

Forces due to Magnetic Fields, Inductances and Magnetic Energy

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Forces Due to Magnetic Fields

There are three ways in which force due to magnetic fields can be experienced:

a. *Force on a charged particle*: The force on a stationary or moving charged particle in an electric field is :

$$\mathbf{F}_e = QE \quad (1)$$

A magnetic field can exert a force only on a moving charge and it is calculated by using eq. (2) which was originally derived from experimental results.

$$\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B} \quad (2)$$

The net force acting on a charged particle moving in presence of both electric and magnetic fields is:

$$\boxed{\mathbf{F} = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B})} \quad (3)$$

which is also called *Lorentz force*.

b. *Force on a current element:*

Analogy between a moving charge and a current carrying element is expressed as:

$$\boxed{I d\mathbf{l} = dQ \mathbf{u}} \quad (4)$$

The net force acting on the line current carrying element is given by using eqs. (2) and (4):

$$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B} \quad (5)$$

By integrating eq. (5), the net force acting on the entire current carrying body can be determined as

$$\boxed{\mathbf{F} = \oint_L I d\mathbf{l} \times \mathbf{B}} \quad (6)$$

The magnetic field of a current carrying element does not exert force on itself just like a charge does not exert force on itself.

Magnetic forces on surface or volume current carrying elements can be calculated by using eq. (6)

c. *Forces between two current elements*: Like any two charges exert force on each other, two current elements exert force on each other.

This is one the most important phenomena to understand since it helps in calculating intertwining forces in transformers, forces between contacts of circuit breakers, etc.

The force between two current carrying loops can be calculated by using the following equation:

$$\mathbf{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\mathbf{l}_1 \times (d\mathbf{l}_2 \times \mathbf{a}_{R_{21}})}{R_{21}^2} \quad (7)$$

Magnetic Torque and Moment

- An important concept for understanding operation of rotating machines (motors).
- For a small current carrying loop in Fig. 1, the net torque can be calculated as:

$$T = BIS \sin \alpha \quad (7)$$

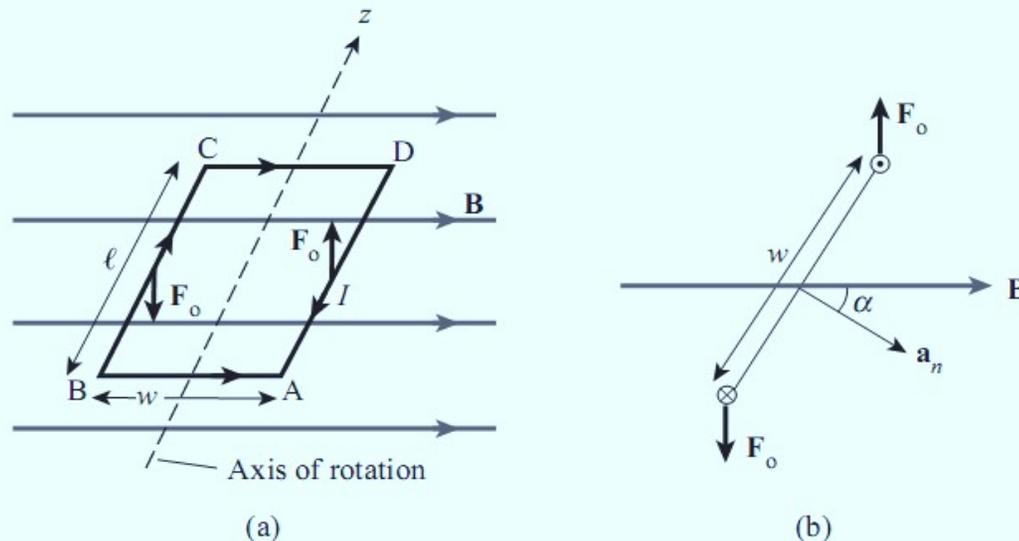


Figure 1 (a) Rectangular planar loop in a uniform magnetic field.
(b) Cross-sectional view of part (a).

Magnetic torque can also be defined as the cross product of the magnetic moment and the magnetic flux density,

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

Magnetic Dipole

A bar magnet or small filamentary current loop is referred to as a magnetic dipole.

The product of the current and the area of the loop is defined as the magnetic dipole moment:

$$\mathbf{m} = IS\mathbf{a}_n \quad (8)$$

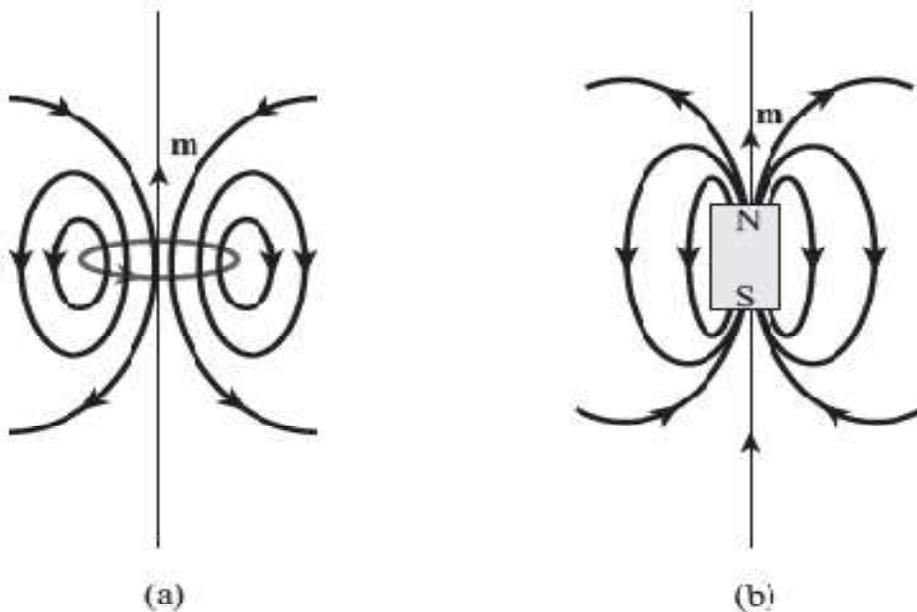


Figure 2 The \mathbf{B} lines due to magnetic dipoles: (a) a small current loop with $\mathbf{m} = IS$, (b) a bar magnet with $\mathbf{m} = Q_m \ell$.

Magnetization in Materials

- Magnetization in a material in a magnetic field is similar to polarization of a material in an electric field.
- Each material is made up of atoms and each atom has electrons rotating around nuclei.
- Rotating electrons are equivalent to a current carrying loop which produces a magnetic field and acts as a magnetic dipole.
- In absence of any external magnetic field, the fields generated by electrons will tend to cancel each other resulting in magnetic neutrality.
- The net magnetic field inside a magnetic material in a magnetic field is given by:

$$\boxed{\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})} \quad (9)$$

- This equation is applicable for all materials
- For linear magnetic materials, \mathbf{M} and \mathbf{H} are related as:

$$\boxed{\mathbf{M} = \chi_m \mathbf{H}} \quad (10)$$

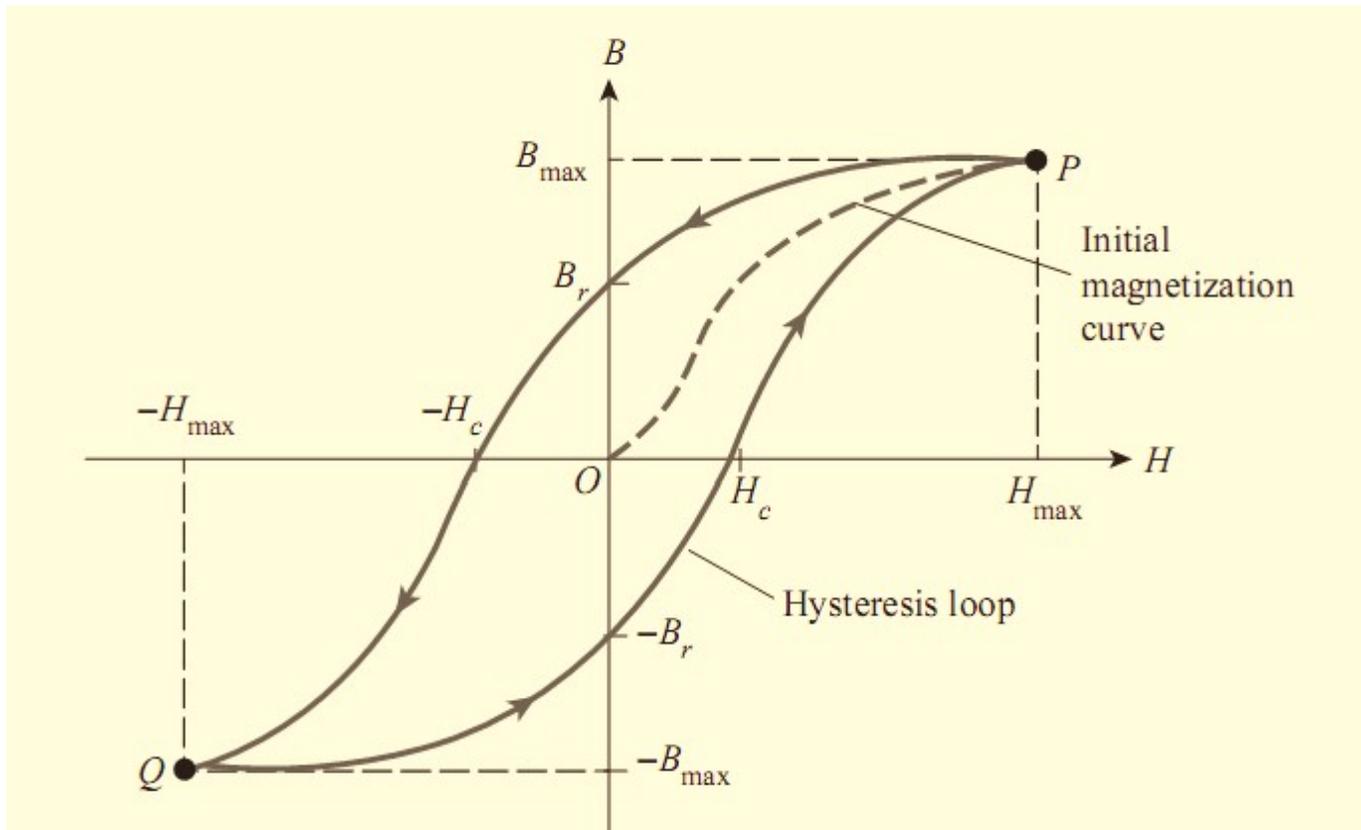
and thus, the net magnetic permeability in a linear magnetic material can be determined by :

$$\boxed{\mu_r = 1 + \chi_m = \frac{\mu}{\mu_0}}$$

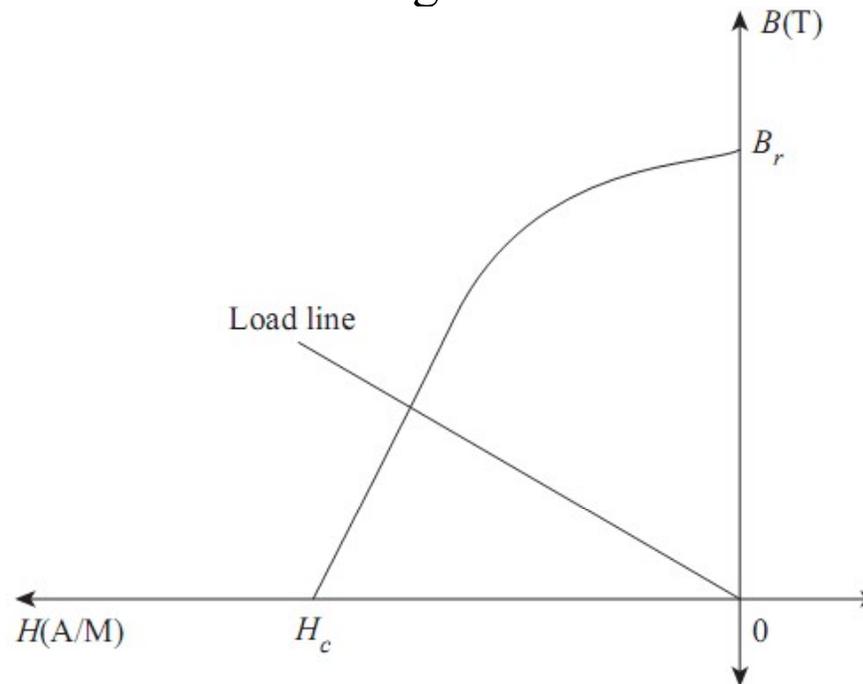
Classification of Materials:

- *Diamagnetic materials*: Magnetic susceptibility of these materials is less than zero and their permeability is slightly less 1
- *Paramagnetic materials*: Permeability is slightly greater than 1
- *Ferromagnetic materials*: These have strong magnetization capability, they exhibit hysteresis behaviour, they retain magnetization when the external field is removed, and they lose ferromagnetism above the Curie temperature

A hysteresis loop of a typical ferromagnetic material is shown in Fig. 3:



Permanent magnets: The BH loop of a permanent magnet is wider as shown in Fig. 4 with a high coercive field and a high residual flux density:



- The remanence (B_r) field is defined as the maximum flux density generated by the permanent magnet without any excitation.
- The coercive field is the magnetic field (H_c) required to reduce the magnetic flux density inside the magnet to zero.

Magnetic Boundary Conditions

- Boundary conditions between two magnetic media as shown in Fig. 5 can be derived by using Gauss's law of magnetic fields and Ampère's circuit law

$$\boxed{\mathbf{H}_{1t} = \mathbf{H}_{2t}} \quad \text{or} \quad \frac{\mathbf{B}_{1t}}{\mu_1} = \frac{\mathbf{B}_{2t}}{\mu_2}$$

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2}$$

$$B_1 \cos \theta_1 = B_{1n} = B_{2n} = B_2 \cos \theta_2$$

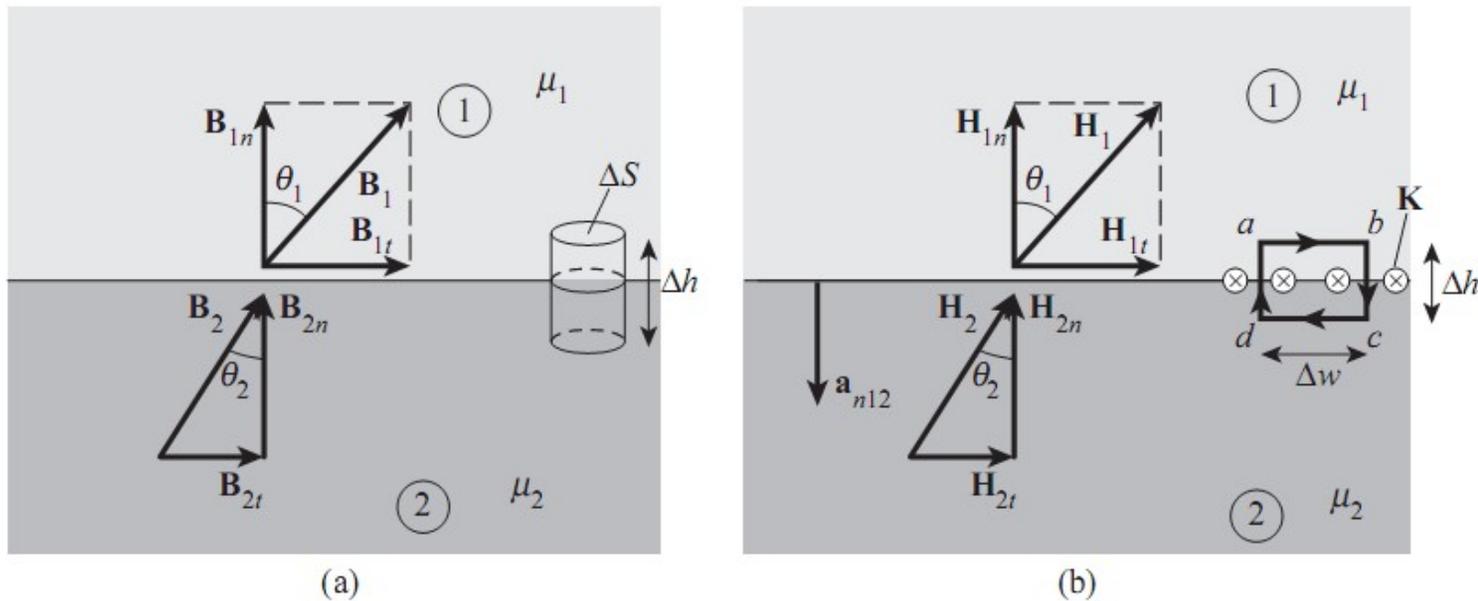


Figure 5 Boundary conditions between two magnetic media: (a) for \mathbf{B} , (b) for \mathbf{H} .

Inductors and Inductances

- **Inductance:** flux linkages per unit current as given in eq. (11):

$$L = \frac{\lambda}{I} = \frac{N\Psi}{I}$$

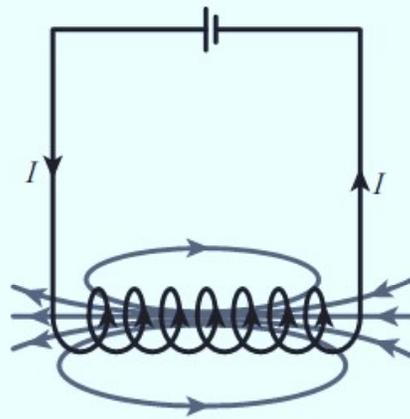


Figure 6 Magnetic field **B** produced by a circuit.

- **Mutual inductance:** flux linkages of coil 2 per unit current in coil 1 as defined in eq. (12):

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2\Psi_{21}}{I_1}$$

Magnetic Energy

- Magnetic energy associated with an inductor is given by

$$W_m = \frac{1}{2} LI^2 \quad (13)$$

- However, it is difficult to determine inductances of complicated structures.
- A simpler approach is:

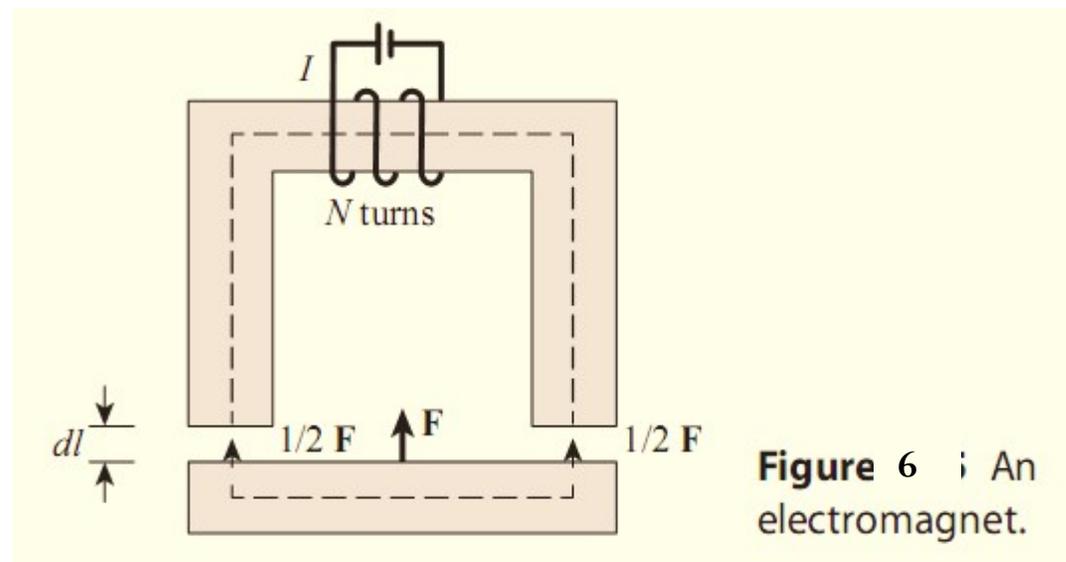
$$W_m = \frac{1}{2} \int \mathbf{B} \cdot \mathbf{H} dv = \frac{1}{2} \int \mu H^2 dv \quad (14)$$

Magnetic Circuits

- Analysis of a magnetic device is easier if we convert it into a magnetic circuit and the analogy between electric and magnetic circuits is exploited.
- Unlike electric circuits, magnetic circuits have the following properties:
 1. The current flows in an electric circuit, the magnetic flux does not flow in a magnetic circuit
 2. Conductivity is independent of J in electric circuits, whereas permeability depends on B in magnetic circuits with ferromagnetic materials

Force on Magnetic Materials

- Applications: relays, magnetic levitation



- Force across a single air gap in the above figure is given by

$$F = -\frac{B^2 S}{2\mu_0} \quad (15)$$