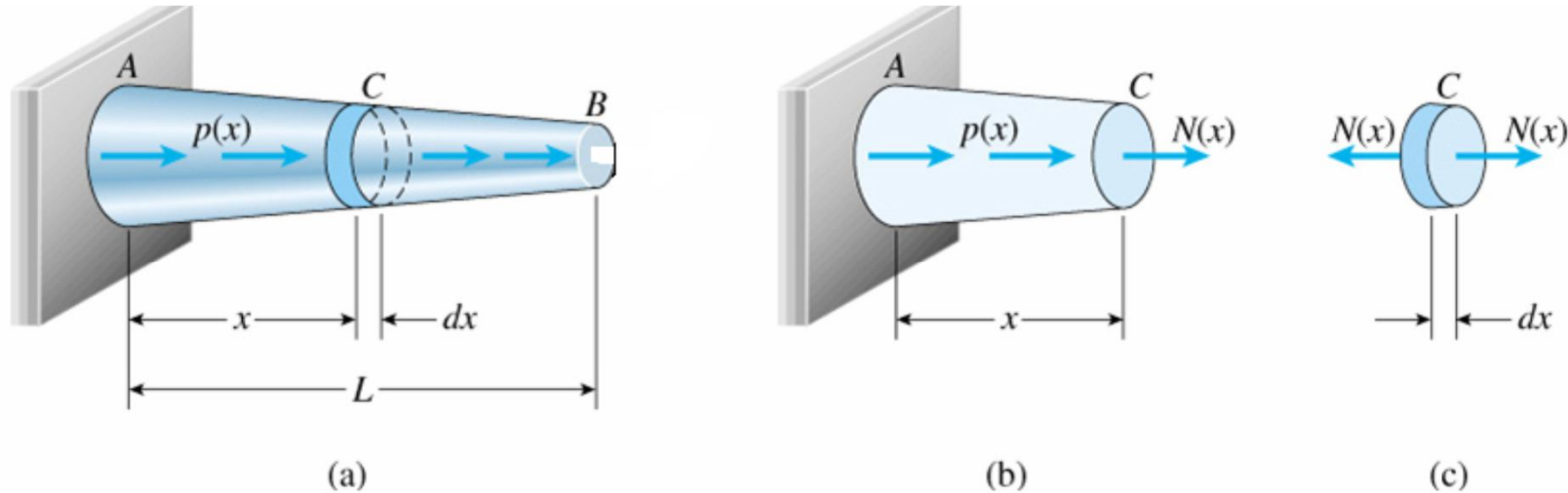
Varying axial force, i.e., distributed force *p(x)* having units of force per unit length. Distributed force may be caused by centrifugal force (helicopter/turbine blade, friction, or simply weight of a bar hanging in a vertical position) .

What is the elongation of the bar?

MECHANICS OF MATERIALS

Bar with Non-Uniform Cross-section/ Loading

(a) (b) (c)
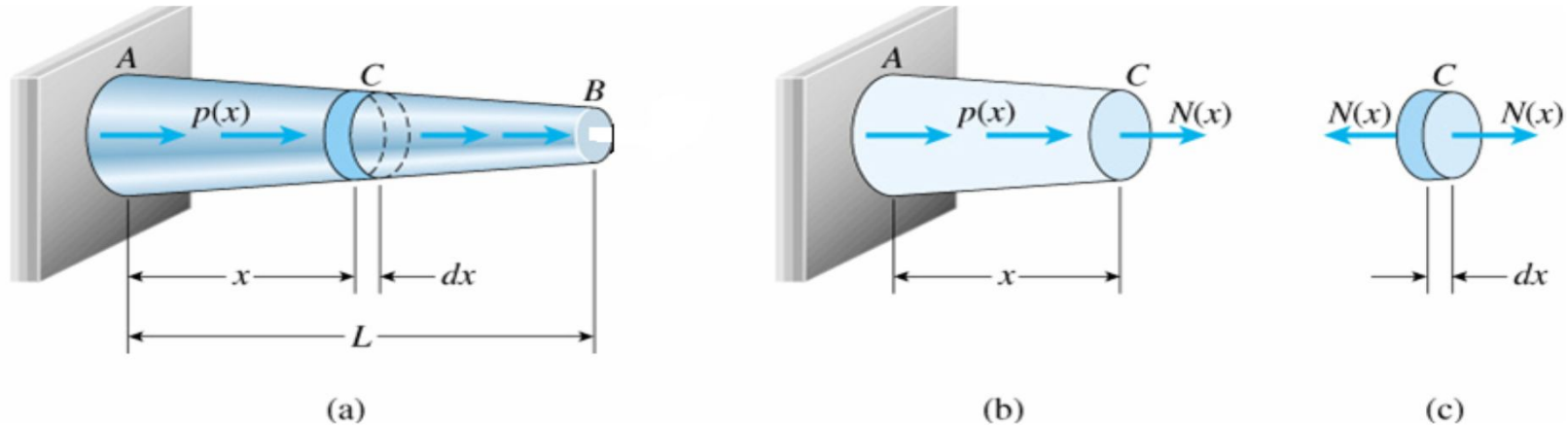
Need to solve this problem by considering a differential element, dx, at a distance x, and obtain the elongation, dδ, of that differential element. Assume that the force acting on the element is $N(x)$.

$$d\delta = \frac{N(x)dx}{A(x)E} \Rightarrow \delta = \int_0^L d\delta = \int_0^L \frac{N(x)dx}{A(x)E}$$

Recall:

$$\delta = \frac{PL}{AE}$$

So you need to find the internal force $N(x)$ acting at a distance x, on the segment. In this case

$$N(x) = \int_x^L p(x)dx$$

Example 2.3

Calculate elongation of bar due to its own weight and load (P) applied

Note: the weight of the bar itself produces a varying internal axial force which is zero at the lower end and is maximum at the upper end

Internal force at a distance x from the upper end
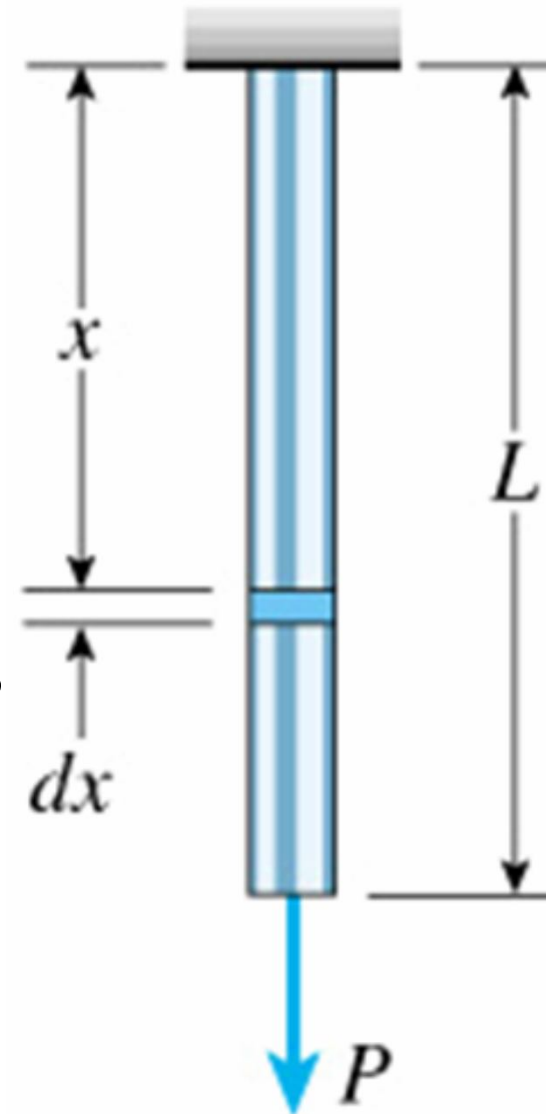
$$N(x) = \int_{x}^{L} \gamma A dx + P = \gamma A(L - x) + P$$

x ↳ **Weight density of the material**

$$\delta = \int_{0}^{L} \frac{N(x)dx}{A(x)E} = \frac{\gamma L^2}{2E} + \frac{PL}{AE}$$

Static Indeterminacy
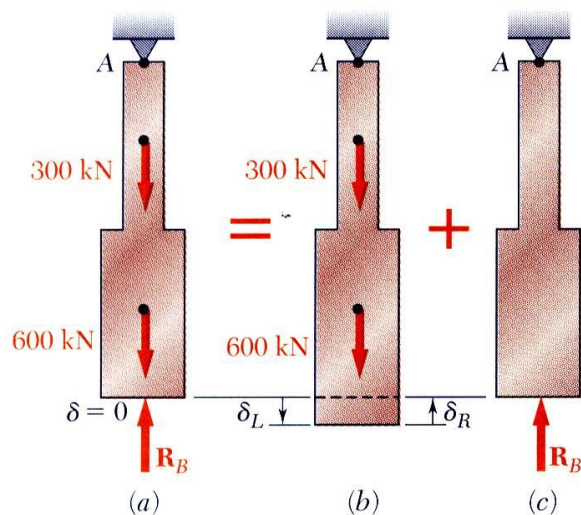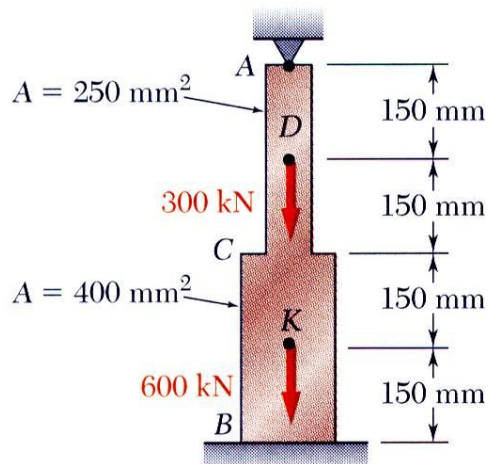
- Structures for which internal forces and reactions cannot be determined from statics alone are *statically indeterminate*.

- Structure will be statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.

- Redundant (i.e., excess) reactions are replaced with unknown loads which along with the other loads must produce compatible deformations.

- Deformations due to actual loads and redundant reactions are determined separately and then *added* or *superposed* to obtain the *compatibility* equation.

$$\delta = \delta_L + \delta_R = 0$$

Example 2.4

$A = 250 \text{ mm}^2$

$A = 400 \text{ mm}^2$

300 kN

600 kN

150 mm
150 mm
150 mm
150 mm

A
D
C
K
B
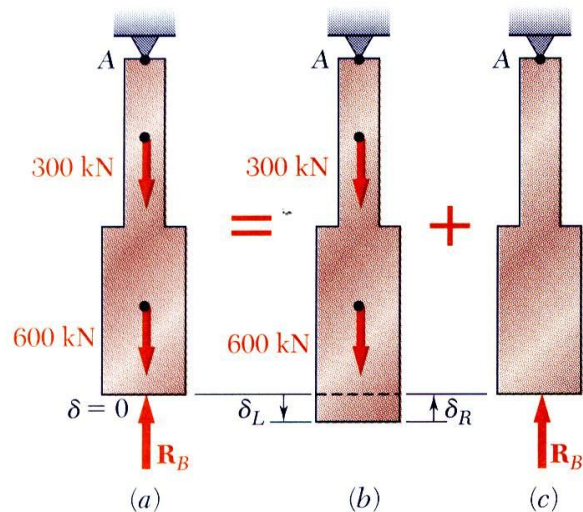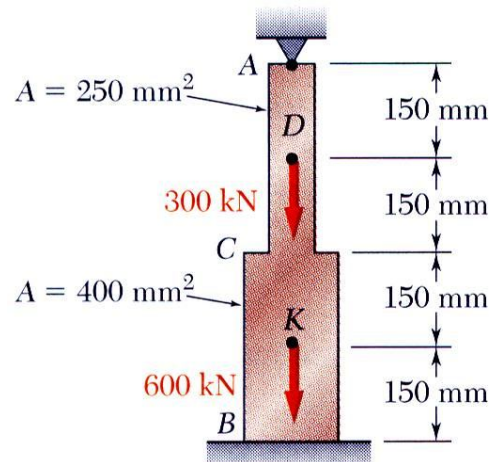
Determine reactions at A and B for steel bar with loading shown, assuming close fit at both supports before loads applied.

SOLUTION:

- Consider reaction at B as redundant, release bar from that support, and solve for the displacement at B due to applied loads.

- Solve for displacement at B due to redundant reaction at B.

- Require that displacements due to loads and due to redundant reaction be compatible, i.e., in this case require that their sum be zero. Solve compatibility equation for R_B

- Solve for the reaction at A due to the applied loads and the reaction found at B.

Example 2.4
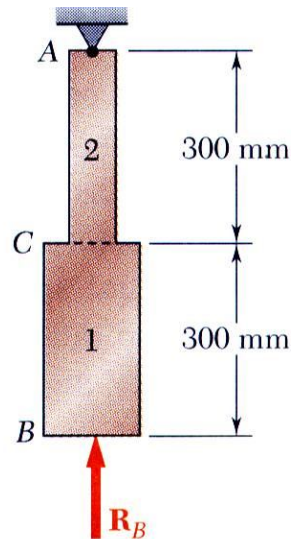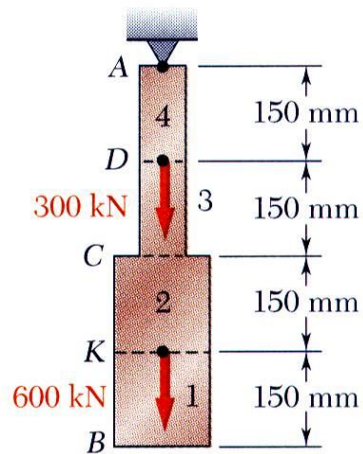
SOLUTION:

- Solve displacement at B due to applied loads, with redundant constraint/reaction released,

$$P_1 = 0 \quad P_2 = P_3 = 600 \times 10^3 \, \text{N} \quad P_4 = 900 \times 10^3 \, \text{N}$$

$$A_1 = A_2 = 400 \times 10^{-6} \, \text{m}^2 \quad A_3 = A_4 = 250 \times 10^{-6} \, \text{m}^2$$

$$L_1 = L_2 = L_3 = L_4 = 0.150 \, \text{m}$$

$$\delta_L = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1.125 \times 10^9}{E}$$

- Solve displacement at B due to redundant constraint/reaction,

$$P_1 = P_2 = -R_B$$

$$A_1 = 400 \times 10^{-6} \, \text{m}^2 \quad A_2 = 250 \times 10^{-6} \, \text{m}^2$$

$$L_1 = L_2 = 0.300 \, \text{m}$$

$$\delta_R = \sum_i \frac{P_i L_i}{A_i E_i} = -\frac{\left(1.95 \times 10^3\right) R_B}{E}$$

Example 2.4
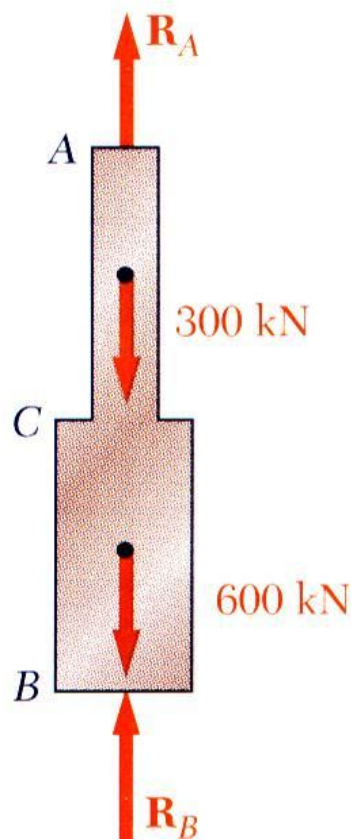
- Require that displacements due to loads and due to redundant reaction be compatible,

$$\delta = \delta_L + \delta_R = 0 \qquad (\delta \text{ non-zero if initial gap at } B)$$

$$\delta = \frac{1.125 \times 10^9}{E} - \frac{\left(1.95 \times 10^3\right) R_B}{E} = 0$$

$$R_B = 577 \times 10^3 \, \text{N} = 577 \, \text{kN}$$

- Find reaction at A due to the loads and the reaction at B

$$\sum F_y = 0 = R_A - 300 \, \text{kN} - 600 \, \text{kN} + 577 \, \text{kN}$$

$$R_A = 323 \, \text{kN}$$

$$\boxed{R_A = 323 \, \text{kN}}$$
$$\boxed{R_B = 577 \, \text{kN}}$$
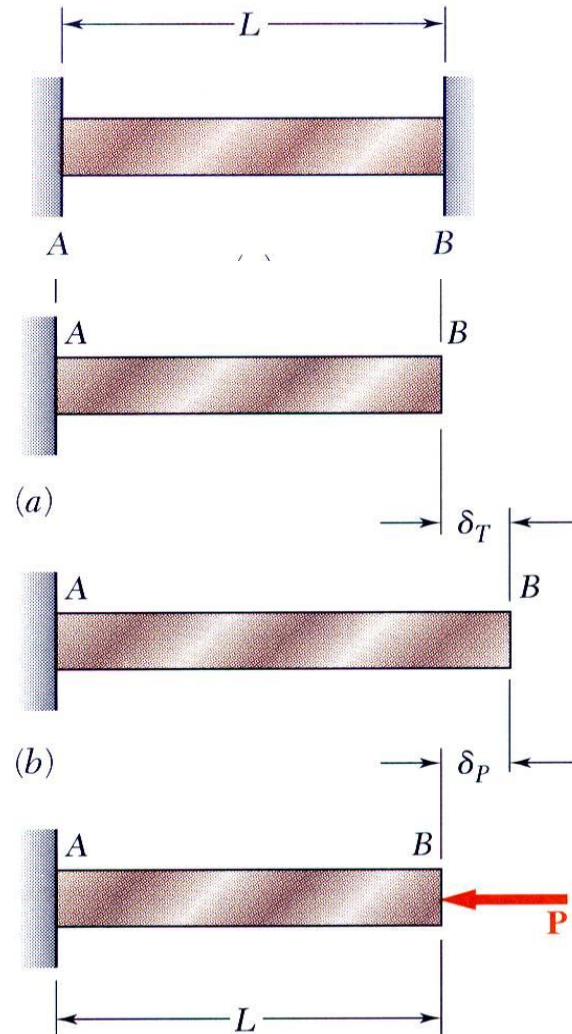
Thermal Stresses

- Temperature change results in change in length, i.e., *thermal strain*. No stress associated with thermal strain, unless elongation restrained by supports.

- Treat additional support as redundant and apply the principle of superposition.

$$\delta_T = \alpha(\Delta T)L \qquad\qquad \delta_P = \frac{PL}{AE}$$

$$\alpha = \text{thermal expansion coef.}$$

- Thermal deformation and deformation from redundant support must be compatible, i.e.,

$$\delta = \delta_T + \delta_P = 0 \quad \Rightarrow \quad \alpha(\Delta T)L + \frac{PL}{AE} = 0$$

$$P = -AE\alpha(\Delta T) \quad \Rightarrow \quad \sigma = \frac{P}{A} = -E\alpha(\Delta T)$$

(a)

(b)
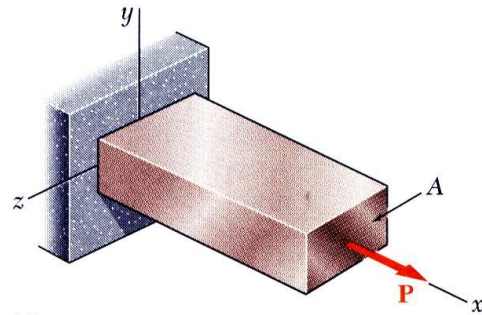
Poisson's Ratio

- For slender bar subjected to axial loading:

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad \sigma_y = \sigma_z = 0$$

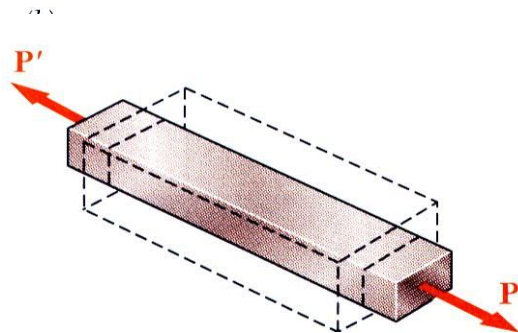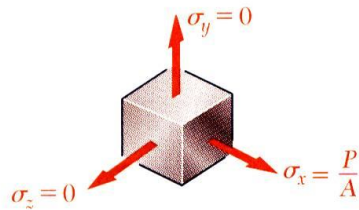- Elongation in x- (i.e., longitudinal/axial) direction is accompanied by contraction in the other (i.e., lateral) directions.

$$\varepsilon_y = \varepsilon_z \neq 0$$

- Poisson's ratio defined as

$$\nu = \left| \frac{\text{lateral strain}}{\text{longitudin al strain}} \right| = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

- For common materials (metals) Poisson's ratio is positive, in the range 0.25-0.35. Positive for polymer foams.

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Only two properties (eg., E and **ν**) needed to characterize an isotropic material

Generalized Hooke's Law
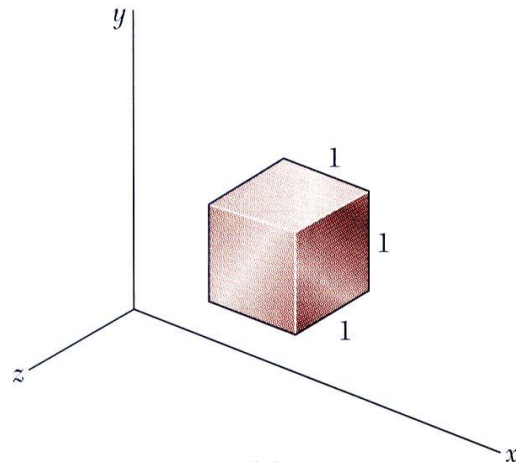
(a)
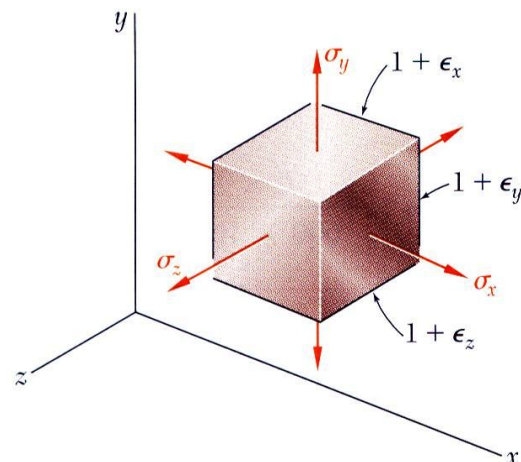
(b)

- For element subjected to multi-axial loading, the normal strain components resulting from stress components may be determined from *principle of superposition*. This requires:

  1) strain is linearly related to stress
  2) deformations are small

- Thus:

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

Dilatation: Bulk Modulus
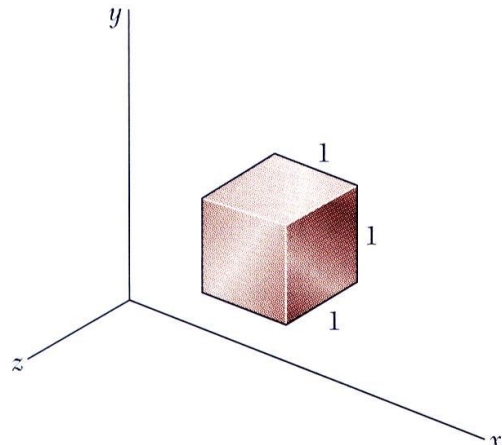
(a)
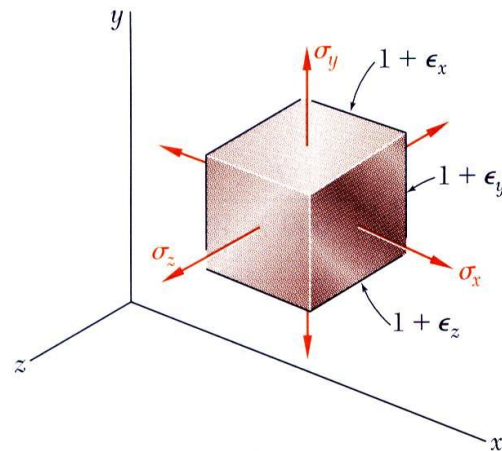
(b)

- Relative to the unstressed state, the change in volume is (neglect nonlinear strain terms, ε<<1 so ε² neglected)

$$e = 1 - \left[(1 + \varepsilon_x)(1 + \varepsilon_y)(1 + \varepsilon_z) \right] = 1 - \left[1 + \varepsilon_x + \varepsilon_y + \varepsilon_z \right]$$

$$= \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \frac{1 - 2v}{E} \left(\sigma_x + \sigma_y + \sigma_z \right)$$

$$= \text{dilatation (change in volume per unit volume)}$$

- For element subjected to uniform hydrostatic pressure,

$$e = -p \frac{3(1 - 2v)}{E} = -\frac{p}{k}$$

$$k = \frac{E}{3(1 - 2v)} = \text{bulk modulus}$$

- Subjected to uniform pressure, dilatation must be negative, therefore

$$0 < v < \frac{1}{2}$$

Shearing Strain
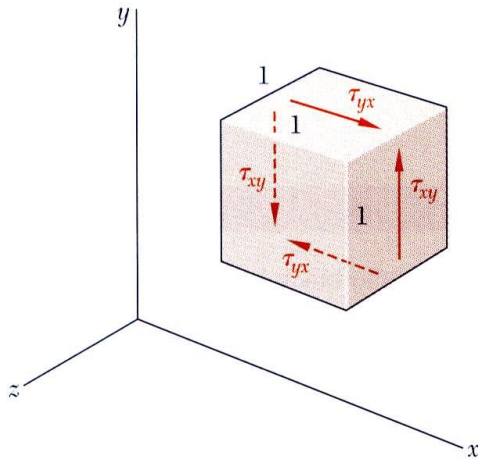
Fig. 2.46
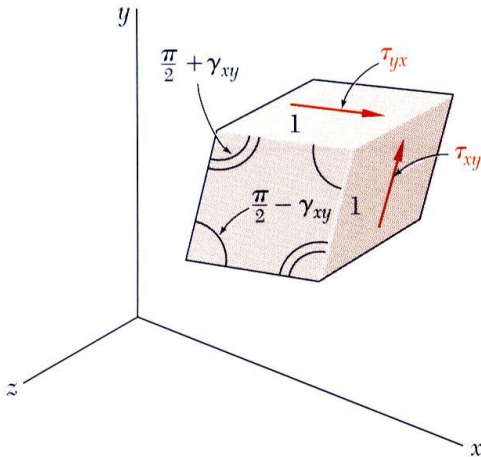
Fig. 2.47

- A cubic element subjected to shear stress will deform into a skewed parallelopiped. The corresponding *shear* strain is defined as change in angle between adjacent sides.

- For isotropic material, shear stress related to shear strain,

$$\tau_{xy} = f(\gamma_{xy})$$

- Plot of shear stress vs. shear strain is similar to plot of normal stress vs. normal strain, except that (yield, ultimate) strength values are approximately half. For small strains,

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{zx} = G\gamma_{zx}$$

where G is modulus of rigidity or shear modulus.

Example 2.5

Rectangular block of material with $G = 90$ ksi is bonded to two rigid plates. Lower plate fixed, upper plate moves 0.04 in. due to applied horizontal force P. Find a) average shearing strain in the material, and b) force P exerted on plate.
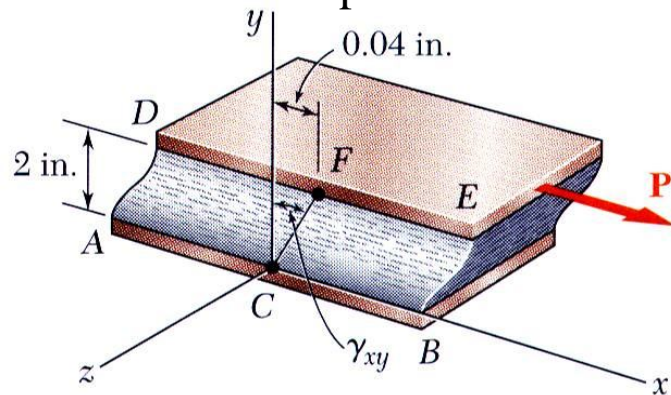
- Determine the average angular deformation or shearing strain of the block.

$$\gamma_{xy} \approx \tan\gamma_{xy} = \frac{0.04\,\text{in.}}{2\,\text{in.}} \qquad \gamma_{xy} = 0.020\,\text{rad}$$

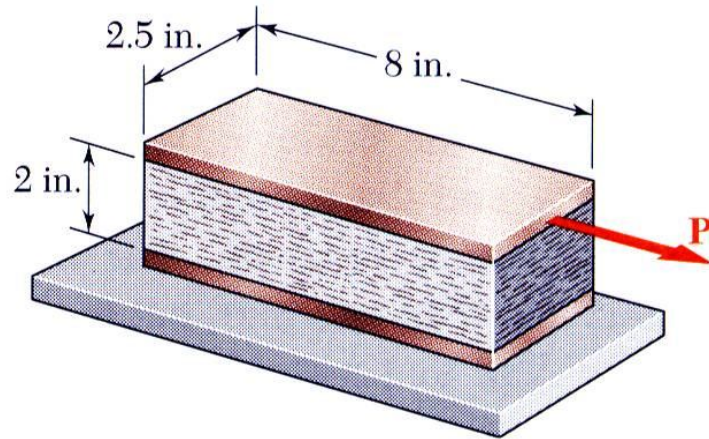- Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.

$$\tau_{xy} = G\gamma_{xy} = \left(90\times10^3\,\text{psi}\right)\left(0.020\,\text{rad}\right) = 1800\,\text{psi}$$

- Use the definition of shearing stress to find the force P.

$$P = \tau_{xy}A = \left(1800\,\text{psi}\right)\left(8\,\text{in.}\right)\left(2.5\,\text{in.}\right) = 36\times10^3\,\text{lb}$$

$$\boxed{P = 36.0\,\text{kips}}$$

Relation Among *E*, *v*, and *G*

(a)
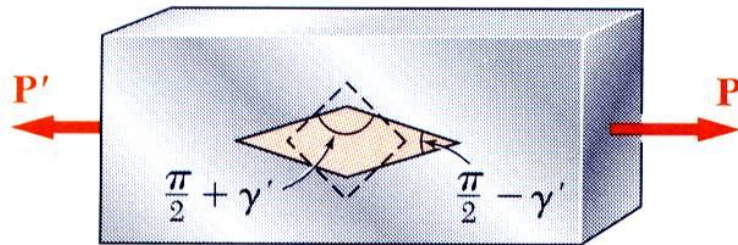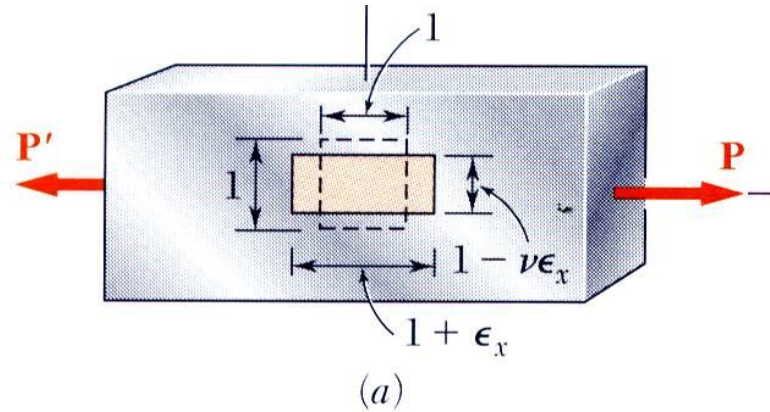
- Consider axially loaded bar with initial cube and rotated-cube elements as shown. Cubic undergoes normal strain, rotated-cubic undergoes shear strain. Diagonal of cube and side of rotated -cube are similar. Compare their angles wrt *x*-axis.

$$\tan\left(\frac{\pi}{4} - \frac{\gamma'}{2}\right) = \frac{1 - \upsilon\varepsilon_x}{1 + \varepsilon_x},$$

note small strains γ' and ε_x, $\tan(\gamma') \cong \gamma'$,

do Taylor series of denominato rs, get

$$\frac{1 - \frac{\gamma'}{2}}{1 + \frac{\gamma'}{2}} = \frac{1 - \upsilon\varepsilon_x}{1 + \varepsilon_x} \Rightarrow 1 - \gamma' = 1 - (1 + \upsilon)\varepsilon_x,$$

use stress $-$ strain law,

$$\frac{\tau'}{G} = (1 + \upsilon)\frac{\sigma_x}{E}, \quad \text{for } 45^0 \text{ plane } \tau' = \frac{P}{2A}, \quad \sigma_x = \frac{P}{A}$$

Result $\quad \dfrac{E}{2G} = (1 + v) \quad \Longleftarrow$

Example 2.6
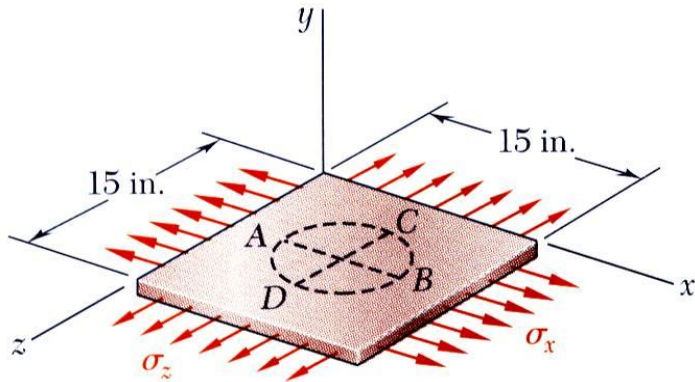
Circle of dia. $d = 9$ in. is scribed on unloaded aluminum plate of thickness $t = 3/4$ in. Forces acting in plane of plate cause normal stresses $\sigma_x = 12$ ksi and $\sigma_z = 20$ ksi.

$E = 10 \times 10^6$ psi and $\nu = 1/3$

Determine change in:

a) length of diameter AB,

b) length of diameter CD,

c) thickness of the plate, and

d) volume of the plate.

SOLUTION: Example 2.6

- Apply the generalized Hooke's Law to find the three components of normal strain.

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{1}{10\times10^6\,\text{psi}}\left[(12\,\text{ksi}) - 0 - \frac{1}{3}(20\,\text{ksi})\right]$$

$$= +0.533\times10^{-3}\,\text{in./in.}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= -1.067\times10^{-3}\,\text{in./in.}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$= +1.600\times10^{-3}\,\text{in./in.}$$

- Evaluate the deformation components.

$$\delta_{B/A} = \varepsilon_x d = \left(+0.533\times10^{-3}\,\text{in./in.}\right)(9\,\text{in.})$$

$$\boxed{\delta_{B/A} = +4.8\times10^{-3}\,\text{in.}}$$

$$\delta_{C/D} = \varepsilon_z d = \left(+1.600\times10^{-3}\,\text{in./in.}\right)(9\,\text{in.})$$

$$\boxed{\delta_{C/D} = +14.4\times10^{-3}\,\text{in.}}$$

$$\delta_t = \varepsilon_y t = \left(-1.067\times10^{-3}\,\text{in./in.}\right)(0.75\,\text{in.})$$

$$\boxed{\delta_t = -0.800\times10^{-3}\,\text{in.}}$$

- Find the change in volume

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 1.067\times10^{-3}\,\text{in}^3/\text{in}^3$$

$$\Delta V = eV = 1.067\times10^{-3}(15\times15\times0.75)\,\text{in}^3$$

$$\boxed{\Delta V = +0.187\,\text{in}^3}$$

Generalized Hooke's law in 3D

(Stress-strain relationships)

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\sigma_x = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_x + \upsilon(\varepsilon_y + \varepsilon_z)\right]$$

$$\sigma_y = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_y + \upsilon(\varepsilon_x + \varepsilon_z)\right]$$

$$\sigma_z = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_z + \upsilon(\varepsilon_x + \varepsilon_y)\right]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

$$\tau_{xy} = G\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx}$$

E= elastic modulus

ν = poisson's ratio

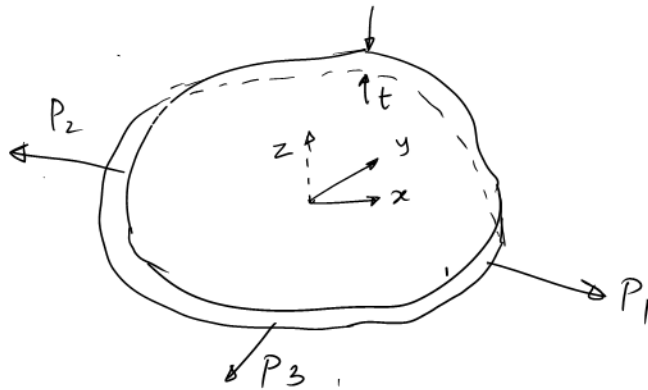**G= shear modulus**

$$G = \frac{E}{2(1+\upsilon)}$$

Hooke's law in 2D

Plane Stress Problem

A thin plate subject to in plane (x-y) loading

Assumptions:

$$\sigma_z = 0; \tau_{xz} = 0; \tau_{yz} = 0$$

So $\gamma_{xz} = \dfrac{\tau_{xz}}{G} = 0, \quad \gamma_{yz} = \dfrac{\tau_{yz}}{G} = 0$

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E}$$

$$\varepsilon_z = -\upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\sigma_x = \frac{E}{(1-\upsilon^2)}\left[\varepsilon_x + \upsilon\varepsilon_y\right]$$

$$\sigma_y = \frac{E}{(1-\upsilon^2)}\left[\varepsilon_y + \upsilon\varepsilon_x\right]$$
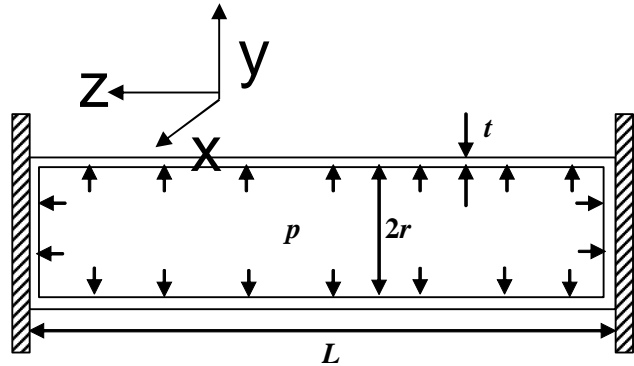
$$\tau_{xy} = G\gamma_{xy}$$

Fig. 1

A cylindrical pressure tube constrained between rigid walls

Hooke's law

Plane Strain Problem

Assumptions:

$$\varepsilon_z = 0; \gamma_{xz} = 0; \gamma_{yz} = 0$$

So $\tau_{xz} = G\gamma_{xz} = 0, \quad \tau_{yz} = G\gamma_{yz} = 0$

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_z}{E}$$

$$0 = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\sigma_x = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_x + \upsilon\varepsilon_y\right]$$

$$\sigma_y = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_y + \upsilon\varepsilon_x\right]$$
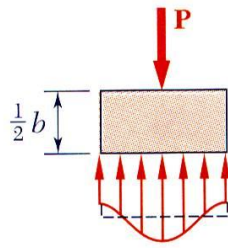
$$\sigma_z = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[\upsilon(\varepsilon_x + \varepsilon_y)\right]$$
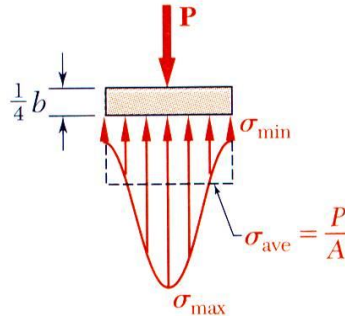
$$\tau_{xy} = G\gamma_{xy}$$

Saint-Venant's Principle
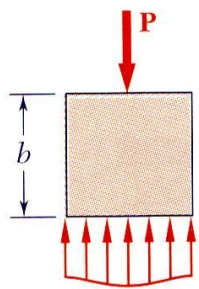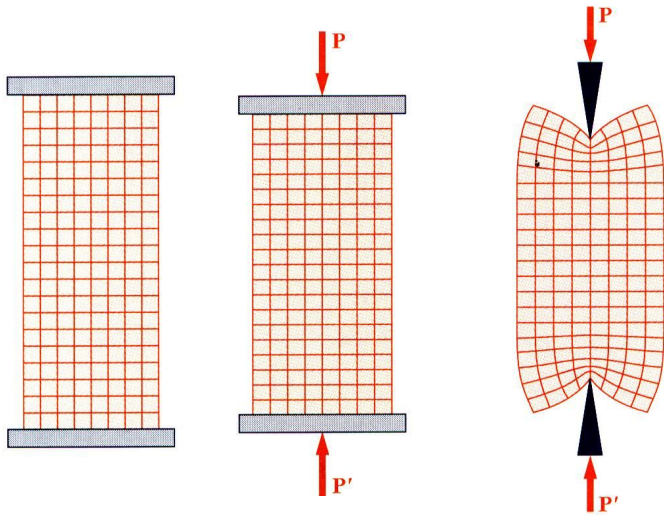
$$\sigma_{min} = 0.973\sigma_{ave}$$
$$\sigma_{max} = 1.027\sigma_{ave}$$

$$\sigma_{min} = 0.668\sigma_{ave}$$
$$\sigma_{max} = 1.387\sigma_{ave}$$

$$\sigma_{min} = 0.198\sigma_{ave}$$
$$\sigma_{max} = 2.575\sigma_{ave}$$

- Loads transmitted through rigid plates result in uniform distribution of stress and strain.

- Concentrated loads result in large stresses in the vicinity of the load application point.

- Stress and strain distributions become uniform at a relatively short distance from the load application points (plate width is b).

- **Saint-Venant's Principle:** Stress distribution may be assumed independent of the method of load application except in the immediate vicinity of load application points.

- Note: Loads used to find far away stresses should be statically eqvt.
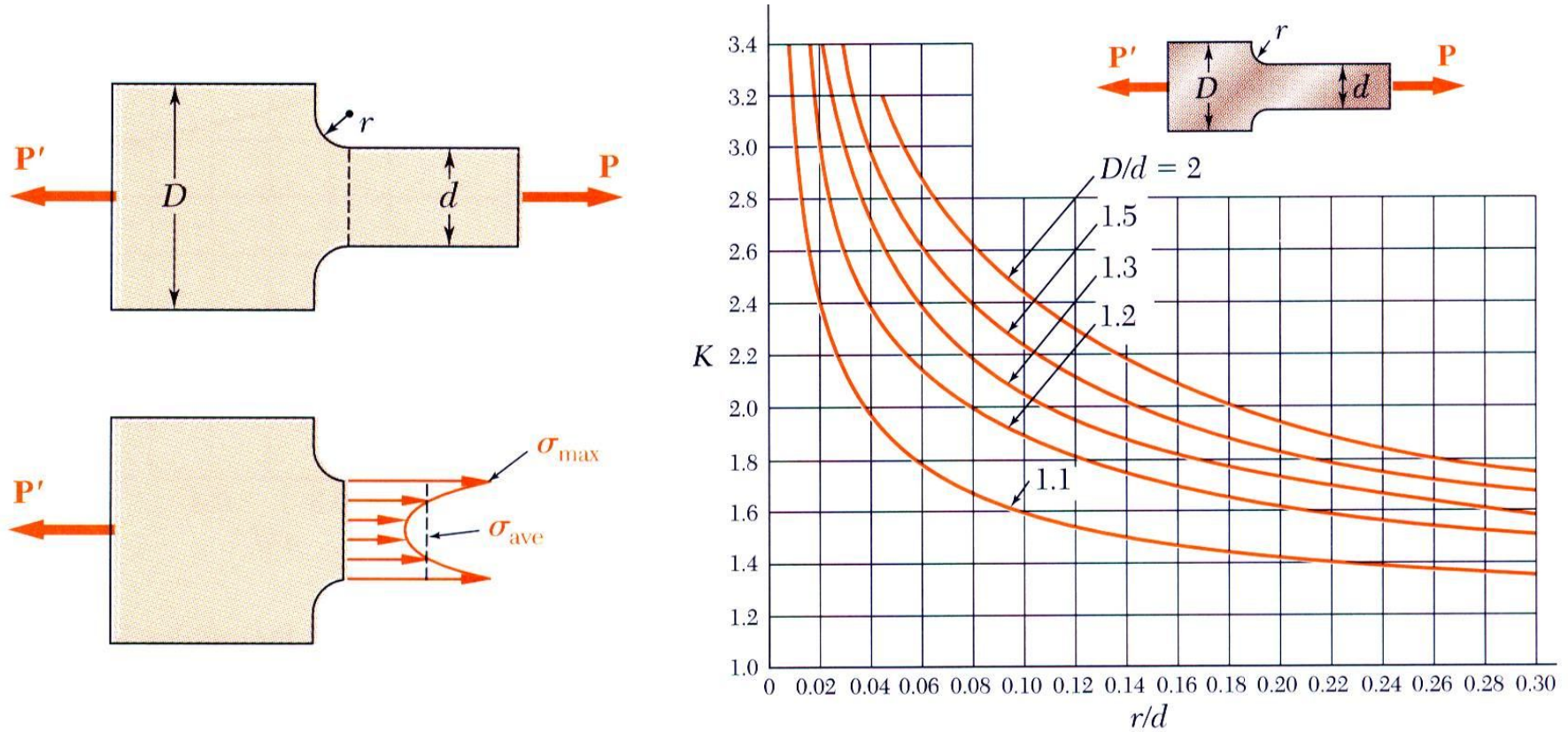
Stress Concentration: Hole
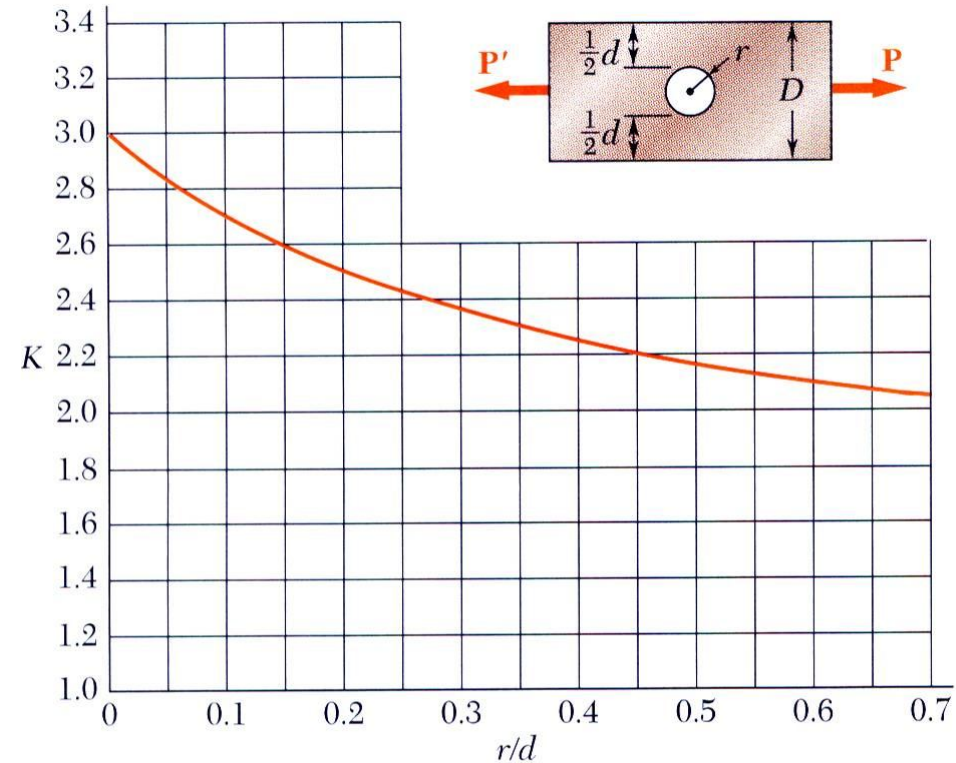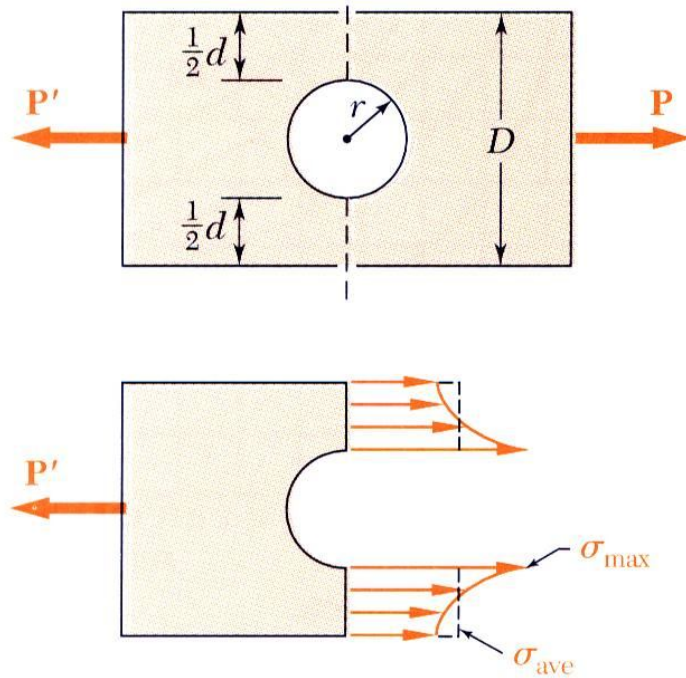
(a) Flat bars with holes

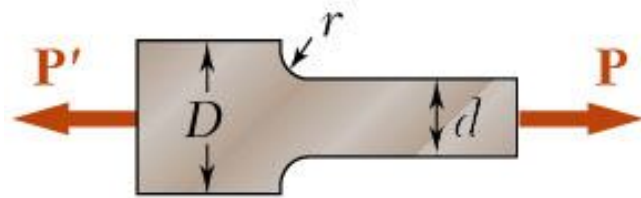Discontinuities of cross section may result in high localized or *concentrated* stresses.

$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

Stress Concentration: Fillet

(b) Flat bars with fillets

Example 2.7

Determine the largest axial load P that can be safely supported by a flat steel bar consisting of two portions, both 10 mm thick, and respectively 40 and 60 mm wide, connected by fillets of radius $r = 8$ mm. Assume an allowable normal stress of 165 MPa.

- Determine the geometric ratios and find the stress concentration factor from Fig. 2.64b.

$$\frac{D}{d} = \frac{60\,\text{mm}}{40\,\text{mm}} = 1.50 \qquad \frac{r}{d} = \frac{8\,\text{mm}}{40\,\text{mm}} = 0.20$$

$$K = 1.82$$

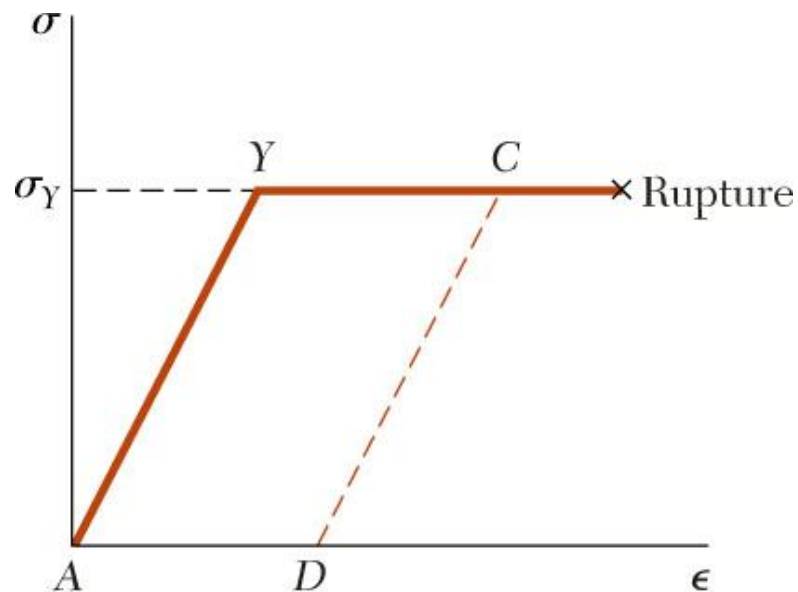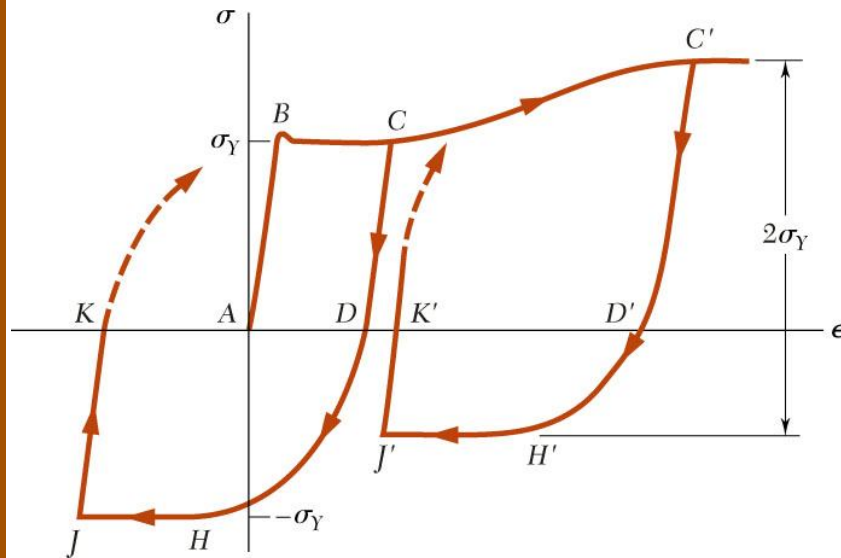- Find the allowable average normal stress using the material allowable normal stress and the stress concentration factor.

$$\sigma_{ave} = \frac{\sigma_{max}}{K} = \frac{165\,\text{MPa}}{1.82} = 90.7\,\text{MPa}$$

- Apply the definition of normal stress to find the allowable load.

$$P = A\sigma_{ave} = (40\,\text{mm})(10\,\text{mm})(90.7\,\text{MPa})$$

$$= 36.3 \times 10^3\,\text{N}$$

$$\boxed{P = 36.3\,\text{kN}}$$

Elastoplastic Materials

- So far analyses based on assumption of linear stress-strain relationship, i.e., stresses below the yield stress
- Assumption good for brittle materials which rupture without yielding
- If yield stress of ductile material is exceeded, plastic deformation occurs

- Analysis of plastic deformations is simplified by assuming an idealized *elastoplastic material (elastic-perfectly plastic)*
- Deformations of an elastoplastic material are divided into elastic and plastic ranges
- Permanent deformations result from loading beyond the yield stress

Plastic Deformations

$$P = \sigma_{ave} A = \frac{\sigma_{max} A}{K}$$

(a)

$$P_Y = \frac{\sigma_Y A}{K}$$

(b)

(c)

$$P_U = \sigma_Y A$$
$$= K P_Y$$

(d)
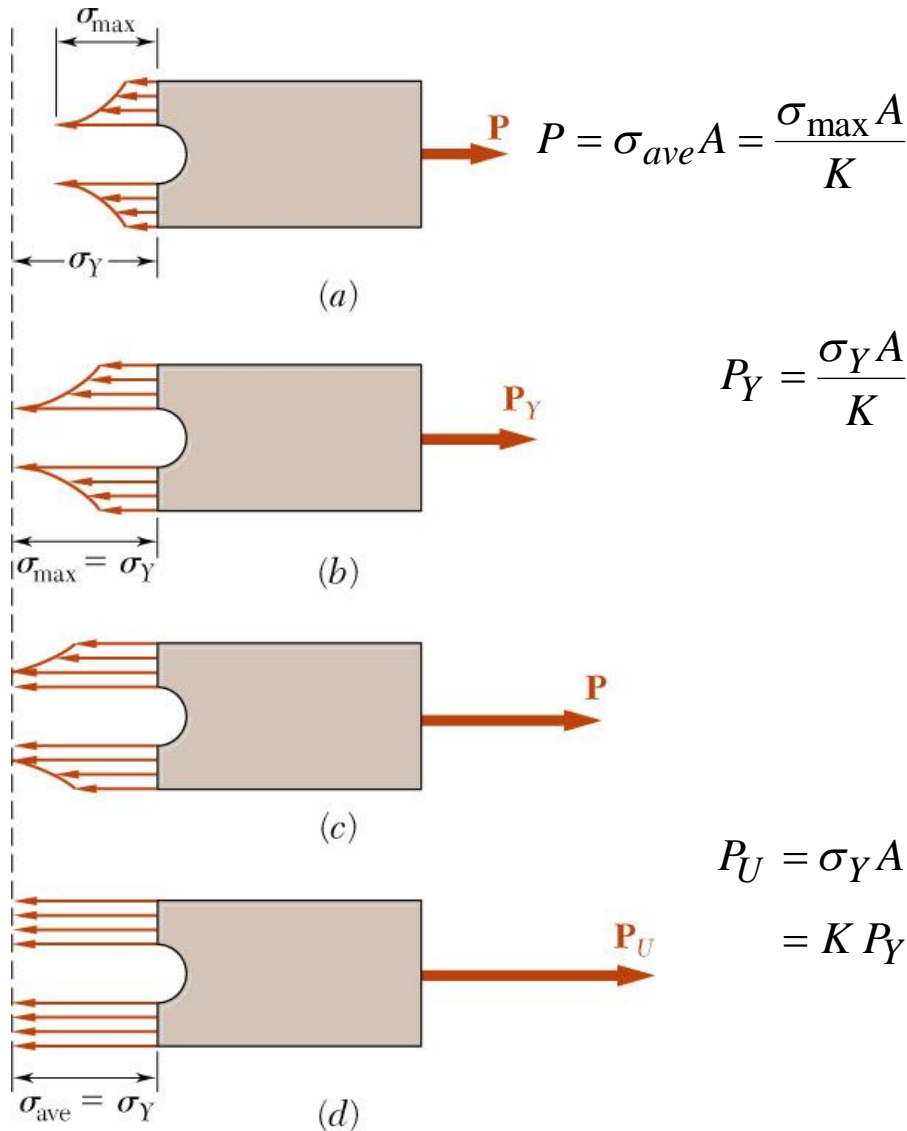
- Elastic deformation while maximum stress is less than yield stress

- Maximum stress is equal to the yield stress at the maximum elastic loading

- At loadings above the maximum elastic load, a region of plastic deformations develop near the hole

- As the loading increases, the plastic region expands until the section is at a uniform stress equal to the yield stress

MECHANICS OF MATERIALS

Residual Stresses

- When a <u>single</u> structural element is loaded uniformly beyond its yield stress and then unloaded, it is permanently deformed but all stresses disappear. This is not so for a <u>system</u>.

- *Residual stresses* will remain in a structure after loading and unloading if
  - only part of the structure undergoes plastic deformation
  - different parts of the structure undergo different plastic deformations

- Residual stresses also result from the uneven heating or cooling of structures or structural elements

Example 2.8

Cylindrical rod placed inside tube of same length. Ends of rod and tube are attached to a rigid support on one side and a rigid plate on the other. Load on the assembly increased from 0 to 25 kN and decreased back to 0.

a) draw load-deflection diagram for assembly
b) find max elongation
c) find permanent set
d) calculate residual stresses in rod and tube.

Tube

Plate

Rod

P

0.75 m
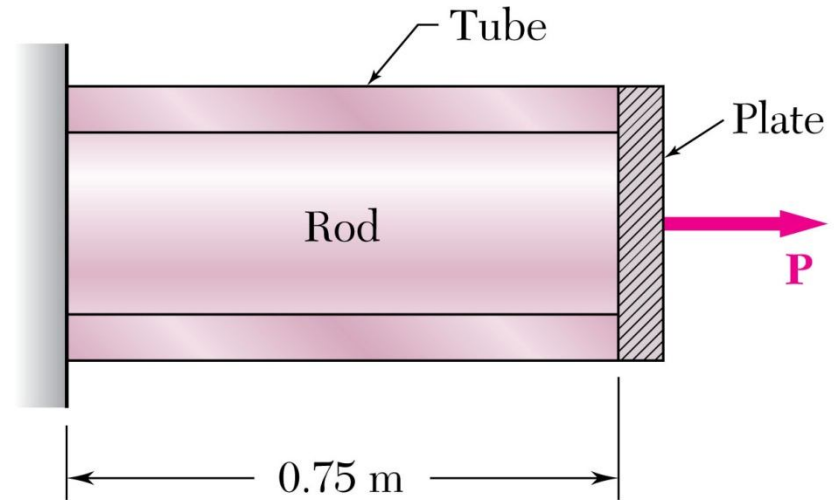
$A_r = 48$ mm^2 $A_t = 62$ mm^2

$E_r = 210$ GPa $E_t = 105$ GPa

$(\sigma_r)_Y = 250$ MPa $(\sigma_t)_Y = 310$ MPa