

Reverse Curve

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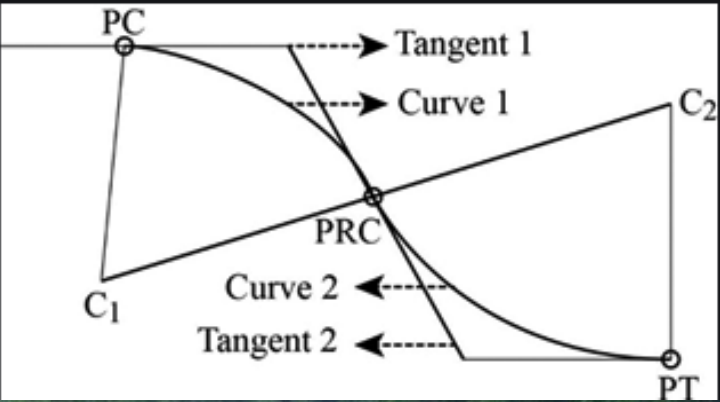
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Reverse curves: Consist of 2 arcs of equal or unequal radii, bending in opposite direction with common tangent at their junction (meeting Point), their center lying on the opposite sides of the curve.

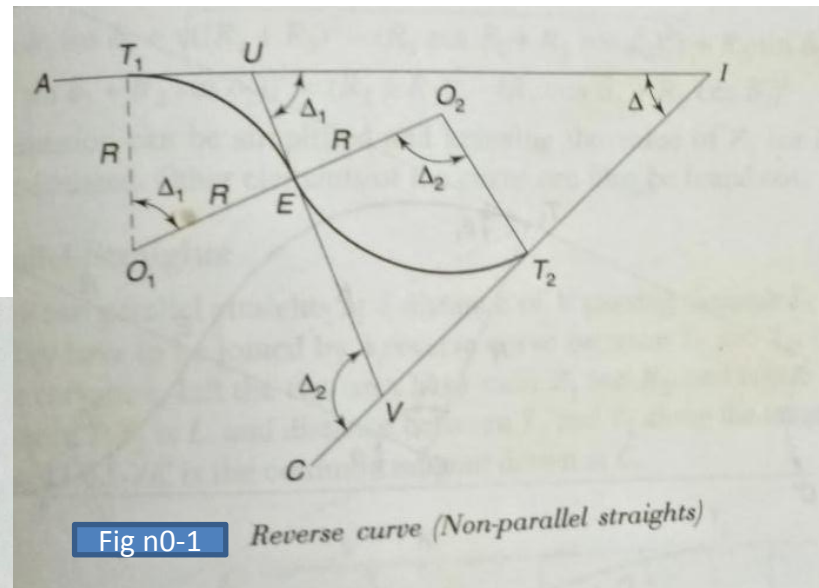


Usage of Reverse curve

Reverse curves are used when either the two straights are parallel or their angle of intersection is too small. These are used in hilly terrains and in railway sidings as cross-overs. These are also used on highways and railways designed for low speed. As far as possible, they should not be used on main highways and railways designed for high speed because of the following reasons:

1. Sudden change of superelevation is needed from one edge to the other one, near the point of reverse curvature.
2. The sudden change of curvature and direction reduces the life of the vehicle and gives discomfort to the passengers.
3. On highways, steering becomes uncomfortable.
4. It is not possible to provide proper superelevation at the point of reverse curvature.

Elements of Reverse curve



1.4.1 Elements of a reverse curve are:

1. Radii R_1 and R_2 of the two circular arcs.
2. Angle of total deflection Δ between straights.
3. Central angle or angle of deflections (Δ_1 and Δ_2) of the common tangent.
4. Angle (δ_1 and δ_2) between the straight and the line T_1T_2 .

1.4.2 Non-Parallel Straights

Case I
Central angles Δ_1 and Δ_2 and length of common tangent are given, $\Delta_2 > \Delta_1$
To find out the common radius R and chainage of T_1 , E and T_2 .

11.31

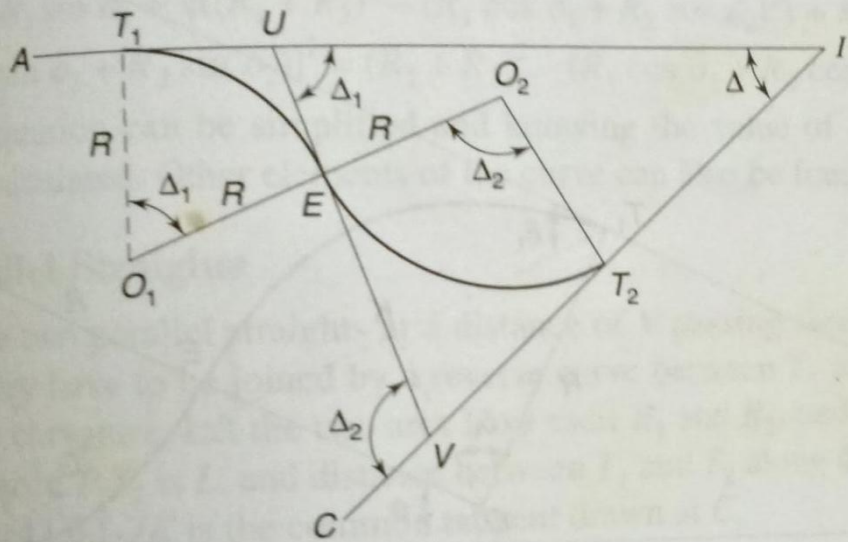


Fig no-2

Reverse curve (Non-parallel straights)

$$IT_1 = UT_1 + UI = R \tan \frac{\Delta_1}{2} + \frac{d \sin \Delta_2}{\sin \Delta}$$

Chainage of T_1 = chainage of $I - IT_1$

Chainage of E = chainage of T_1 + length of first arc $\left(\frac{\pi R \Delta_1}{180^\circ} \right)$

Chainage of T_2 = chainage of E + length of second arc $\left(\frac{\pi R \Delta_2}{180^\circ} \right)$

The first arc can be set out from T_1 and the second arc from E by the method of deflection angles.

$$R_1 = R_2 = R$$

$$T_1U = UE \text{ and } \angle IUE = \Delta_1$$

$$T_2V = VE \text{ and } \angle CVE = \Delta_2$$

$$UV = d = UE + EV$$

$$UV = d = R \tan \frac{\Delta_1}{2} + R \tan \frac{\Delta_2}{2}$$

$$R = \frac{d}{\left(\tan \frac{\Delta_1}{2} + \tan \frac{\Delta_2}{2} \right)}$$

$$\Delta = \Delta_2 - \Delta_1$$

$$IU = UV \frac{\sin \Delta_2}{\sin \Delta} = d \frac{\sin \Delta_2}{\sin \Delta}$$

or

Hence,

From triangle IUV ,

Case II

Given $T_1T_2 = L$, δ_1 and δ_2 , find common R .

Condition: $R_1 = R_2 = R$

Refer to Fig. 11.32.

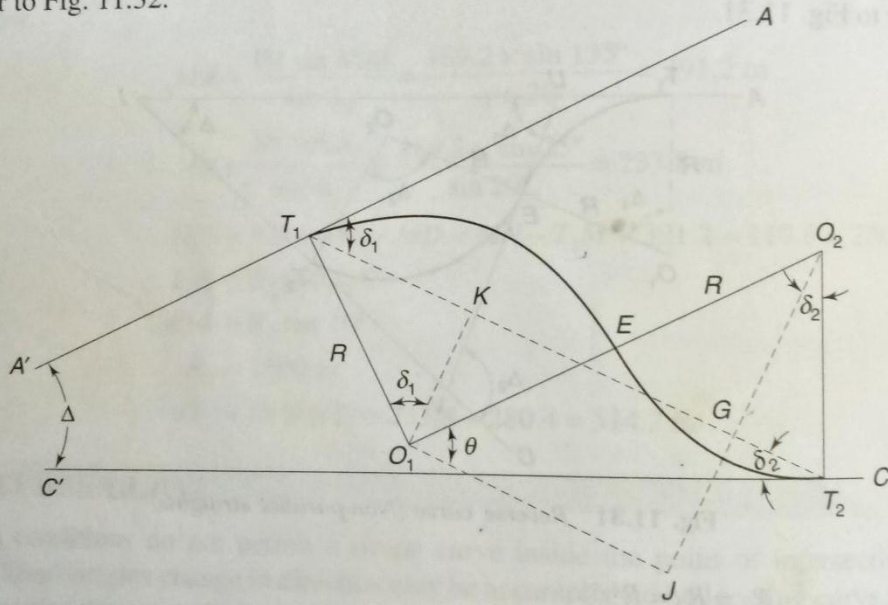


Fig 3

Drop perpendicular O_1K from O_1 and perpendicular O_2G from O_2 on T_1T_2 . Then,

$$O_1J = KG \quad \text{Let } \angle O_2O_1J = \theta$$

$$O_1K = GJ = R \cos \delta_1, \quad O_2G = R \cos \delta_2, \quad O_1O_2 = 2R$$

From triangle O_1JO_2 ,

$$\sin \theta = \frac{O_2J}{O_1O_2} = \frac{R \cos \delta_1 + R \cos \delta_2}{2R} = \frac{1}{2} (\cos \delta_1 + \cos \delta_2)$$

or,
$$\theta = \sin^{-1} \frac{1}{2} (\cos \delta_1 + \cos \delta_2) \quad (1)$$

$$\begin{aligned} L = T_1T_2 &= T_1K + KG + GT_2 \\ &= R \sin \delta_1 + 2R \cos \theta + R \sin \delta_2 \end{aligned}$$

$$R = \frac{L}{(\sin \delta_1 + 2 \cos \theta + \sin \delta_2)}$$

of.

Central angle for first arc, $\Delta_1 = \delta_1 + (90 - \theta)$

Central angle for second arc, $\Delta_2 = \delta_2 + (90 - \theta)$

Knowing R , Δ_1 and Δ_2 , the length of arcs can be calculated.

Case III
 Given $T_1 T_2 = L$, δ_1 and δ_2 , and R_1 (or R_2); find R_2 ($R_2 > R_1$)

Refer to Fig. **4**

$$T_1 K = R_1 \sin \delta_1, \quad T_2 G = R_2 \sin \delta_2$$

$$KG = O_1 J = \sqrt{(O_1 O_2)^2 - O_2 J^2}$$

$$= \sqrt{((R_1 + R_2)^2 - (R_1 \cos \delta_1 + R_2 \cos \delta_2)^2)}$$

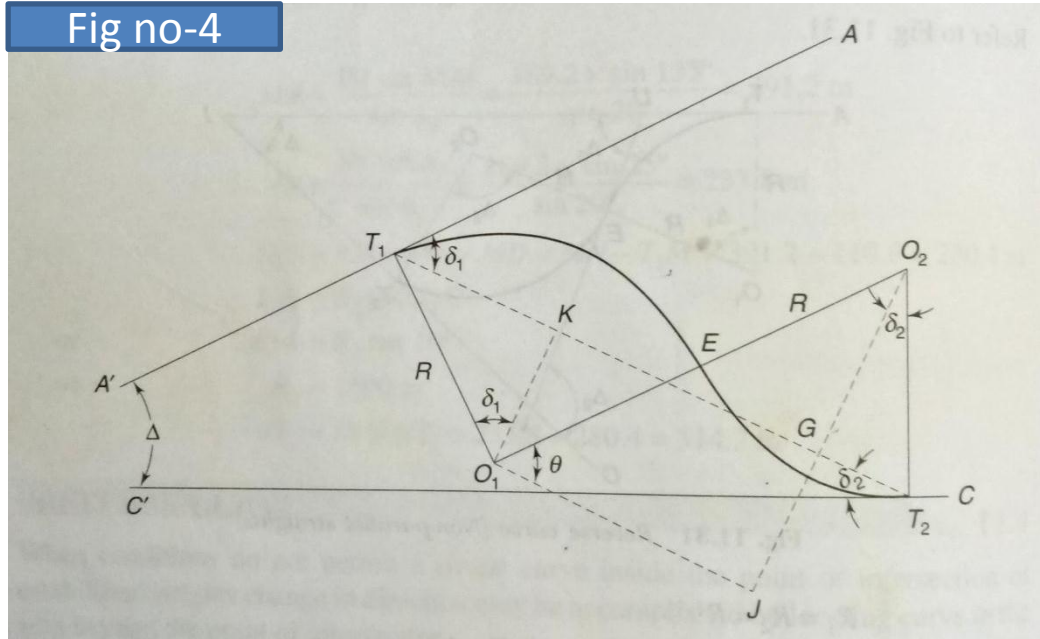
$$T_1 T_2 = L = T_1 K + KG + GT_2$$

$$= R_1 \sin \delta_1 + \sqrt{((R_1 + R_2)^2 - (R_1 \cos \delta_1 + R_2 \cos \delta_2)^2)} + R_2 \sin \delta_2$$

$$\text{or } [L - (R_1 \sin \delta_1 + R_2 \sin \delta_2)]^2 = (R_1 + R_2)^2 - (R_1 \cos \delta_1 + R_2 \cos \delta_2)^2$$

The above equation can be simplified and knowing the value of R_1 (or R_2), R_2 (or R_1) can be calculated. Other elements of the curve can also be found out.

Fig no-4



Parallel Straights

T_1J and T_2K are two parallel straight lines at a distance of V passing through T_1 and T_2 respectively. They have to be joined by a reverse curve between T_1 and T_2 . C is the point of reverse curvature. Let the two arcs have radii R_1 and R_2 , and central angles Δ_1 and Δ_2 . Distance T_1T_2 is L , and distance between T_1 and T_2 along the tangent is D as shown in Fig. 5. JK is the common tangent drawn at C .

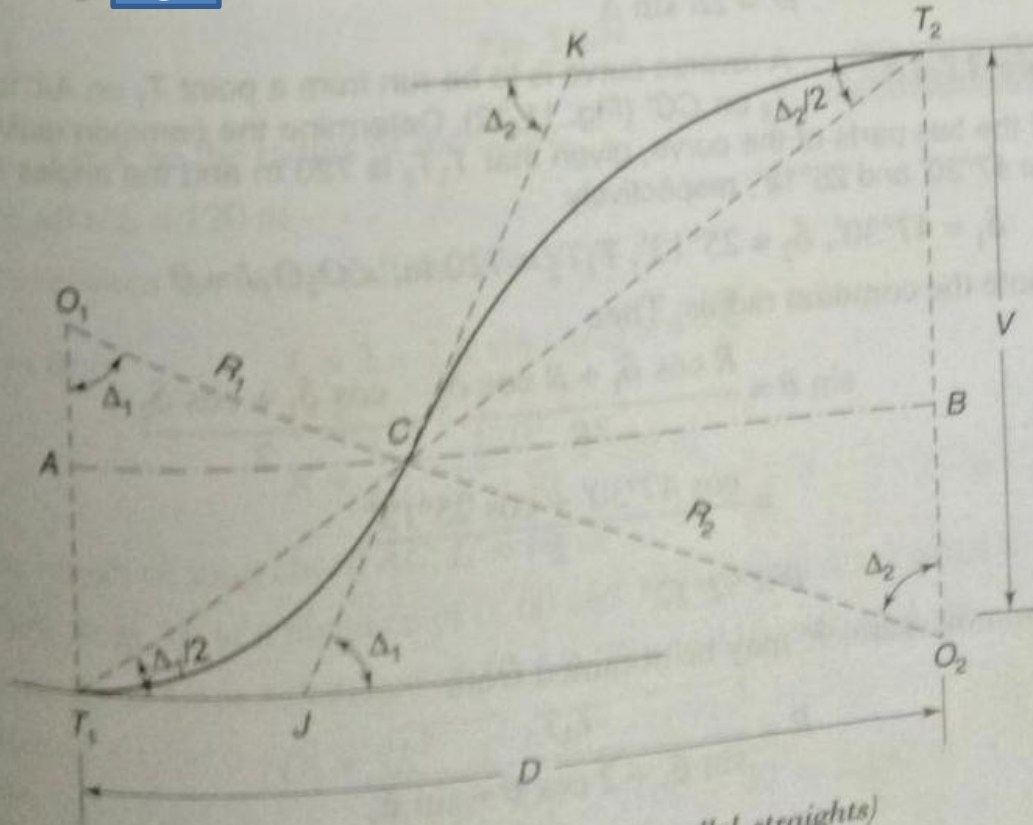


Fig no 5

Reverse curve (Parallel straight)

$\Delta_1 = \Delta_2 = \Delta$ (say). Hence O_1T_1 and O_2T_2 are parallel and O_1CO_2 joins them.

$$\begin{aligned}
 V &= T_1A + BT_2 \\
 &= (O_1T_1 - O_1A) + (O_2T_2 - O_2B) \\
 &= (R_1 - R_1 \cos \Delta_1) + (R_2 - R_2 \cos \Delta_2) \\
 &= R_1 (1 - \cos \Delta_1) + R_2 (1 - \cos \Delta_2) \\
 V &= (R_1 + R_2) (1 - \cos \Delta)
 \end{aligned}$$

$$\begin{aligned}
 \overline{T_1T_2} = L &= T_1C + CT_2 \\
 &= 2R_1 \sin (\Delta_1/2) + 2R_2 \sin (\Delta_2/2) \\
 L &= 2 (R_1 + R_2) \sin (\Delta/2)
 \end{aligned}$$

Using $\sin \frac{\Delta}{2} = \frac{V}{L}$ in Eq. (2) we get

$$L = 2 (R_1 + R_2) \times \frac{V}{L}$$

or $L^2 = 2 (R_1 + R_2) V$

or $L = \sqrt{2V(R_1 + R_2)}$ (3)

Also, $D = AC + CB$

$$\begin{aligned}
 &= R_1 \sin \Delta_1 + R_2 \sin \Delta_2 \\
 D &= (R_1 + R_2) \sin \Delta \quad (4)
 \end{aligned}$$

Special cases If, as a special case, $R_1 = R_2 = R$ (say), the above Eqs (1) to (4) are transformed to:

$$V = 2R (1 - \cos \Delta)$$

$$L = 4R \sin (\Delta/2)$$

$$L = 2 \sqrt{RV}$$

$$D = 2R \sin \Delta$$

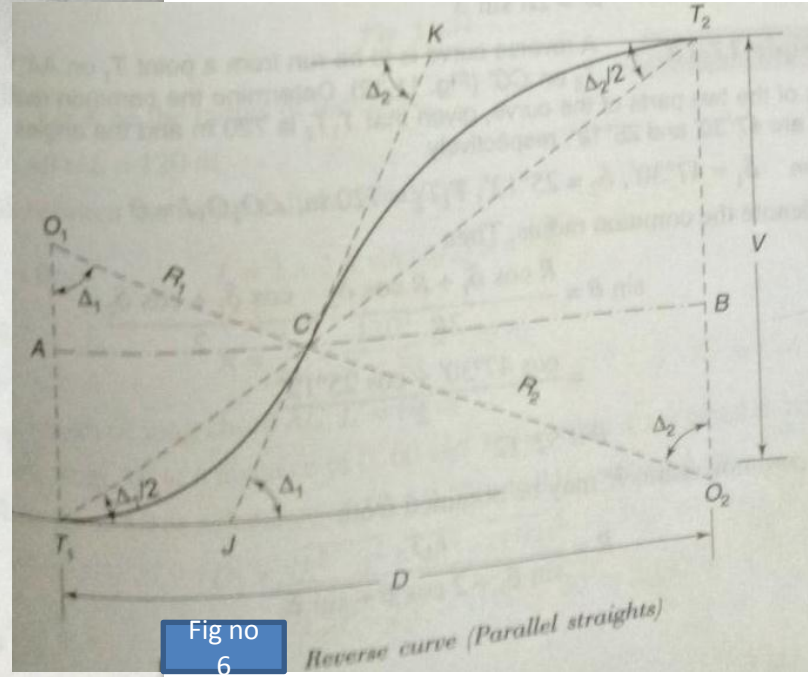


Fig no 6

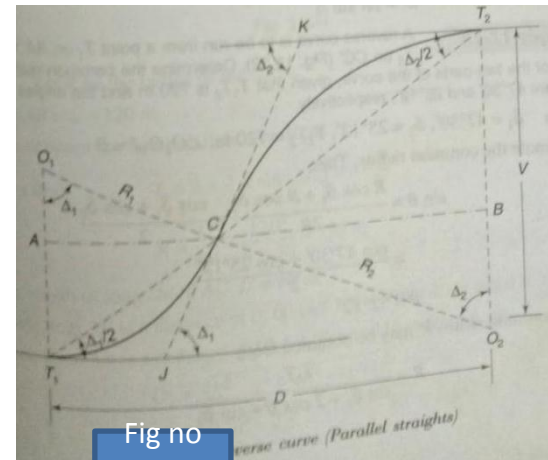


Fig no

7

Example

1

A reverse curve is to be run from a point T_1 on AA' to the point T_2 on CC' (Fig. 7). Determine the common radius and the lengths of the two parts of the curve, given that T_1T_2 is 720 m and the angles AT_1T_2 and T_2T_2C' are $47^\circ 30'$ and $25^\circ 12'$, respectively.

7

Solution $\delta_1 = 47^\circ 30'$, $\delta_2 = 25^\circ 12'$, $T_1T_2 = 720$ m, $\angle O_2O_1J = \theta$

Let R denote the common radius. Then

$$\sin \theta = \frac{R \cos \delta_1 + R \cos \delta_2}{2R} = \frac{\cos \delta_1 + \cos \delta_2}{2}$$

$$= \frac{\cos 47^\circ 30' + \cos 25^\circ 12'}{2}$$

\therefore

$$\theta = 52^\circ 12'$$

The common radius R may be obtained from

$$R = \frac{T_1T_2}{\sin \delta_1 + 2 \cos \theta + \sin \delta_2}$$

$$= \frac{720}{\sin 47^{\circ}30' + 2 \cos 52^{\circ}12' + \sin 25^{\circ}12'} = 301.496 \text{ m}$$

Now, the central angle T_1O_1E (α_1) of the first arc

$$= \delta_1 + 90^{\circ} - \theta = 47^{\circ}30' + 90^{\circ} - 52^{\circ}12' = 85^{\circ}18'$$

The central angle T_2O_2E (α_2) of the second arc

$$= \delta_2 + 90^{\circ} - \theta = 25^{\circ}12' + 90^{\circ} - 52^{\circ}12' = 63^{\circ}$$

$$\text{Length of the first arc} = \frac{\pi R \alpha_1}{180^{\circ}} = \frac{\pi \times 301.496 \times 85^{\circ}18'}{180^{\circ}} = 448.86 \text{ m}$$

$$\text{Length of the second arc} = \frac{\pi R \alpha_2}{180^{\circ}} = \frac{\pi \times 301.496 \times 63^{\circ}}{180^{\circ}} = 331.51 \text{ m}$$

Example

2

A reverse curve ACB is to be set out between two parallel straights 30 m apart (Fig. 11.34). If $R_1 = R_2$ and the distance between tangent points A and B is 120 m, calculate the radius. Also, calculate the length of the offsets if the whole curve is to be laid by means of offsets, from the long chord at 10 m intervals.

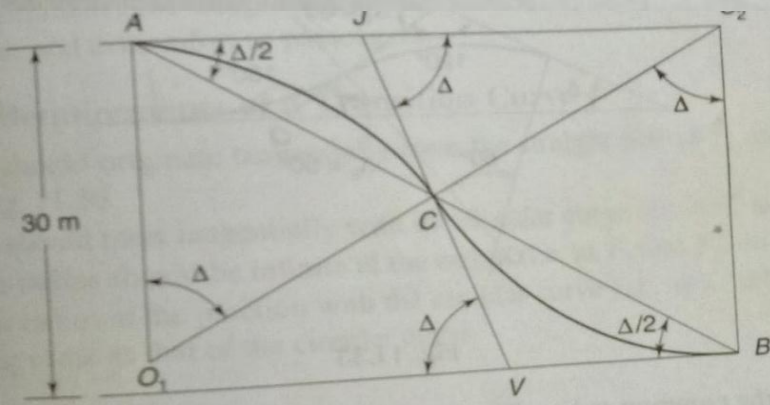


Fig. 8

Solution Let R be the radius of the curve.

Distance $AB = L = 120$ m

Distance between the two parallels, $V = 30$ m

We know that

$$L = 2 \times 2 R \sin (\Delta/2) = 4 \frac{RV}{L}$$

$$R = \frac{L^2}{4V} = \frac{120^2}{4 \times 30} = 120 \text{ m}$$

Now, length of long chord AC , $L' = 60$ m

Offsets from AB at a distance of 0, 60 and 120 m from A are equal to zero.

Offsets from AB at a distance of 10, 50, 70 and 110 m from A are:

$$O_1 = \sqrt{R^2 - x^2} - \sqrt{R^2 - (L'/2)^2} = \sqrt{120^2 - 20^2} - \sqrt{120^2 - 30^2} = 2.132 \text{ m}$$

Offsets from AB at a distance of 20, 40, 80 and 100 m are:

$$O_2 = \sqrt{120^2 - 10^2} - \sqrt{120^2 - 30^2} = 3.393 \text{ m}$$

Offsets from AB at a distance of 30 m and 90 m from A are:

$$O_3 = \sqrt{120^2 - 0} - \sqrt{120^2 - 30^2} = 3.81 \text{ m}$$

Example 3

A reverse curve is to join two straights having a very acute angle of intersection. The common tangent (140 m) makes an angle of intersection of 120° and 130° with the main straights. Calculate the suitable common radius.

Solution Refer to Fig. 9

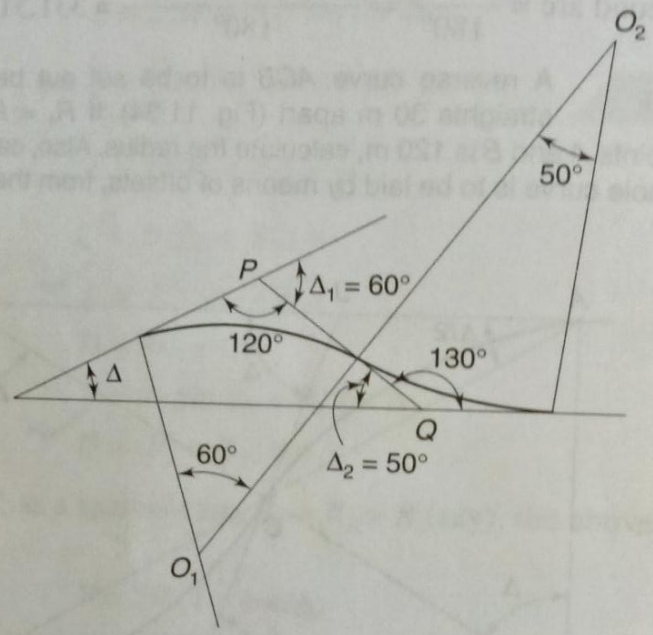


Fig. 9

Let R be the common radius. The deflection angles are

$$\Delta_1 = 60^\circ, \quad \Delta_2 = 50^\circ, \quad \Delta = \Delta_1 - \Delta_2 = 10^\circ$$

Length of the common tangent (PQ) = 140 m

Common tangent,

$$PQ = R \left(\tan \frac{60^\circ}{2} + \tan \frac{50^\circ}{2} \right)$$

or

$$R (\tan 30^\circ + \tan 25^\circ) = 140 \text{ m}$$

or

$$R = \frac{140}{\tan 30^\circ + \tan 25^\circ} = 134.143 \text{ m}$$

Example 4 While surveying along the alignment of a road, the magnetic bearings of the lines AB, BC and CD are measured as 80° , 110° and 60° respectively. The length of BC is 200 m, and the chainage of B is 950.00 m. Calculate the necessary data for setting out a reverse curve connecting the lines AB and CD by taking BC as the common tangent. The radii of both curves may be assumed equal.

Solution Consider Fig.

10

$$\phi_1 = 110^\circ - 80^\circ = 30^\circ$$

$$\begin{aligned} \phi_2 &= \text{BB of BC} - (180^\circ + 60^\circ) \\ &= 290^\circ - 240^\circ = 50^\circ \end{aligned}$$

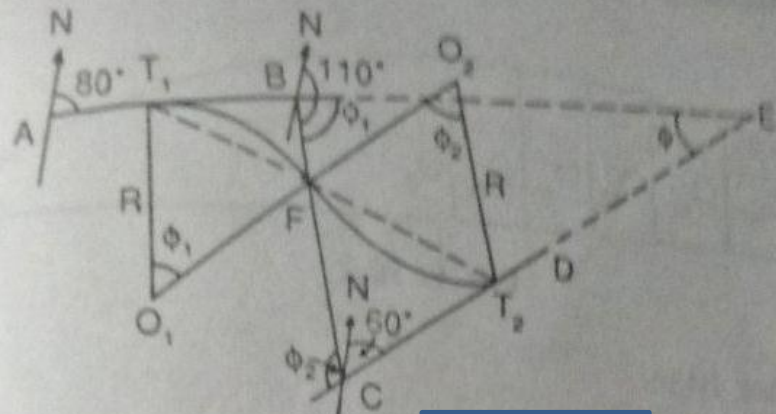


Fig. 1

10

Given data:

$$BC = 200 \text{ m}$$

$$\text{Chainage of B} = 950.00 \text{ m}$$

Let F be the point of reverse curvature.

Now

$$BC = BF + CF = R \tan \frac{\phi_1}{2} + R \tan \frac{\phi_2}{2}$$

That is,

$$200 = R \tan 15^\circ + R \tan 25^\circ = 0.7342R$$

$$R = 272.40 \text{ m}$$

∴ 1. Tangent length $T_1B = R \tan 15^\circ = 272.40 \times 0.2679 = 72.97 \text{ m}$

2. Chainage of $T_1 = 950.00 - 72.97 = 877.03 \text{ m}$

3. First curve length $= \frac{\pi R \phi_1^\circ}{180^\circ} = \frac{\pi \times 272.40 \times 30^\circ}{180^\circ} = 142.62 \text{ m}$

4. Chainage of F $= 877.03 + 142.62 = 1,019.65 \text{ m}$

5. Second curve length $= \frac{\pi R \phi_2^\circ}{180^\circ} = \frac{\pi \times 272.40 \times 50^\circ}{180^\circ} = 237.71 \text{ m}$

6. Chainage of $T_2 = 1,019.65 + 237.71 = 1,257.36 \text{ m}$

7. Length of long chord for small curve,

$$T_1F = 2R \sin \frac{\phi_1}{2} = 2 \times 272.40 \times \sin 15^\circ = 141.00 \text{ m}$$

8. Length of long chord for large curve,

$$T_2F = 2R \sin \frac{\phi_2}{2} = 2 \times 272.40 \times \sin 25^\circ = 230.24 \text{ m}$$

Setting the first curve: The long chord T_1F is divided into two equal parts, each of length 70.5 m.

Offsets for the left half are calculated, taking peg intervals of 10 m.

$$\text{Mid-ordinate, } O_0 = R - \sqrt{R^2 - (L/2)^2} = 272.40 - \sqrt{(272.40)^2 - (70.5)^2} = 9.28 \text{ m}$$

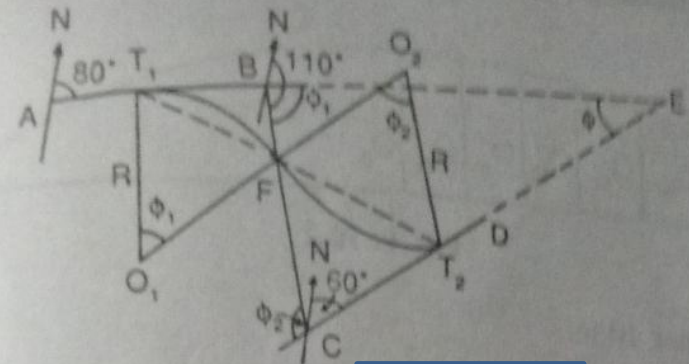


Fig. 10

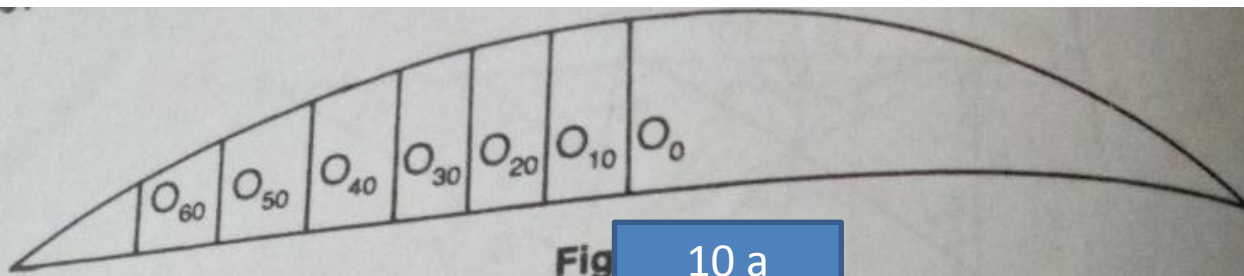


Fig 10 a

Ordinate at distance 10 m,

$$O_{10} = \sqrt{(R^2 - x^2) - (R - O_0)}$$

$$= \sqrt{(272.40)^2 - (10)^2} - (272.40 - 9.28) = 9.09 \text{ m}$$

$$O_{20} = \sqrt{(272.40)^2 - (20)^2} - 263.12 = 8.54 \text{ m}$$

$$O_{30} = \sqrt{(272.40)^2 - (30)^2} - 263.12 = 7.62 \text{ m}$$

$$O_{40} = \sqrt{(272.40)^2 - (40)^2} - 263.12 = 6.32 \text{ m}$$

$$O_{50} = \sqrt{(272.40)^2 - (50)^2} - 263.12 = 4.65 \text{ m}$$

$$O_{60} = \sqrt{(272.40)^2 - (60)^2} - 263.12 = 2.58 \text{ m}$$

The offsets for the right half are similar to those for the left half.

Setting the second curve: The long chord T_2F is divided into left and right halves. The offsets for the left half are calculated exactly as described above. The offsets of the right half are similar to those of the left half. Then both the halves are set out in a similar manner.