

Vertical Curve

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Profiles:

Curve a: Crest Vertical Curve (concave downward)

Curve b: Sag Vertical Curve (concave upward)

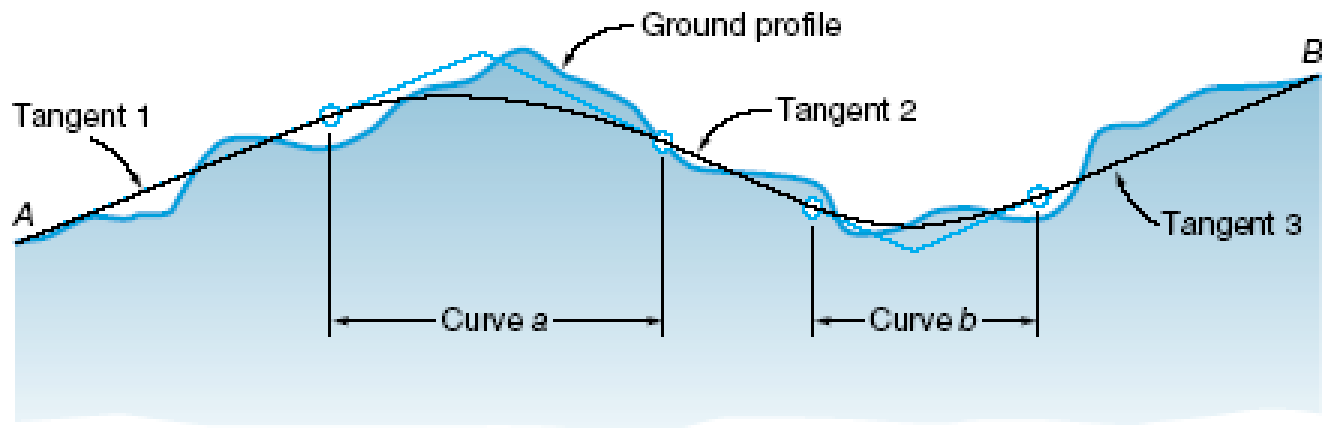


Figure 25-1 Grade line and ground profile of a proposed highway section.

Tangents: Constant Grade (Slope)

Equal-Tangent Vertical Parabolic Curve:

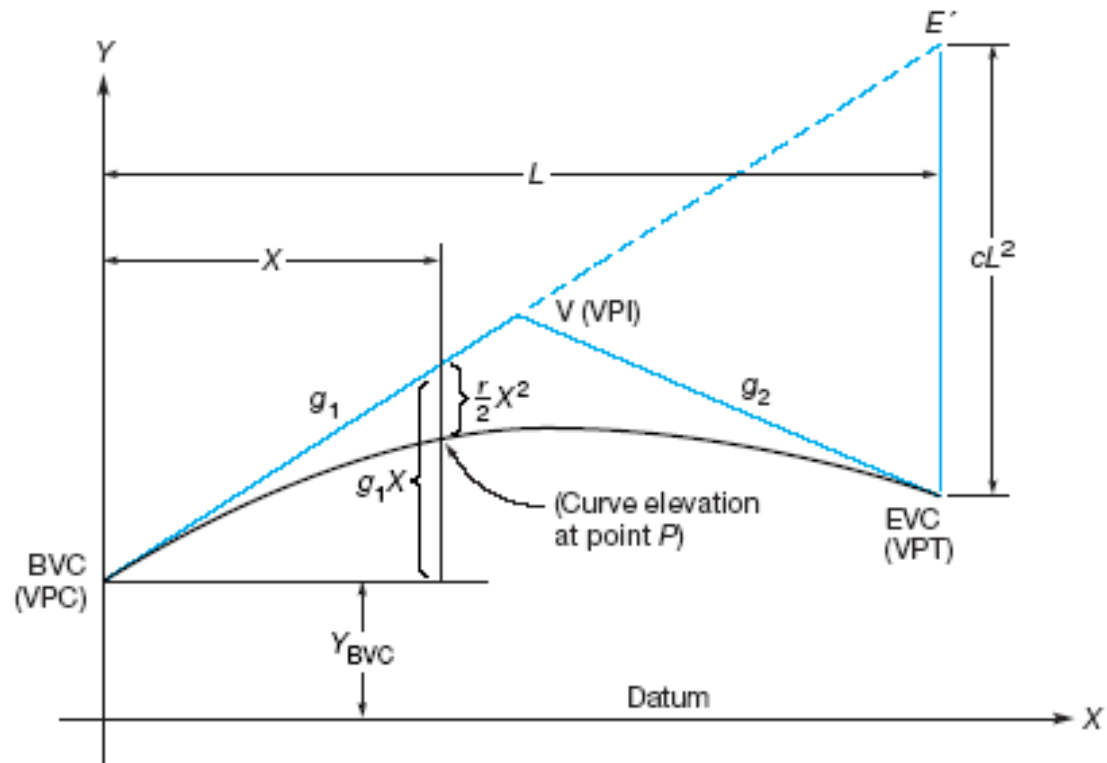


Figure 25-3 Vertical parabolic curve relationships.

Equations:

$$r = (g_2 - g_1)/L$$

where:

g_2 & g_1 - in percent (%)

L – in stations

and

$$Y = Y_{BVC} + g_1x + (r/2)x^2$$

where:

Y_{BVC} – elevation of the BVC in feet

Example: Equal-Tangent Vertical Curve

Given the information show below, compute and tabulate the curve for stakeout at full 100' stations.

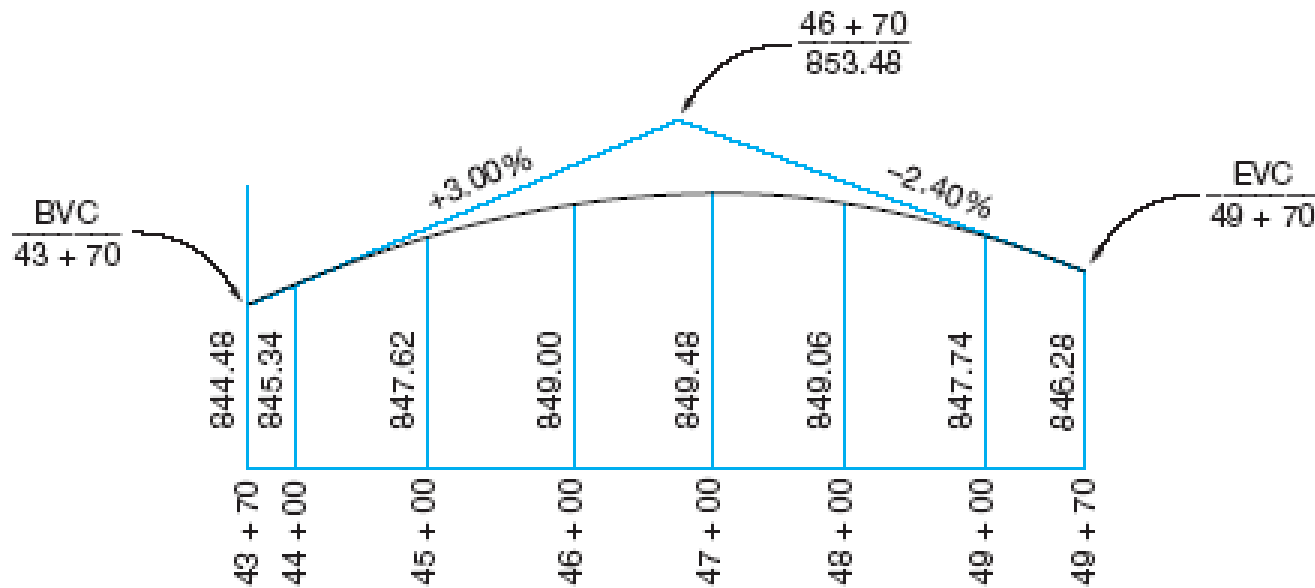


Figure 25-4 Crest curve of Example 25-1.

Solution:

$$L = STA_{EVC} - STA_{BVC}$$

$$L = 4970 - 4370 = 600'$$

or 6 *full* stations

$$r = (g_2 - g_1) / L$$

$$r = (-2.4 - 3) / 6$$

$$r = -0.90$$

$$r/2 = -0.45 \% \text{ per station}$$

$$STA_{BVC} = STA_{\text{Vertex}} - L / 2 = 4670 - 600/2 = STA_{BVC} = STA 43 + 70$$

$$STA_{EVC} = STA_{\text{Vertex}} + L / 2 = 4670 + 600/2 = STA_{EVC} = STA 49 + 70$$

$$Elev_{BVC} = Elev_{\text{vertex}} - g_1 (L/2) = 853.48 - 3.00 (3) = 844.48'$$

$$Elev_{EVC} = Elev_{\text{vertex}} - g_2 (L/2) = 853.48 - 2.40 (3) = 846.28'$$

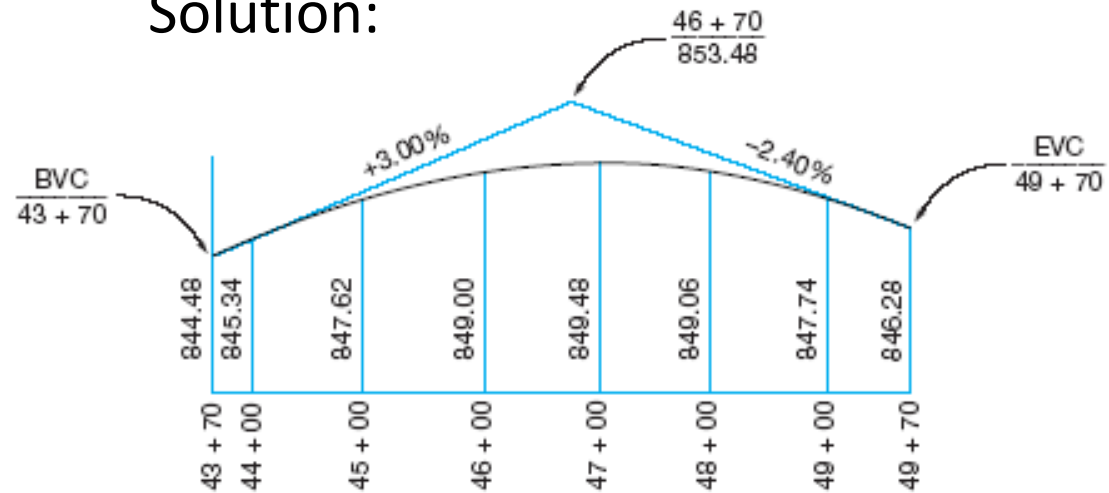


Figure 25-4 Crest curve of Example 25-1.

Solution:
(continued)

$r/2 = -0.45$ % per station

$$\text{Elev}_x = \text{Elev}_{\text{BVC}} + g_1x + (r/2)x^2$$

$$\text{Elev}_{44+00} = 844.48 + 3.00(0.30) - 0.45(0.30)^2 = 845.34'$$

$$\text{Elev}_{45+00} = 844.48 + 3.00(1.30) - 0.45(1.30)^2 = 847.62'$$

$$\text{Elev}_{46+00} = 844.48 + 3.00(2.30) - 0.45(2.30)^2 = 849.00'$$

etc.

$$\text{Elev}_{49+00} = 844.48 + 3.00(5.30) - 0.45(5.30)^2 = 847.74'$$

$$\text{Elev}_{49+70} = 844.48 + 3.00(6.00) - 0.45(6.00)^2 = 846.28' \text{ (CHECKS)}$$

Solution:
(continued)

Station	x (stations)	g_1x	$r/2 x^2$	Curve Elevation
43 + 70 BVC	0.0	0.00	0.00	844.48
44 + 00	0.3	.90	-0.04	845.34
45 + 00	1.3	3.90	-0.76	847.62
46 + 00	2.3	6.90	-2.38	849.00
47 + 00	3.3	9.90	-4.90	849.48
48 + 00	4.3	12.90	-8.32	849.06
49 + 00	5.3	15.90	-2.64	847.74
49 + 70 EVC	6.0	18.00	-6.20	<u>846.28</u>

High and Low Points on Vertical Curves

Sag Curves:

Low Point defines location of catch basin for drainage.

Crest Curves:

High Point defines limits of drainage area for roadways.

Also used to determine or set elevations based on minimum clearance requirements.

Equation for High or Low Point on a Vertical Curve:

$$y = y_{BVC} + g_1x + (r/2)x^2$$

Set $dy/dx = 0$ and solve for x to locate turning point

$$0 = 0 + g_1 + r x$$

Substitute $(g_2 - g_1) / L$ for r

$$-g_1 = x (g_2 - g_1) / L$$

$$-g_1 L = x (g_2 - g_1)$$

$$x = (-g_1 L) / (g_2 - g_1)$$

or

$$x = (g_1 L) / (g_1 - g_2) = g_1/r$$

x – distance from BVC to HP or LP

Example: High Point on a Crest Vertical Curve

From previous example:

$$g_1 = + 3 \%, g_2 = - 2.4\%, L = 600' = 6 \text{ full stations}, r/2 = - 0.45,$$

$$\text{Elev}_{\text{BVC}} = 844.48'$$

$$x = (g_1 L) / (g_1 - g_2)$$

$$x = (3)(6) / (3 + 2.4) = 3.3333 \text{ stations or } 333.33'$$

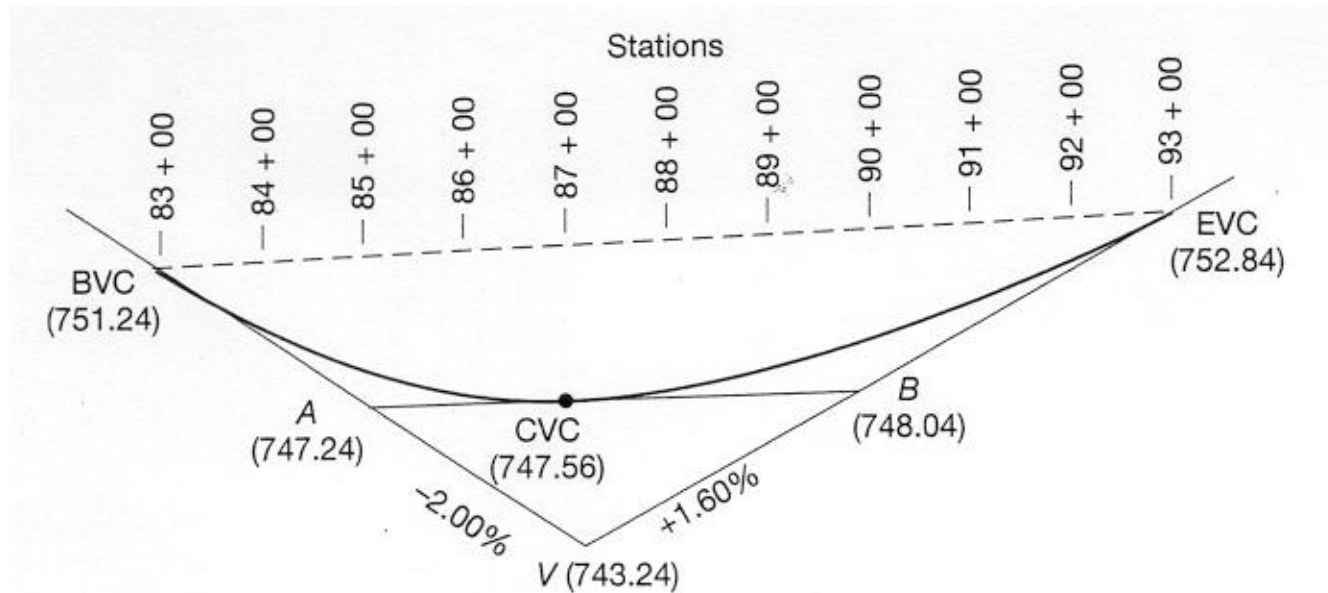
$$\text{HP STA} = \text{BVC STA} + x$$

$$\text{HP STA} = 4370 + 333.33 = \text{HP STA } 47 + 03.33$$

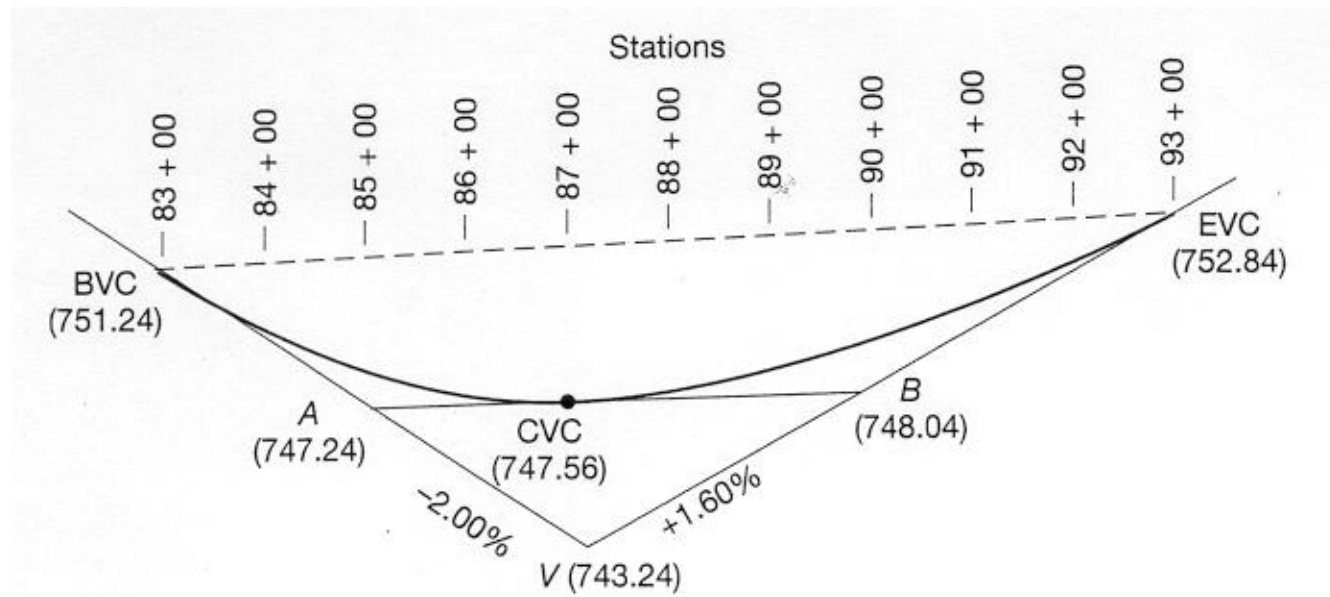
$$\text{ELEV}_{\text{HP}} = 844.48 + 3.00(3.3333) - 0.45(3.3333)^2 = 849.48'$$

Check table to see if the computed elevation is reasonable!

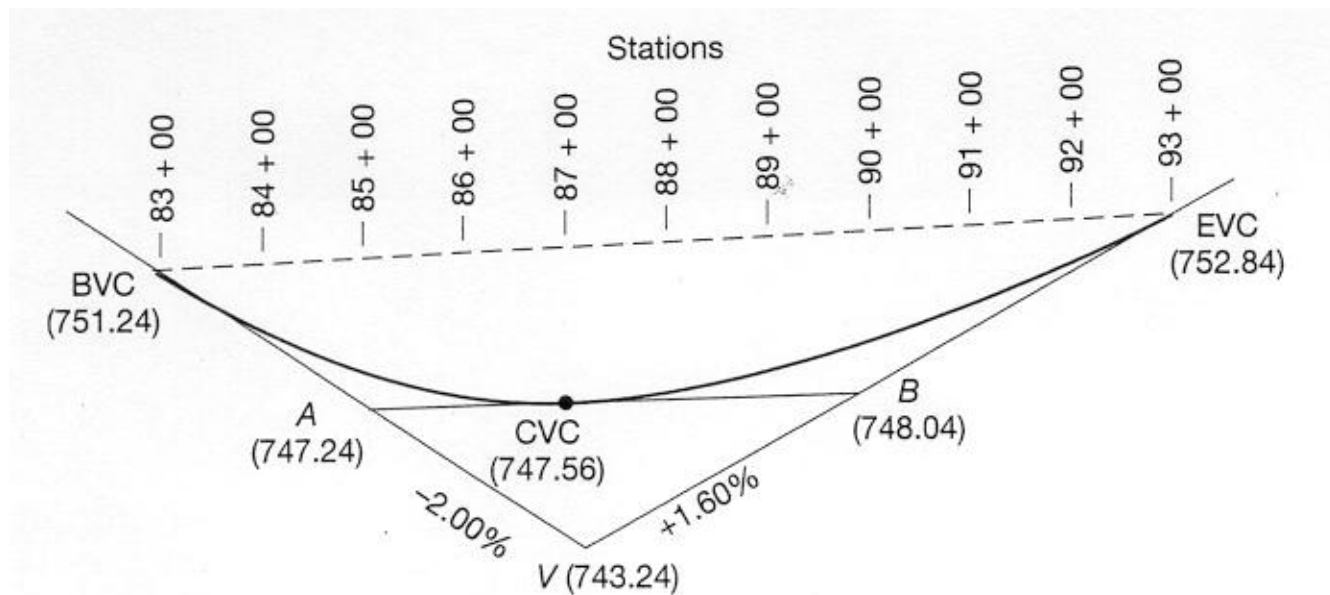
Unequal-Tangent Parabolic Curve



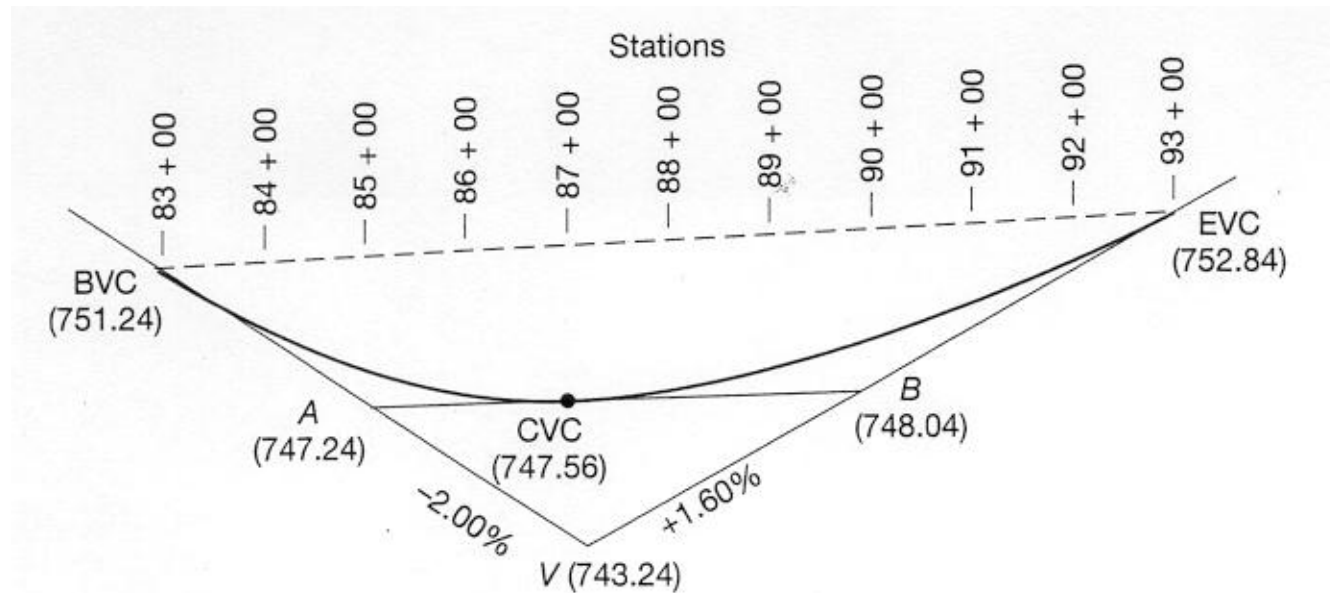
Solution:



Solution (continued):



Solution (continued):



Computation of values for g_1x and g_2x

	<u>STATION</u>	<u>x</u>	<u>g_1x</u>	<u>$(r/2)x^2$</u>	<u>Curve Elevation</u>
BVC	83 + 00	0	0	0	751.24'
	84 + 00	1	-2.00		
	85 + 00	2			
	86 + 00	3			
CVC	87 + 00	4			747.56'
	88 + 00	1	0.16		
	89 + 00	2			
	90 + 00	3			
	91 + 00	4			
	92 + 00	5			
EVC	93 + 00	6			
			$g_1x = -2 (1) = -2.00$		
			$g_2x = .16(1) = 0.16$		

Computation of values for $(r_1/2)x^2$ and $(r_2/2)x^2$

	<u>STATION</u>	<u>x</u>	<u>g₁x</u>	<u>(r/2)x²</u>	<u>Curve Elevation</u>
BVC	83 + 00	0	0	0	751.24'
	84 + 00	1	-2.00	0.27	
	85 + 00	2	-4.00		
	86 + 00	3	-6.00		
CVC	87 + 00	4	-8.00		747.56'
	88 + 00	1	0.16	0.12	
	89 + 00	2	0.32		
	90 + 00	3	0.48		
	91 + 00	4	0.64		
	92 + 00	5	0.80		
EVC	93 + 00	6	0.96		
				$(r_1/2)x^2 = (0.27)(1)^2 = 0.27$	
				$(r_2/2)x^2 = (0.12)(1)^2 = 0.12$	

Elevation Computations for both Vertical Curves

	<u>STATION</u>	<u>x</u>	<u>g₁x</u>	<u>(r/2)x²</u>	<u>Curve Elevation</u>
BVC	83 + 00	0	0	0	751.24'
	84 + 00	1	-2.00	0.27	
	85 + 00	2	-4.00	1.08	
	86 + 00	3	-6.00	2.43	
CVC	87 + 00	4	-8.00	4.32	747.56'
	88 + 00	1	0.16	0.12	
	89 + 00	2	0.32	0.48	
	90 + 00	3	0.48	1.08	
	91 + 00	4	0.64	1.92	
	92 + 00	5	0.80	3.00	
EVC	93 + 00	6	0.96	4.32	
$Y_1 = 751.24 - 2.00 + 0.27 = 749.51'$					
$Y_2 = 747.56 + 0.16 + 0.12 = 747.84'$					

Designing a Curve to Pass Through a Fixed Point

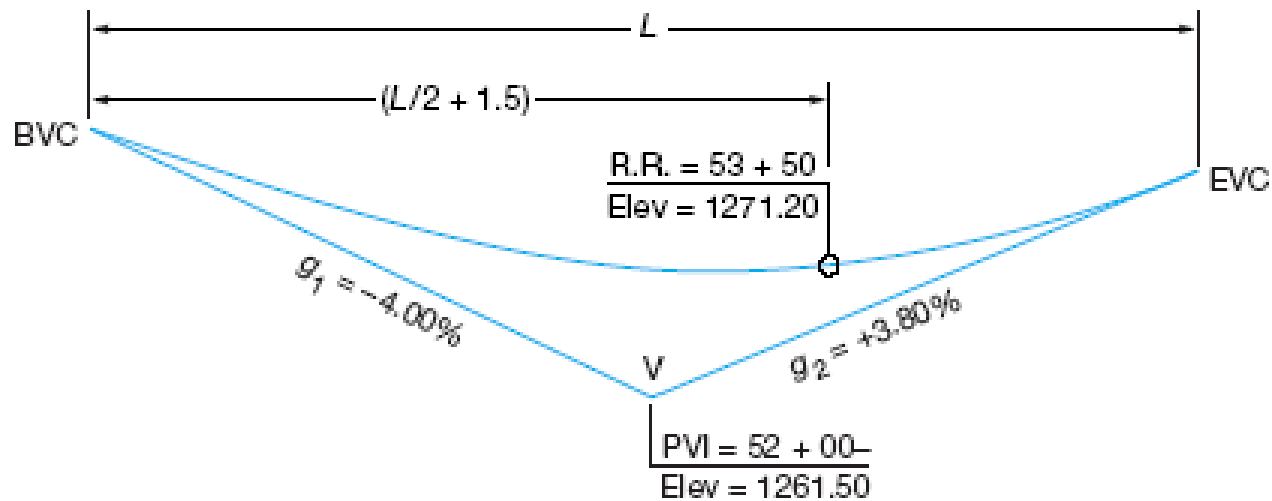


Figure 25-8 Designing a parabolic curve to pass through a fixed point.

Solution:

$$x = \frac{L}{2} + (5350 - 5200) = \frac{L}{2} + 150' = \frac{L}{2} + 1.5 \text{ stations}$$

$$y = y_{BVC} + g_1 x + \frac{r}{2} x^2$$

$$r = \frac{g_2 - g_1}{L}$$

$$Y_{BVC} = 1261.50 + 4.00 \left(\frac{L}{2} \right)$$

$$g_1 x = -4.00 x = -4.00 \left(\frac{L}{2} + 1.5 \right)$$

$$r = \frac{3.80 + 4.00}{L}$$

$$\frac{r}{2} x^2 = \frac{3.80 + 4.00}{2L} \left(\frac{L}{2} + 1.5 \right)^2$$

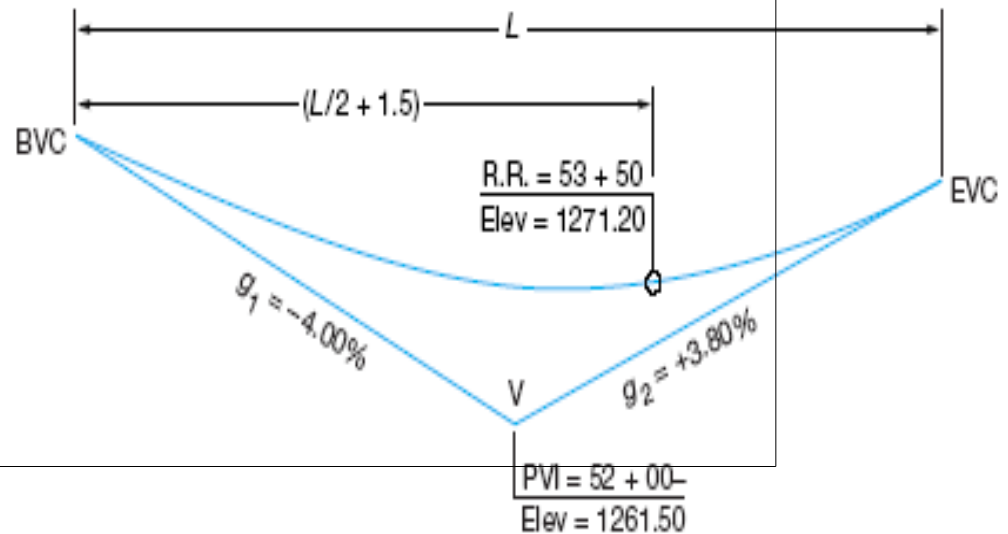


Figure 25-8 Designing a parabolic curve to pass through a fixed point.

Solution (continued):

$$1271.20 = \left[1261.50 + 4.00 \left(\frac{L}{2} \right) \right] + \left[-4.00 \left(\frac{L}{2} + 1.5 \right) \right] + \left[\frac{3.80 + 4.00}{2L} \left(\frac{L}{2} + 1.5 \right)^2 \right]$$

$$0.975L^2 - 9.85L + 8.775 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 0.975$$

$$b = -9.85$$

$$c = 8.775$$

$$L = 9.1152 \text{ stations} = 911.52'$$

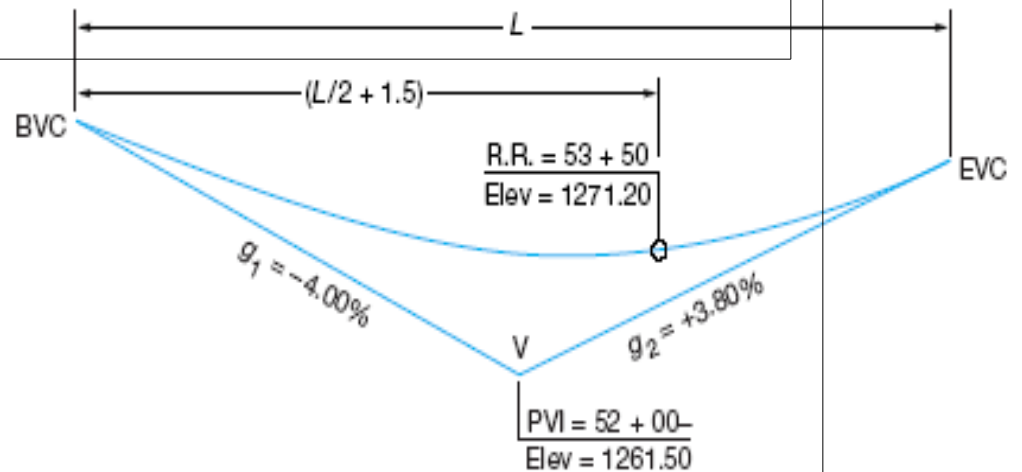


Figure 25-8 Designing a parabolic curve to pass through a fixed point.

Sight Distance

- Defined as “the distance required, for a given design speed to safely stop a vehicle thus avoiding a collision with an unexpected stationary object in the roadway ahead” by AASHTO (American Association of State Highway and Transportation Officials)
- Types
 - Stopping Sight Distance
 - Passing Sight Distance
 - Decision Sight Distance
 - Horizontal Sight Distance

Sight Distance Equations

For Crest Curves

For Sag Curves

$$S \leq L$$

$$L = \frac{S^2 (g_2 - g_1)}{4 + 3.5S}$$

$$S \geq L$$

$$L = 2S - \frac{4 + 3.5S}{g_1 - g_2}$$

