

Electric Potential

Maxwell's Two Equations for Electrostatic Fields

by

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Electric Potential

❖ $\mathbf{F} = Q\mathbf{E}$; Work done in displacing Q by $d\mathbf{l}$: $dW = -\mathbf{F} \cdot d\mathbf{l} = -q\mathbf{E} \cdot d\mathbf{l}$

$$W = -Q \int_A^B \mathbf{E} \cdot d\mathbf{l}$$

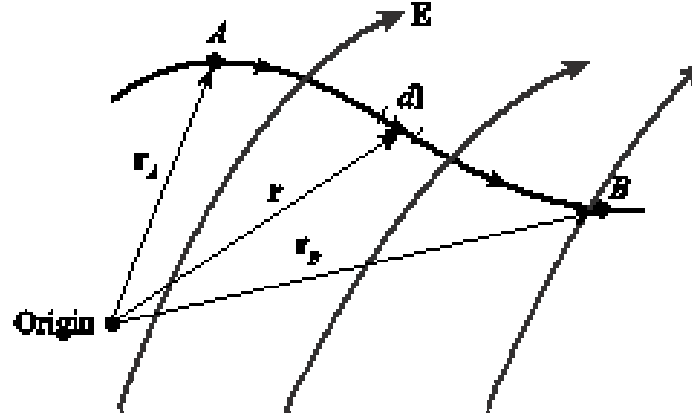


Figure 4

- ❖ This is the total work done in moving the point charge Q from A to B in the electric field.
- ❖ The negative sign indicates that an external agent does the work.
- ❖ The potential difference between the A and B points is defined as the potential energy per unit charge:

$$V_{AB} = -\int_A^B \mathbf{E} \cdot d\mathbf{l} = \frac{W}{Q} = V_B - V_A$$

Electric Potential (Cont'd.)

- In figure 4, if the \mathbf{E} field is due to a point charge Q located at the origin, then

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2} \mathbf{a}_r \qquad V_{AB} = - \int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2} \mathbf{a}_r \cdot d\mathbf{r} \mathbf{a}_r$$

- Points A and B lie on two different equipotential contours-circles with radii r_A and r_B

$$V_{AB} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right] = V_B - V_A$$

- \mathbf{E} is in the radial direction, so any contribution from a displacement in the θ or ϕ direction is zero.

Electric Potential (Cont'd.)

- Thus, V_{AB} that is the potential at B reference to that at A
- If we choose infinity as reference with its potential equal to zero, $V_A=0$ as $r_A \rightarrow$ infinity, the potential at any point ($r_B = r$) due to Q at the origin is:

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Electric Potential (Cont'd.)

- The potential at any point is the potential difference between that point and a chosen reference point at which the potential is zero.
- The principle of superposition applies to determining potential at a point with position vector \mathbf{r} due to point charges Q_1, Q_2, \dots, Q_n located at points with position vectors $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n$:

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{Q_k}{|\mathbf{r} - \mathbf{r}_k|}$$

- For continuous charge distributions:

- Line charge

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_L \frac{\rho_L(r') dl'}{|\mathbf{r} - \mathbf{r}'|}$$

- Surface charge

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\rho_S(r') dS'}{|\mathbf{r} - \mathbf{r}'|}$$

- Volume charge

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_v(r') dv'}{|\mathbf{r} - \mathbf{r}'|}$$

Relationship between \mathbf{E} and V – Maxwell's Equations

- ❖ The potential at \mathbf{r} due to a point charge at \mathbf{r}'

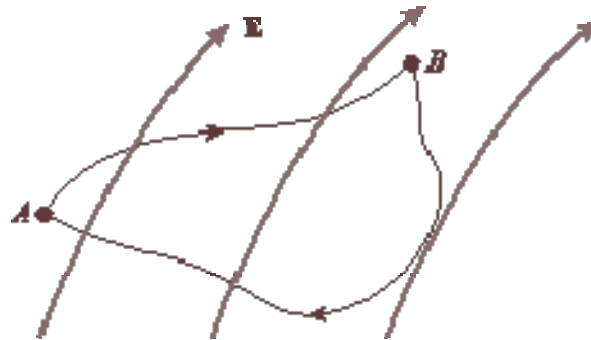


Figure 5

$V_{BA} = -V_{AB}$, so $V_{AB} + V_{BA} = \oint \mathbf{E} \cdot d\mathbf{l} = 0 \Rightarrow$ conservative electrostatic field. No net work is done in moving a charge along a closed path.

- ❖ For the potential field, the corresponding electric field is: $\mathbf{E} = -\nabla V$

Relationship between \mathbf{E} and V – Maxwell's Equations

- Applying Stokes's theorem:

$$\oint_L \mathbf{E} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = 0 \Rightarrow \nabla \times \mathbf{E} = 0$$

- Any curl-free field like the electrostatic field above is called conservative or irrotational field.
- For such a conservative field, its line integral does not depend on the path of integration
- The above is Maxwell's equation for electrostatics

$$V = -\int \mathbf{E} \cdot d\mathbf{l} \Rightarrow dV = -\mathbf{E} \cdot d\mathbf{l} = -E_x dx - E_y dy - E_z dz$$

but

$$\begin{aligned} dV &= \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz \\ \Rightarrow E_x &= -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z} \\ \Rightarrow \mathbf{E} &= -\nabla V \end{aligned}$$

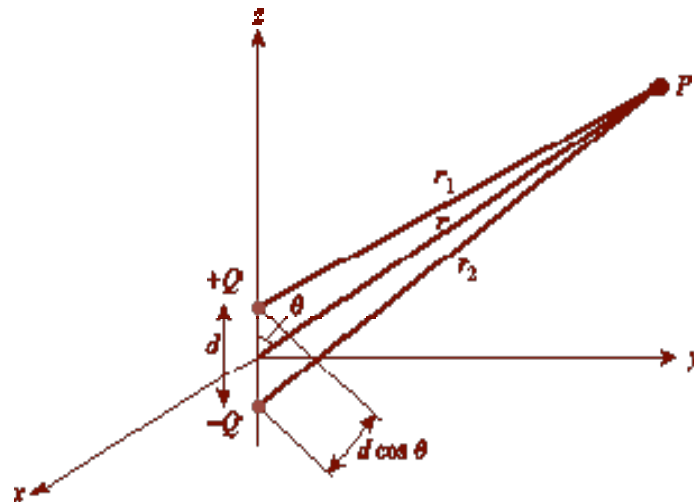
Relationship between \mathbf{E} and V – Maxwell's Equations (Cont'd.)

- Gradient: from a lower to higher level.
- \mathbf{E} : from a higher potential to a lower one.
- $\Rightarrow \mathbf{E}$ and ∇V are directed in opposite directions.

An Electric dipole and Flux lines

- ❖ An **electric dipole** is formed when two point charges of equal magnitude but with opposite signs are separated by a small distance

- ❖ Figure 6



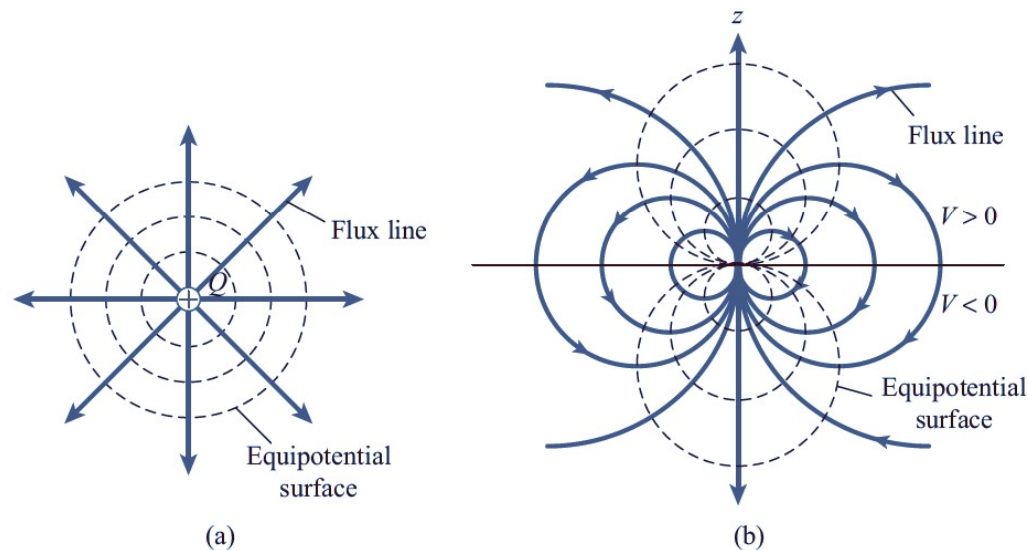
$$V(\mathbf{r}) = \frac{\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

- ❖ For an electric dipole centered at \mathbf{r}' with dipole moment \mathbf{p} , the potential at \mathbf{r} is given by

An Electric dipole and Flux lines

- ❖ Flux density \mathbf{D} is tangential to the electric flux lines at every point.
- ❖ An equipotential surface (line) is one on which $V = \text{constant}$
- ❖ An equipotential line is orthogonal to an electric flux line.

Figure 7



An electric flux line is an imaginary path in such a way that its direction at any point is the direction of the electric field at any point

Torque on a dipole

- The two forces \mathbf{F}_A and \mathbf{F}_B create a couple and produce a torque which acts on the dipole

$$|\mathbf{T}| = |\mathbf{p}||\mathbf{E}|\sin\theta$$

$$\mathbf{T} = \mathbf{p} \times \mathbf{E}$$

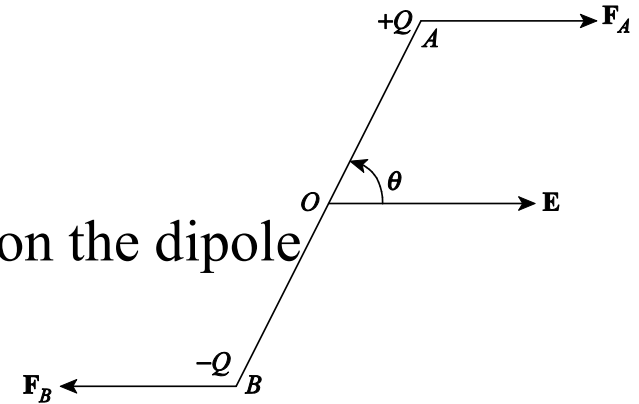


Figure 8

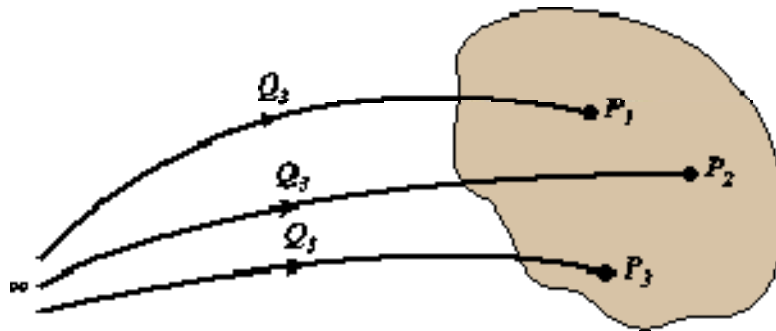
Direction of \mathbf{T} : perpendicular to the plane containing \mathbf{E} and the dipole moment \mathbf{p}

Energy Density in Electrostatic Fields

- ❖ Electrostatic energy due to n point charges:

$$W_E = \frac{1}{2} \sum_{k=1}^n Q_k V_k$$

- ❖ Figure 9



- ❖ For a continuous volume charge distribution

$$W_E = \frac{1}{2} \int_v \mathbf{D} \cdot \mathbf{E} \, dv = \frac{1}{2} \int_v \epsilon_0 |\mathbf{E}|^2 \, dv$$