

Growth models: von Bertalanffy's growth equation

Growth models

von Bertalanffy's growth equation

Introduction

The study of fish growth and phenomena related to growth such as maturation, migration, food and feeding habits is a central theme in fisheries biology. The study of growth means basically the determination of the body size as a function of age. For fish stock assessment methods, the age composition data is of prime importance.

Growth

The simplest definition of growth of fish is that it is just an increase in size i.e. length or weight. During growth, the annual growth increment in length or weight steadily decreases through successive years of age while the length or weight itself keeps on increasing with age. The example given below demonstrates that the growth increment in length decreases continuously with the age of the fish.

Age in years (increase)	Length in mm (increase)	Interval between age	Annual increment in mm (decrease)
0	0	-	-
1	150	0 - 1	$150 - 0 = 150$
2	220	1 - 2	$220 - 150 = 70$
3	240	2 - 3	$240 - 220 = 20$
4	250	3 - 4	$250 - 240 = 10$

Growth parameters

The growth parameters that are usually estimated in the fish stock assessment include

- Length infinity or Asymptotic length (L_{∞})
- Growth coefficient or curvature parameter (K)
- Initial condition parameter / Arbitrary origin of growth (t_0)
- L_{max}
- Growth rate ($\frac{\Delta L}{\Delta t}$)

Length infinity or Asymptotic length (L_{∞})

The growth increment during growth continues to decrease and attain virtually zero increment between two adjacent years of age in some future. The length of the fish corresponding to zero increment is called 'length infinity' or 'asymptotic length' and is designated by L_{∞} . This is the theoretical maximum length beyond which the fish cannot grow. In nature, the fish seldom attains this value but tend to grow towards it, as they die due to natural causes or due to fishing. L_{∞} is the mean length of the fish would reach if they were to grow to a very old age. It is one of the three parameters of the von Bertalanffy growth function.

Growth coefficient or curvature parameter (K)

This is a parameter of von Bertalanffy growth function, expressing the rate at which the asymptotic length is approached. Short lived species, particularly, tropical fishes attain their L_{∞} in a year or two and such species have high 'K' value. The long lived species usually have low 'K' value and need many years to attain their L_{∞} . Such long lived species have flat growth curve.

Initial condition parameter / Arbitrary origin of growth (t_0)

It is the initial condition parameter. " t_0 " is the hypothetical age, the fish would have had at zero length if they had always grown according to the VBGF equation. t_0 should be read as t zero. Usually ' t_0 ' has a negative value. t_0 is also called as 'arbitrary origin of growth'. Biologically ' t_0 ' has no meaning because the growth begins at hatching when the larva already has certain length, which may be called L_0 when we put $t = 0$ at the day of birth.

L max

L max is the length of the largest fish reported from a well sampled stock and T_{max} is the age of the largest fish reported from a well sampled stock. The L_{max} / T_{max} will be more accurate when the exploitation of the stock in question is low.

Growth rate

The growth rate is denoted by

$$\frac{\Delta L}{\Delta t} = \frac{L(t + \Delta t) - L(t)}{\Delta t} \dots\dots\dots (1)$$

Time (or age), t is usually expressed in units of years.

The mathematical relationship between the length of a fish and the growth rate at a given time is a linear function.

$$\frac{\Delta L}{\Delta t} = a + b \cdot L(t) \dots\dots\dots (2)$$

Using VBGF equation, linear relationship can be derived as follows :

$$\frac{\Delta L}{\Delta t} = K \cdot (L_{\infty} - L(t)) \text{ cm/year} \dots\dots\dots (3)$$

where $K = -b$ and $L_{\infty} = -a/b$

The mean growth rate (in length) can be calculated using the equation

$$\overline{L_t} = \frac{L(t + \Delta t) + L(t)}{2} \dots\dots\dots (4)$$

von Bertalanffy's growth equation

Length based von Bertalanffy's growth equation

$$L_t = L_{\infty} [1 - \exp(-K(t - t_0))] \dots\dots\dots(5)$$

L_{∞} = Asymptotic length

K = Growth coefficient or curvature parameter

t_0 = Initial condition parameter or arbitrary origin of growth

(t_0 = should be read as 't' Zero).

e = Base of natural logarithm

t = time

Weight based von Bertalanffy's growth equation

$$W(t) = W_{\infty} [1 - \exp(-K(t - t_0))]^3 \dots\dots\dots(6)$$

W_{∞} = Asymptotic weight

K, t_0 and e and t are defined as above (Refer equation 5)

