

# Ampere's Circuit Law, Magnetic Flux Density, Magnetic Scalar and Vector Potentials

by

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## Ampere's Circuit Law: Maxwell's Equation

**Ampère's circuit law** states that the line integral of  $\mathbf{H}$  around a *closed* path is the same as the net current  $I_{\text{enc}}$  enclosed by the path.

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}}$$

The above equation is always valid irrespective of whether current distribution is symmetrical or not

However, the equation is used to find  $H$  when a symmetrical current distribution exists

Ampere's law is a special case of Biot-Savart's law

## Magnetic Flux Density - Maxwell's Equation

The flux passing through an open surface:

$$\boxed{\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}}$$

The total (net) flux passing through a closed surface is always zero:

$$\boxed{\oint_S \mathbf{B} \cdot d\mathbf{S} = 0}$$

Applying the divergence theorem to the above equation, we obtain

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{B} dv = 0 \quad \longrightarrow \quad \boxed{\nabla \cdot \mathbf{B} = 0}$$

An isolated magnetic charge does not exist

## Magnetic Scalar and Vector Potentials

Magnetic scalar potential can be defined in current free regions.

$$\boxed{\mathbf{H} = -\nabla V_m} \quad \text{if } \mathbf{J} = \mathbf{0}$$

$$\mathbf{J} = \nabla \times \mathbf{H} = \nabla \times (-\nabla V_m) = \mathbf{0}$$

$$\nabla^2 V_m = 0, \quad (\mathbf{J} = \mathbf{0})$$

For using the magnetic scalar potential

- ❑ current should be zero throughout the region where the magnetic scalar potential is defined
- ❑ any arbitrary loop in the region should not enclose any current distribution

## Magnetic Scalar and Vector Potentials (Cont'd.)

$$\nabla \cdot \mathbf{B} = 0$$

Since the divergence of a curl is always zero, we can express  $\mathbf{B}$  as:

$$\mathbf{B} = \nabla \times \mathbf{A}$$

where the magnetic vector potential,  $\mathbf{A}$ , is:

$$\mathbf{A} = \int_L \frac{\mu_0 I d\mathbf{l}}{4\pi R} \quad \text{for line current}$$

$$\mathbf{A} = \int_s \frac{\mu_0 \mathbf{K} dS}{4\pi R} \quad \text{for surface current}$$

$$\mathbf{A} = \int_v \frac{\mu_0 \mathbf{J} dv}{4\pi R} \quad \text{for volume current}$$

## Magnetic Scalar and Vector Potentials (Cont'd.)

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_L \mathbf{A} \cdot d\mathbf{l}$$

$$\boxed{\Psi = \oint_L \mathbf{A} \cdot d\mathbf{l}}$$

The above formula is particularly useful for calculating flux passing through an area

# Analogy between Electric and Magnetic Fields

Table .1 Analogy between Electric and Magnetic Fields'

Term	Electric	Magnetic
Basic laws	$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \mathbf{a}_r$	$d\mathbf{B} = \frac{\mu_0 I d\mathbf{l} \times \mathbf{a}_r}{4\pi R^2}$
	$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{enc}$	$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}$
Force law	$\mathbf{F} = Q\mathbf{E}$	$\mathbf{F} = Q\mathbf{u} \times \mathbf{B}$
Source element	$dQ$	$dQu = Id\mathbf{l}$
Field intensity	$\mathbf{E} = \frac{V}{\ell} (\text{V/m})$	$\mathbf{H} = \frac{I}{\ell} (\text{A/m})$
Flux density	$\mathbf{D} = \frac{\Psi}{S} (\text{C/m}^2)$	$\mathbf{B} = \frac{\Psi}{S} (\text{Wb/m}^2)$
Relationship between fields	$\mathbf{D} = \epsilon \mathbf{E}$	$\mathbf{B} = \mu \mathbf{H}$
Potentials	$\mathbf{E} = -\nabla V$	$\mathbf{H} = -\nabla V_m (I = 0)$
	$V = \int \frac{\rho_L dl}{4\pi\epsilon R}$	$A = \int \frac{\mu Idl}{4\pi R}$
Flux	$\psi = \int \mathbf{D} \cdot d\mathbf{S}$	$\psi = \int \mathbf{B} \cdot d\mathbf{S}$
	$\psi = Q = CV$	$\psi = LI$
	$I = C \frac{dV}{dt}$	$V = L \frac{dI}{dt}$
Energy density	$w_E = \frac{1}{2} \mathbf{D} \cdot \mathbf{E}$	$w_m = \frac{1}{2} \mathbf{B} \cdot \mathbf{H}$
Poisson's equation	$\nabla^2 V = -\frac{\rho_v}{\epsilon}$	$\nabla^2 A = -\mu \mathbf{J}$