

23-03-20

MODULE - 5 . RANDOM SIGNAL THEORY

⇒ Random variable → Probability → Random experiment.
⇒ Random process. → Sample space
→ event.

RANDOM EXPERIMENT

Tossing A COIN:-

Sample space (S) = { H, T }.

↓
Sample points / events.

EVENT

⇒ The ^{possible desired} ~~collected~~ outcomes of an experiment is called an event.

SAMPLE SPACE

⇒ The collections of all possible outcomes of an experiment is called sample space.

PROBABILITY OF UNCERTAINTY

⇒ Let an event be E which represents only head.
E = { E }. Then, probability of getting only.

S = { H, T }.

head

$P(E) = P(H) = \frac{1}{2}$ (because number of sample points is 2)

RANDOM EXPERIMENT

⇒ Random experiment means we will get

: unpredictable outcome of an experiment,

PROBABILITY

→ The probability of an event is defined as the ratio between the favourable number of faces to an event and the total number of outcomes of an experiment.

Example :- $P(S) = 1$ → Number of sure event
[probability of sample space]

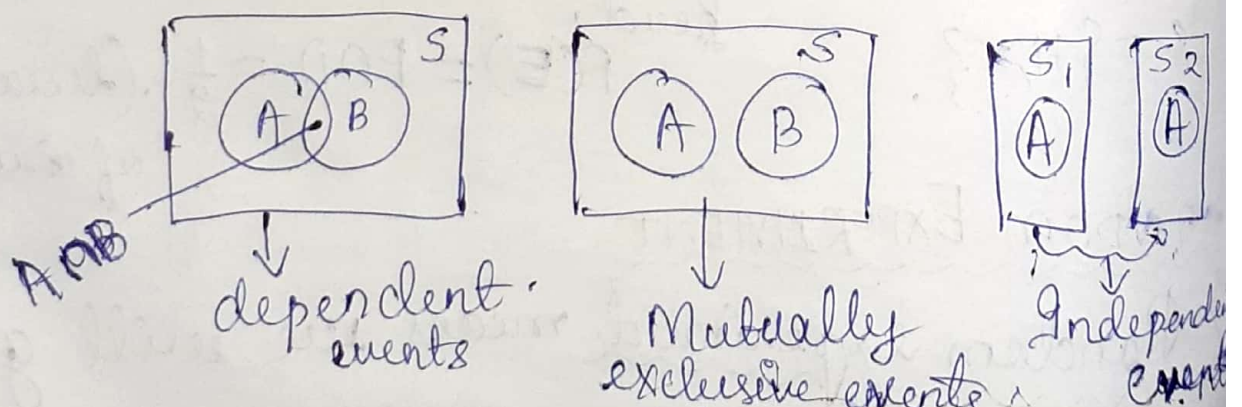
→ Probability of an event $P(E)$ lies between 0 to 1.

$$0 \leq P(E) \leq 1$$

→ There are three condition of probability cases that two events are :-

- Dependent
- Independent (Both events occur in two sample space)
- Mutually exclusive,

→ Suppose S be sample space :-



MUTUALLY EXCLUSIVE EVENTS

→ Mutually exclusive events is defined as the occurrence of an event does not depend on the occurrence of other event in the sample space.

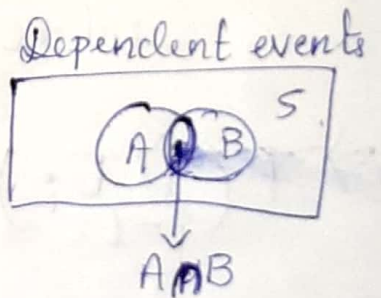
INDEPENDENT EVENTS

→ Independent events are those events in which the occurrence of one event does not depend on the occurrence of other event in different sample space.

DEPENDENT EVENTS

→ Dependent events are those events in which occurrence of one event is depended on occurrence of other event in sample space.

→ Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



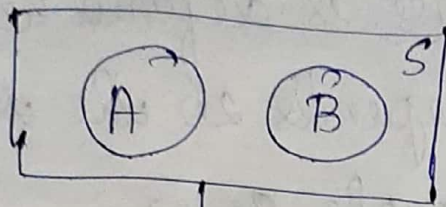
→ In Mutually exclusive case:-

$$P(A \cup B) = P(A) + P(B)$$

→ Probability of empty set/space (Nothing is present)

$$P(\phi) = 0$$

→ In mutually exclusive events.



$$P(A \cap B) = 0.$$

$$P(A+B) = P(A) + P(B)$$

→ In mutually exclusive cases:-

$$\rightarrow P\left(\bigcup_{i=1}^N E_i\right) = \sum_{i=1}^N P(E_i) \rightarrow \text{Probability of individual summation.}$$

$$\rightarrow P(E_1 \cup E_2 \cup E_3 \cup E_4 \cup \dots) = P(E_1) + P(E_2) +$$

$$P(E_3) + \dots$$

→ If events are independent cases:-

$$\rightarrow P\left(\bigcap_{i=1}^N E_i\right) = \prod_{i=1}^N P(E_i)$$

where π = product symbol.

$$\rightarrow P\left(\bigcup_{i=1}^N E_i\right) = \sum_{i=1}^N P(E_i)$$

PROBABILITY DISTRIBUTION FUNCTION (PDF)

→ It is denoted by F,

$$F(x) = P(X \leq x) \quad F_X(x) = P(X \leq x)$$

→ It is also known as CDF i.e. cumulative distribution function.

PDF is always represented graphically.

Suppose, let take an experiment rolling of a dice.

$$S = \{1, 2, 3, 4, 5, 6\}.$$

X = Random variable.

X	$P(X=x)$
1	$\rightarrow 1/6$
2	$\rightarrow 1/6$
3	$\rightarrow 1/6$
4	$\rightarrow 1/6$
5	$\rightarrow 1/6$
6	$\rightarrow 1/6$

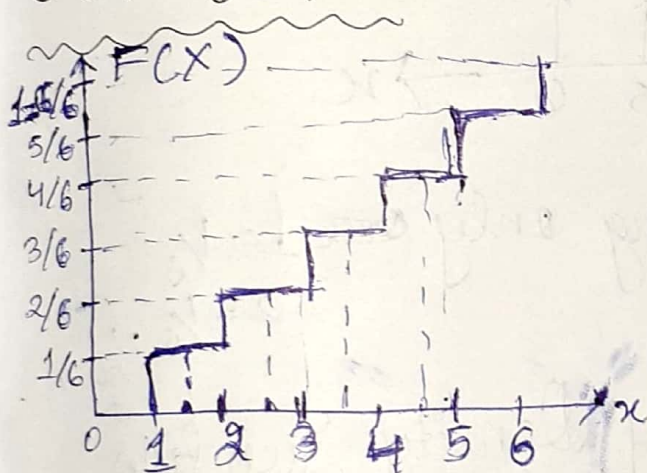
~~$P(X=x)$~~

$$F(X=0.99) = 0.$$

$$F(1.2) = P(X \leq 1.2) = 1/6$$

$$F(5) = P(X \leq 5) = 5/6$$

CDF CURVE



CDF/PDF Graphical Representation

PROBABILITY DENSITY FUNCTION (F)

It is denoted by f ,

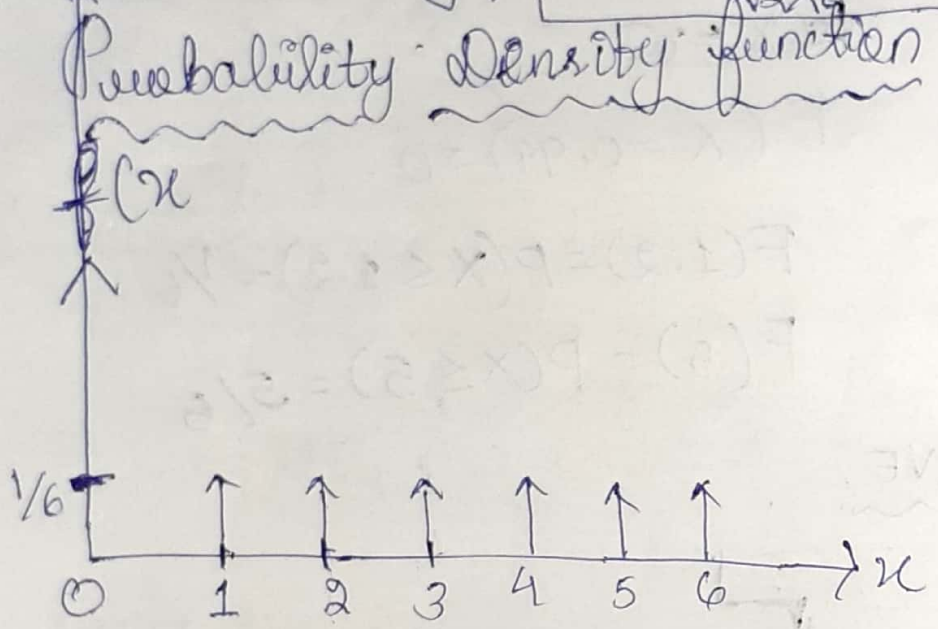
$$f(x) = \frac{d}{dx} \{F(x)\}$$

The probability density function is defined

as the distribution of total probability to various random variables,

→ The probability density function indicates the distribution of total probability to various random variables.

→ Mathematically, $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-a)^2}{2\sigma^2}}$ where $\sigma^2 = \text{variance}$ and $a = \text{mean}$.



Probability of getting only ~~one~~ $1 = 1/6$
 $2 = 1/6 \dots$
soon

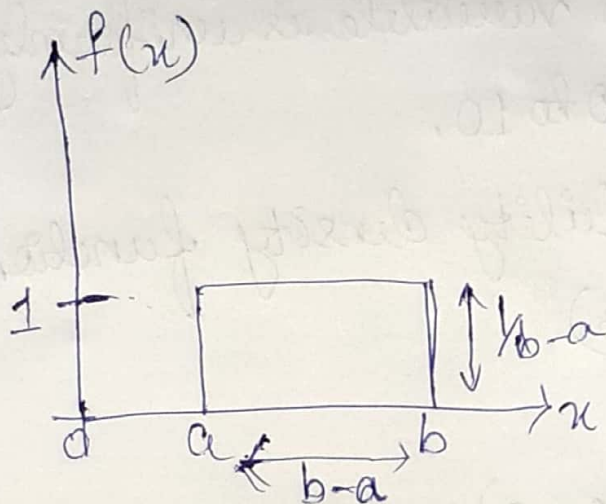
Properties of Probability Density Function

1) We have to find out the ^{total} area under a probability density function,

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

Therefore, total area under probability density function is 1.

① UNIFORM PROBABILITY DENSITY FUNCTION



∴ total area = $\cancel{b-a} \times \frac{1}{b-a} = 1$.

Then, $P(X \leq x_1) = F(x_1) = \int_{-\infty}^{x_1} f(x) dx$.

③ $P(X > x_1) = 1 - F(x_1) = \int_{x_1}^{\infty} f(x) dx$.

④ $P(x_1 < X \leq x_2) = \int_{x_1}^{x_2} f(x) dx$.

UNIFORM PROBABILITY DENSITY FUNCTION

→ Uniform Probability Density Function is defined as the probability density function is constant over a specified range.

Question: - Imp

1) A continuous random variable is uniformly distributed over a range from 0 to 10.

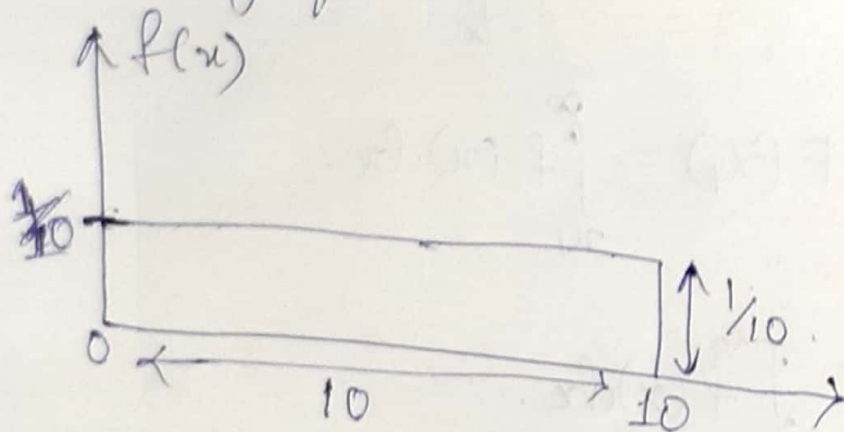
(i) Sketch the probability density function.

(ii) Determine $P(X \leq 3)$

(iii) Find $P(X > 9)$

(iv) Determine $P(4 < X \leq 8)$

Ans (i) ~~Probability~~ It is uniform probability density function:



(1) Area = $\frac{1}{10} \times 10 = 1$.

(2)

(ii) $P(X \leq 3) = \int_0^3 f(x) dx = \int_0^3 \frac{1}{10} \cdot dx = \left[\frac{1}{10} x \right]_0^3$
 $= \frac{1}{10} (3) = \frac{3}{10} = 0.3$ (Ans)

(iii) $P(X > 9) = \int_9^{10} f(x) dx = \int_9^{10} \frac{1}{10} \cdot dx = \left[\frac{1}{10} x \right]_9^{10}$

$$= 10 \cdot \frac{1}{10} (10-9) = \frac{1}{10} = 0.1 \text{ (C.P.)}$$

$$\begin{aligned} \text{(iv) } P(4 < X \leq 8) &= \int_4^8 f(x) dx = \int_4^8 \frac{1}{10} dx = \left[\frac{1}{10} x \right]_4^8 \\ &= \frac{1}{10} (8-4) = \frac{4}{10} = 0.4 \text{ (C.P.)} \end{aligned}$$

STATISTICAL AVERAGES

MEAN

→ Mean is also known as averages which is also known as expected value.

→ We can find mean of a probability density function. Consider a probability density function $f(x)$. Then,

$$\text{Mean} = \bar{X} = E(X) = m_1 = \int_{-\infty}^{\infty} x \cdot f(x) dx.$$

MEAN SQUARE VALUE

→ Mean square value is ~~found~~^{found} after ~~mean~~^{second mean}.
Square value of a probability density function

$$\text{Mean square value} = \bar{X}^2 = E(X^2) = m_2 = \int_{-\infty}^{\infty} x^2 f(x) dx$$

VARIANCE

→ Variance is denoted as σ^2 where σ is standard deviation.

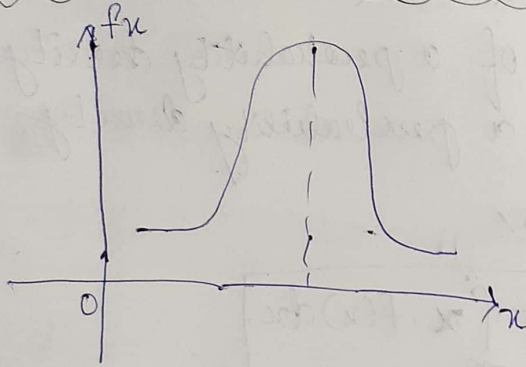
$$\text{Variance} = \sigma^2 = m_2 - m_1^2$$

STANDARD DEVIATION

→ Standard deviation is denoted as σ .

$$\text{Standard Deviation} = \sigma = \frac{1}{\sqrt{\text{Variance}}}$$

GAUSSIAN PROBABILITY DENSITY FUNCTION



→ We know,

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-a)^2}{2\sigma^2}}$$

where $\sigma^2 = \text{variance}$

$a = \text{mean}$

Question

1) Determine the statistical averages of the exponential probability density function.

$f(x) = b e^{-bx}$ Determine :-

- (i) Mean (m_1)
- (ii) Mean square value (m_2)
- (iii) Variance (σ^2)

Ans) Given, $f(x) = b e^{-bx}$

