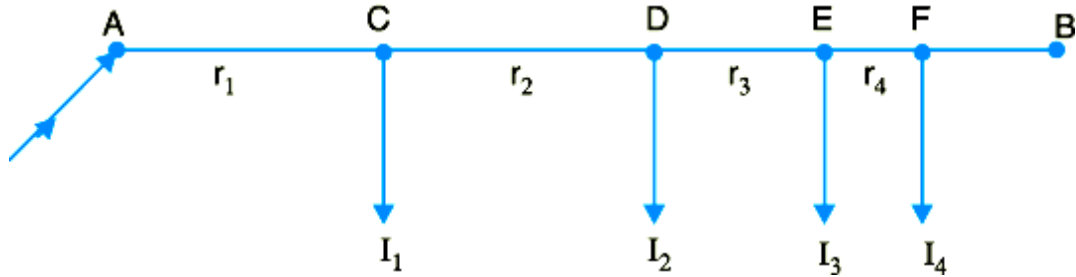


D.C. DISTRIBUTOR FED AT ONE END — CONCENTRATED LOADING

Fig. shows the single line diagram of a 2-wire d.c. distributor A B fed at one end A and having concentrated loads I_1 , I_2 , I_3 and I_4 tapped off at points C, D, E and F respectively.

Let r_1 , r_2 , r_3 and r_4 be the resistances of both wires (go and return) of the sections A C, CD, DE and EF of the distributor respectively



$$\begin{aligned}
\text{Current fed from point } A &= I_1 + I_2 + I_3 + I_4 \\
\text{Current in section } AC &= I_1 + I_2 + I_3 + I_4 \\
\text{Current in section } CD &= I_2 + I_3 + I_4 \\
\text{Current in section } DE &= I_3 + I_4 \\
\text{Current in section } EF &= I_4 \\
\text{Voltage drop in section } AC &= r_1 (I_1 + I_2 + I_3 + I_4) \\
\text{Voltage drop in section } CD &= r_2 (I_2 + I_3 + I_4) \\
\text{Voltage drop in section } DE &= r_3 (I_3 + I_4) \\
\text{Voltage drop in section } EF &= r_4 I_4
\end{aligned}$$

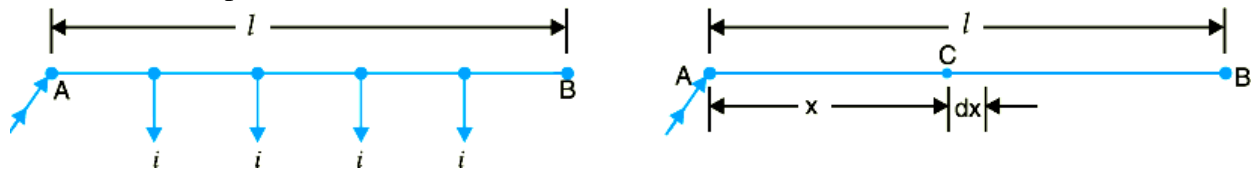
∴ Total voltage drop in the distributor

$$= r_1 (I_1 + I_2 + I_3 + I_4) + r_2 (I_2 + I_3 + I_4) + r_3 (I_3 + I_4) + r_4 I_4$$

It is easy to see that the minimum potential will occur at point *F* which is farthest from the feeding point *A*.

UNIFORMLY LOADED DISTRIBUTOR FED AT ONE END

Fig shows the single line diagram of a 2-wire d.c. distributor *AB* fed at one end *A* and loaded uniformly with *i* amperes per metre length. It means that at every 1 m length of the distributor, the load tapped is *i* amperes. Let *l* metres be the length of the distributor and *r* ohm be the resistance per metre run.



Consider a point *C* on the distributor at a distance *x* metres from the feeding point *A* as shown in Fig. Then current at point *C* is

$$= i l - i x \text{ amperes} = i (l - x) \text{ amperes}$$

Now, consider a small length *dx* near point *C*. Its resistance is *r dx* and the voltage drop over length *dx* is $d v = i (l - x) r dx = i r (l - x) dx$ Total voltage drop in the distributor upto point *C* is

$$v = \int_0^x i r (l - x) dx = i r \left(l x - \frac{x^2}{2} \right)$$

The voltage drop upto point *B* (i.e. over the whole distributor) can be obtained by putting $x = l$ in the above expression.

∴ Voltage drop over the distributor *AB*

$$\begin{aligned}
 &= i r \left(l \times l - \frac{l^2}{2} \right) \\
 &= \frac{1}{2} i r l^2 = \frac{1}{2} (i l) (r l) \\
 &= \frac{1}{2} I R
 \end{aligned}$$

where

$i l = I$, the total current entering at point A

$r l = R$, the total resistance of the distributor

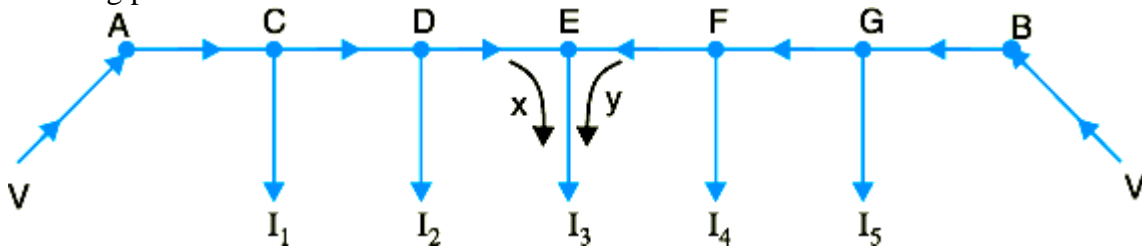
Thus, in a uniformly loaded distributor fed at one end, the total voltage drop is equal to that produced by the whole of the load assumed to be concentrated at the middle point.

DISTRIBUTOR FED AT BOTH ENDS — CONCENTRATED LOADING

Whenever possible, it is desirable that a long distributor should be fed at both ends instead of at one end only, since total voltage drop can be considerably reduced without increasing the cross-section of the conductor. The two ends of the distributor may be supplied with (i) equal voltages (ii) unequal voltages.

(i) **Two ends fed with equal voltages.** Consider a distributor A B fed at both ends with equal voltages V volts and having concentrated loads I_1, I_2, I_3, I_4 and I_5 at points C, D, E, F and G respectively as shown in Fig. As we move away from one of the feeding points, say A, p.d. goes on decreasing till it reaches the minimum value at some load point, say E, and then again starts rising and becomes V volts as we reach the other feeding point B.

All the currents tapped off between points A and E (minimum p.d. point) will be supplied from the feeding point A while those tapped off between B and E will be supplied from the feeding point B.



The current tapped off at point E itself will be partly supplied from A and partly from B. If these currents are x and y respectively, then,

$$I_3 = x + y$$

Therefore, we arrive at a very important conclusion that at the point of minimum potential, current comes from both ends of the distributor.

Point of minimum potential.

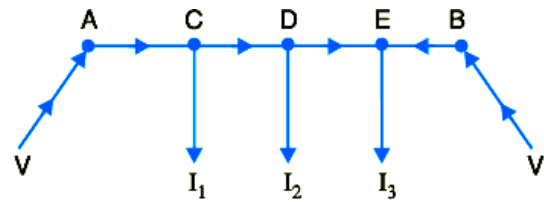
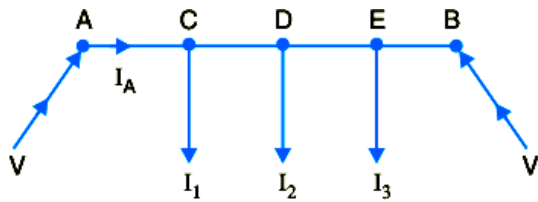
It is generally desired to locate the point of minimum potential. There is a simple method for it. Consider a distributor A B having three concentrated loads I_1, I_2 and I_3 at points C, D and E respectively. Suppose that current supplied by feeding end A is I_a . Then current distribution in the various sections of the distributor can be worked out as shown in Fig.

$$I_{AC} = I_A;$$

$$I_{DE} = I_A - I_1 - I_2;$$

$$I_{CD} = I_A - I_1$$

$$I_{EB} = I_A - I_1 - I_2 - I_3$$



Voltage drop between A and B = Voltage drop over A B

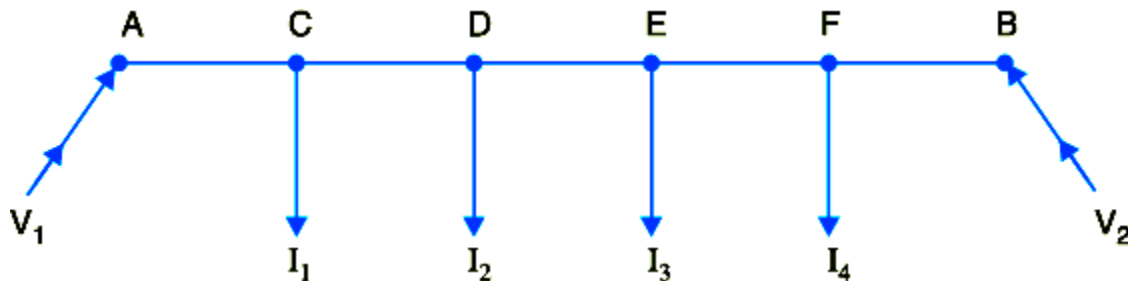
$$\text{or } V - V = I_A R_{AC} + (I_A - I_1) R_{CD} + (I_A - I_1 - I_2) R_{DE} + (I_A - I_1 - I_2 - I_3) R_{EB}$$

From this equation, the unknown I_A can be calculated as the values of other quantities are generally given. Suppose actual directions of currents in the various sections of the distributor are indicated as shown in Fig. The load point where the currents are coming from both sides of the distributor is the point of minimum potential i.e. point E in this case

(ii) Two ends fed with unequal voltages.

Fig. shows the distributor A B fed with unequal voltages end A being fed at V_1 volts and end B at V_2 volts. The point of minimum potential can be found by following the same procedure as discussed above. Thus in this case, Voltage drop between A and B = Voltage drop over A B

$$V_1 - V_2 = \text{Voltage drop over A B}$$



UNIFORMLY LOADED DISTRIBUTOR FED AT BOTH ENDS

We shall now determine the voltage drop in a uniformly loaded distributor fed at both ends. There can be two cases viz. the distributor fed at both ends with (i) equal voltages (ii) unequal voltages. The two cases shall be discussed separately.

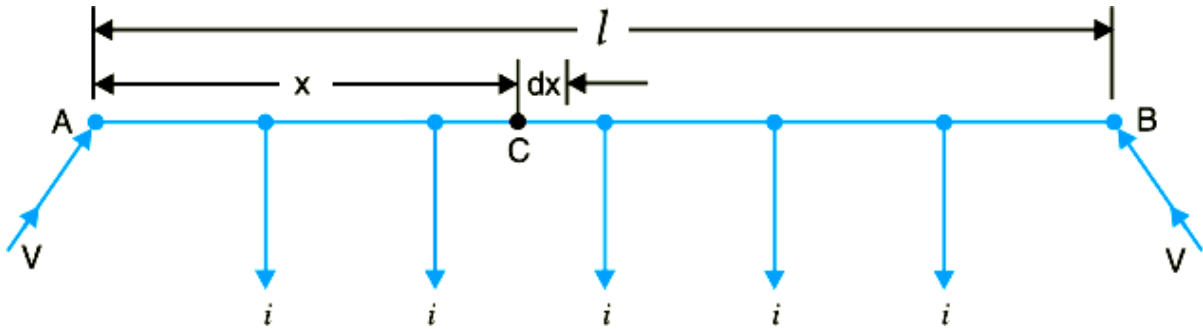
(i) Distributor fed at both ends with equal voltages.

Consider a distributor A B of length l metres, having resistance r ohms per metre run and with uniform loading of i amperes per metre run as shown in Fig. 13.24. Let the distributor be fed at the feeding points A and B at equal voltages, say V volts. The total current supplied to the

distributor is $i l$. As the two end voltages are equal, therefore, current supplied from each feeding point is $i l/2$ i.e.

Current supplied from each feeding point

$$= \frac{i l}{2}$$



Consider a point C at a distance x metres from the feeding point A. Then current at point C is $i l$

$$= \frac{i l}{2} - i x = i \left(\frac{l}{2} - x \right)$$

Now, consider a small length dx near point C . Its resistance is $r dx$ and the voltage drop over length dx is

$$dv = i \left(\frac{l}{2} - x \right) r dx = i r \left(\frac{l}{2} - x \right) dx$$

$$\begin{aligned} \therefore \text{Voltage drop upto point } C &= \int_0^x i r \left(\frac{l}{2} - x \right) dx = i r \left(\frac{l x}{2} - \frac{x^2}{2} \right) \\ &= \frac{i r}{2} (l x - x^2) \end{aligned}$$

Obviously, the point of minimum potential will be the mid-point. Therefore, maximum voltage drop will occur at mid-point i.e. where $x = l/2$.

$$\begin{aligned} \therefore \text{Max. voltage drop} &= \frac{i r}{2} (l x - x^2) \\ &= \frac{i r}{2} \left(l \times \frac{l}{2} - \frac{l^2}{4} \right) && \text{[Putting } x = l/2\text{]} \\ &= \frac{1}{8} i r l^2 = \frac{1}{8} (i l) (r l) = \frac{1}{8} I R \end{aligned}$$

where

$i l = I$, the total current fed to the distributor from both ends

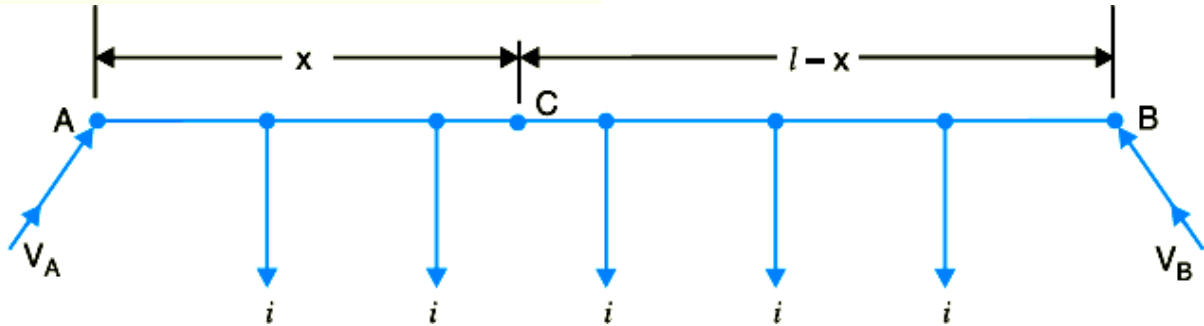
$r l = R$, the total resistance of the distributor

$$\text{Minimum voltage} = V - \frac{I R}{8} \text{ volts}$$

(ii) Distributor fed at both ends with unequal voltages.

Consider a distributor AB of length l metres having resistance r ohms per metre run and with a uniform loading of i amperes per metre run as shown in Fig. Let the distributor be fed from feeding points A and B at voltages V_A and V_B respectively. Suppose that the point of minimum potential C is situated at a distance x metres from the feeding point A. Then current supplied by the feeding point A will be $i x$.

$$\text{Voltage drop in section } AC = \frac{i r x^2}{2} \text{ volts}$$



As the distance of C from feeding point B is $(l-x)$, therefore, current fed from B is $i(l-x)$.

$$\therefore \text{Voltage drop in section } BC = \frac{i r (l-x)^2}{2} \text{ volts}$$

$$\text{Voltage at point C, } V_C = V_A - \text{Drop over } AC$$

$$= V_A - \frac{i r x^2}{2} \quad \dots(i)$$

$$\text{Also, voltage at point C, } V_C = V_B - \text{Drop over } BC$$

$$= V_B - \frac{i r (l-x)^2}{2} \quad \dots(ii)$$

From equations (i) and (ii), we get,

$$V_A - \frac{i r x^2}{2} = V_B - \frac{i r (l-x)^2}{2}$$

Solving the equation for x , we get,

$$x = \frac{V_A - V_B}{i r l} + \frac{l}{2}$$

As all the quantities on the right hand side of the equation are known, therefore, the point on the distributor where minimum potential occurs can be calculated.