

# Gauss's Law and Applications

by

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## Electric Flux Density

- Relationship between electric field (**E**) and electric flux density (**D**) (in free space)

$$\mathbf{D} = \varepsilon_0 \mathbf{E}$$

- Electric flux (in C) through a surface S is :

$$\psi = \int_S \mathbf{D} \cdot d\mathbf{S}$$



Electric flux crossing the surface **S** (open surface integral)

- Electric flux density is also called electric displacement (in C/m<sup>2</sup>)

## Gauss's Law-Maxwell's Equation

- Integral form of Gauss's law:  $\psi = \oint_S \mathbf{D} \cdot d\mathbf{S} = Q_{enc} = \int_v \rho_v dv \Rightarrow$

Electric flux enclosed in the surfaces (closed surface integral)

- Divergence theorem  $\Rightarrow \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{D} dv$

- Comparing the above two volume integrals, we obtain Maxwell's equation:

$$\rho_v = \nabla \cdot \mathbf{D}$$

This is the point or differential form of Gauss's law

## Gauss's Law-Maxwell's Equation (Cont'd.)

- Volume charge density is the same as the divergence of the electric flux density.
- It should be noted that  $\rho_v$  is 'free' volume charge density and not 'bound' volume charge density
- **E** or **D** field lines originate (terminate) from (on) positive (negative) charges.

## Application of Gauss's Law

- Symmetric charge distribution should exist.
- A hypothetic closed surface (called Gaussian surface) is constructed around or in vicinity of the charge distribution such that  $\mathbf{D}$  is normal or tangential to the Gaussian surface.
- $\mathbf{D}$  is normal to the surface:  $\mathbf{D} \cdot d\mathbf{S} = D dS$
- $\mathbf{D}$  is tangential to the surface:  $\mathbf{D} \cdot d\mathbf{S} = 0$   
(the two vectors are perpendicular to each other)
- $\mathbf{D}$  due to a point charge  $Q$   
Consider that the charge  $Q$  is placed at the origin.
- For determining  $\mathbf{D}$  at a point  $P$ , a spherical surface containing  $P$  and centered at the origin satisfies the symmetry requirement.

## Application of Gauss's Law (Cont'd.)

- $\mathbf{D}$  is everywhere normal to the Gaussian surface:

$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_S D_r \mathbf{a}_r \cdot 4\pi r^2 \mathbf{a}_r = D_r 4\pi r^2 \quad \therefore \mathbf{D} = \frac{Q}{4\pi r^2} \mathbf{a}_r$$

- This is expected since  $\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2} \mathbf{a}_r$   
and  $\mathbf{D} = \epsilon_0 \mathbf{E}$

- Similarly,  $\mathbf{D} = \frac{\rho_L}{2\pi\rho} \mathbf{a}_\rho$  for infinite line charge

- $\mathbf{D} = \frac{\rho_s}{2} \mathbf{a}_z$  for infinite sheet of charge

## Application of Gauss's Law (Cont'd.)

- It is useful to find out electric field from the knowledge of enclosed charge and surface.
- It is useful to find out flux,  $\Psi$  or flux density  $D$ , from the knowledge of enclosed charge and surface.
- It is useful to find out the enclosed charge from the knowledge of either  $D$  or  $E$ .

## Limitations of Gauss's Law

- It can not be applied on Non-Gaussian surfaces.
- It can not be applied if the surface does not enclose the volume completely.