

# Continuity Equation and Relaxation Time

by

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## Continuity Equation

- Charge conservation: The net outflow of charge per unit time should be the outward current

$$I_{\text{out}} = \oint \mathbf{J} \cdot d\mathbf{S} = \frac{-dQ_{\text{in}}}{dt}$$

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{J} dv$$

$$\frac{-dQ_{\text{in}}}{dt} = -\frac{d}{dt} \int_v \rho_v dv = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

$$\int_v \nabla \cdot \mathbf{J} dv = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

$$\boxed{\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}}$$

## Relaxation Time

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon}$$

$$\nabla \cdot \sigma \mathbf{E} = \frac{\sigma \rho_v}{\epsilon} = -\frac{\partial \rho_v}{\partial t}$$

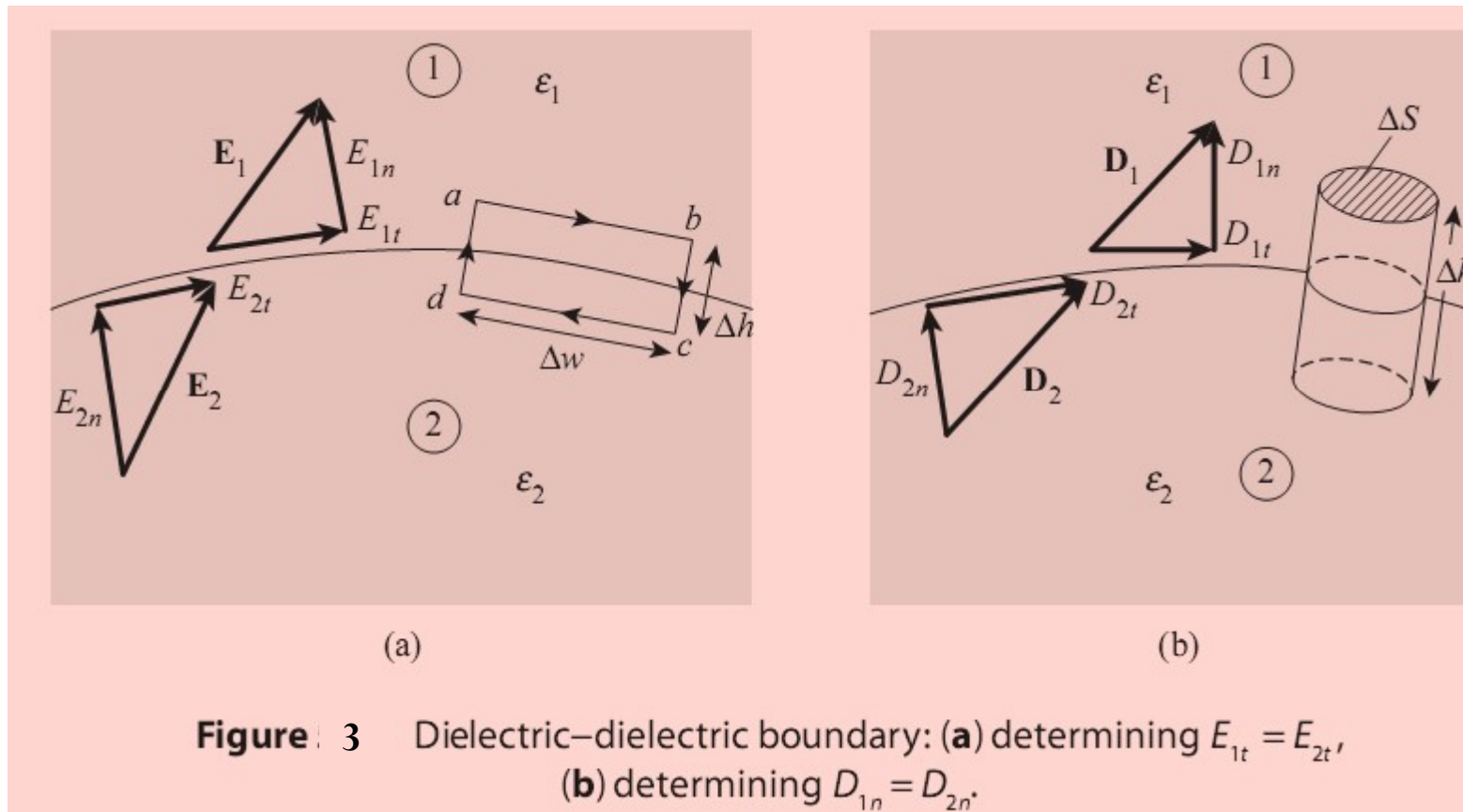
$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon} \rho_v = 0$$

$$\rho_v = \rho_{v0} e^{-t/T_r}$$

$$T_r = \frac{\epsilon}{\sigma}$$

# Boundary Conditions

## Dielectric-Dielectric

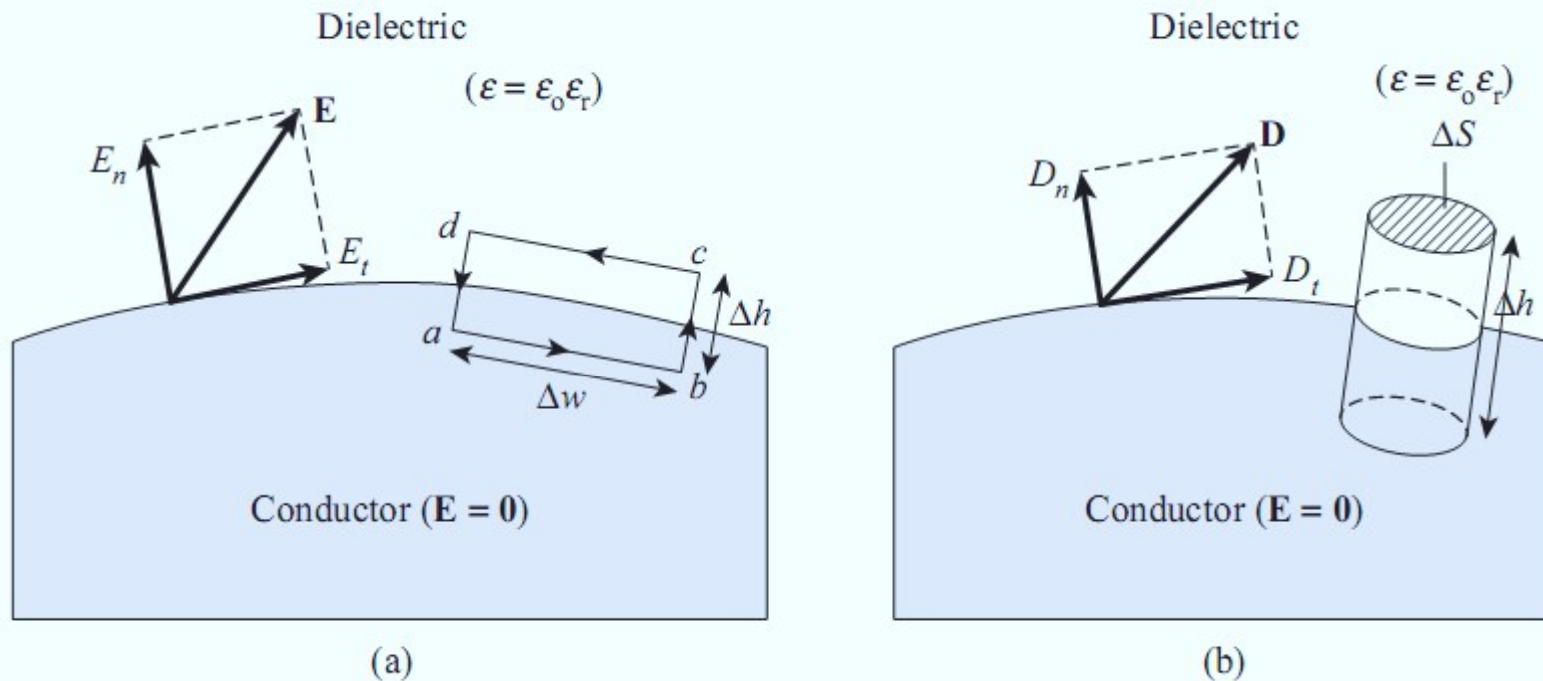


$$E_{1t} = E_{2t}$$

$$D_{1n} = D_{2n}$$

## Boundary Conditions (Cont'd.)

### Conductor-Dielectric



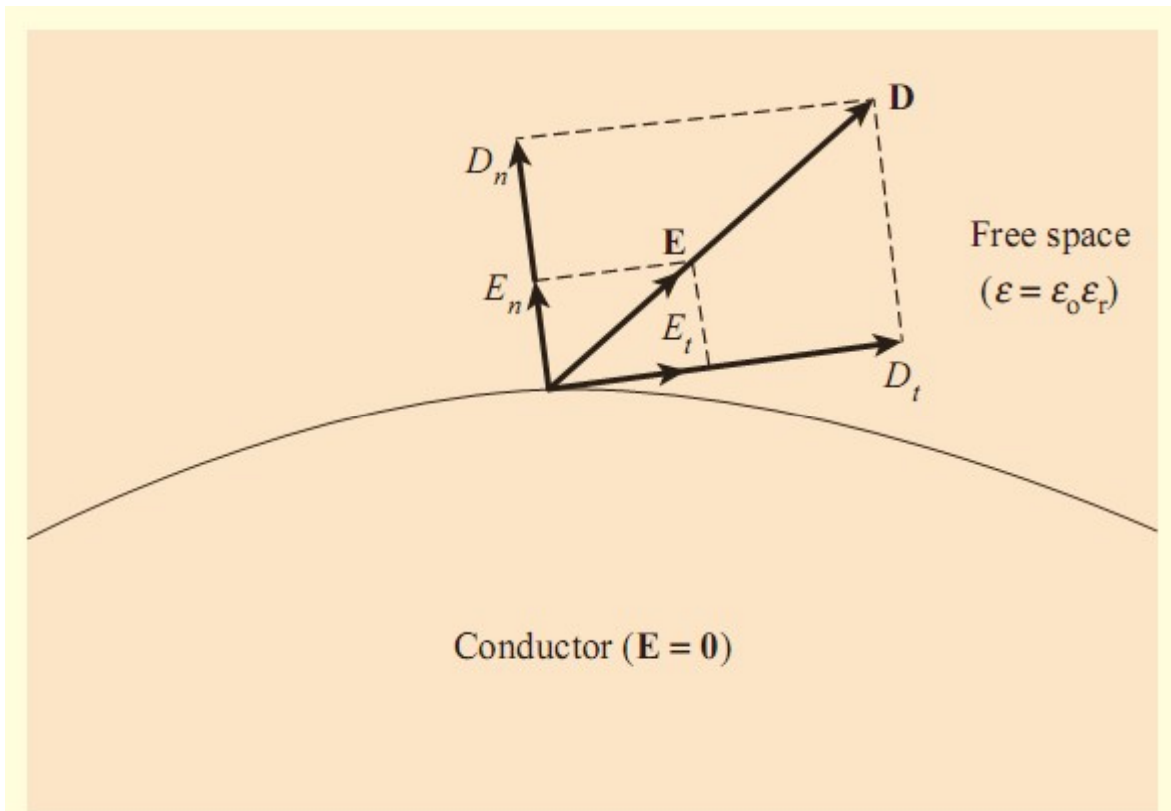
**Figure 4** Conductor–dielectric boundary.

$$\rho_v = 0, \quad \mathbf{E} = \mathbf{0}$$

$$D_t = \epsilon_0 \epsilon_r E_t = 0, \quad D_n = \epsilon_0 \epsilon_r E_n = \rho_S$$

## Boundary Conditions (Cont'd.)

### Conductor-Free Space

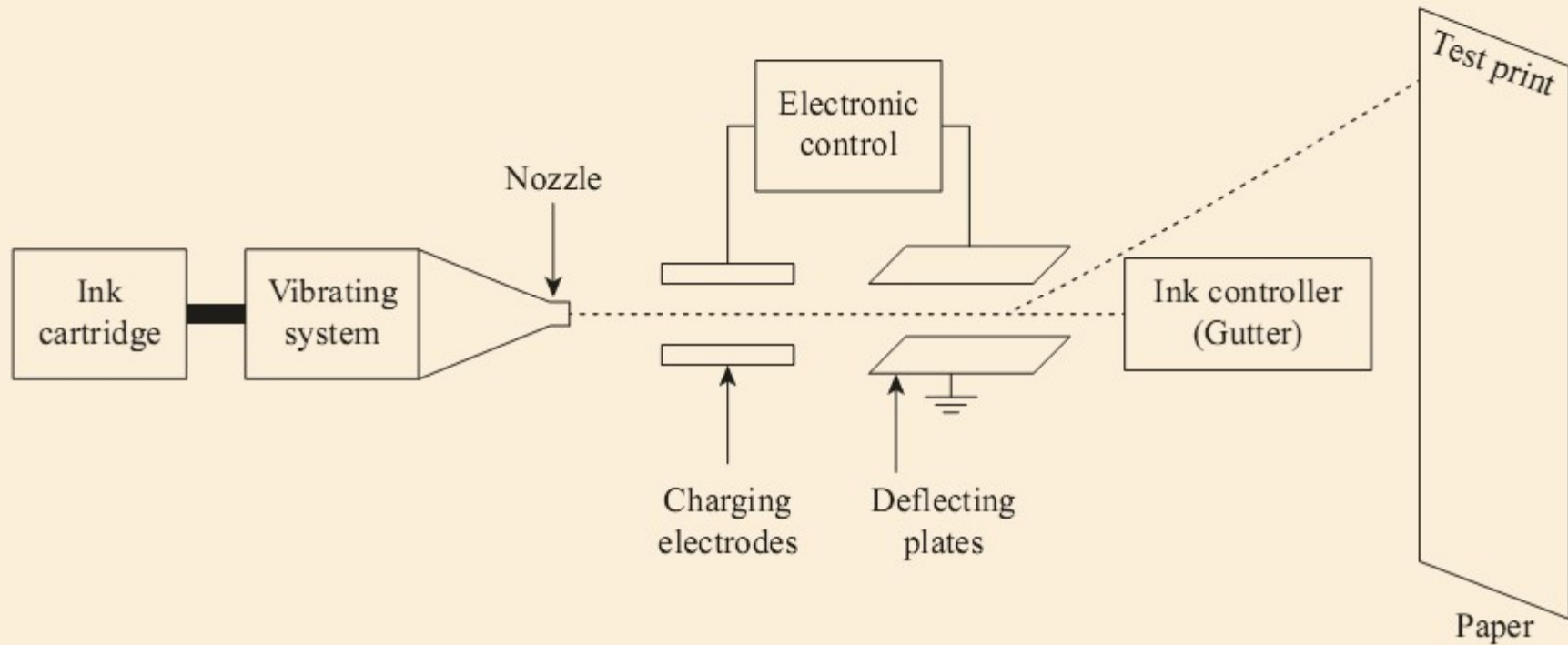


**Figure 5** Conductor-free space boundary.

$$D_t = \epsilon_0 E_t = 0, \quad D_n = \epsilon_0 E_n = \rho_S$$

## Application: Inkjet Printer

- Works on the principle of Coulomb's law.



**Figure 6** Inkjet printer