



Bayes' Theorem

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Bayesian Decision Theory

- Applications of the results of probability theory involves estimating unknown probability
- Making decisions on the basis of new (sample) information
- Bayesian Decision Theory- The purpose is the solution of problems involving decision-making under uncertainty

Bayesian Approach

- Conditional probability takes into account information about the occurrence of one event to predict the probability of another event
- Can be extended to 'revise' probabilities based on new information and to determine the probability that a particular effect was due to specific cause
- Procedures of revising the probabilities-Bayes' Theorem

Bayesian Approach

- British Mathematician Rev. Thomas Bayes
- A unique method for calculating conditional probabilities
- Example-
 - *Use the results of the first examination in statistics (event A) as sample evidence in estimating the probability of getting first class (event B)*
 - *Sample output of 2 defectives in 50 trials (event A) might be used to estimate the probability that a machine is not working correctly (event B)*

Bayesian Approach

- Bayes' theorem is based on the formula for conditional probability
- A_1 and A_2 = The set of events which are mutually exclusive and exhaustive (the combination of two events is the entire experiment)
- $B = A$ a simple event which intersects each of the A events

Bayesian Approach

- The part of B which is within A_1 represents the area 'A₁ and B' and the part of B within A_2 represents the area 'A₂ and B'
- Then the probability of event A_1 given event B is
- $P(A_1/B) = \frac{P(A_1 \text{ and } B)}{P(B)}$
- The probability of event A_2 given B is
- $P(A_2/B) = \frac{P(A_2 \text{ and } B)}{P(B)}$

Bayesian Approach

- Where
- $P(B) = P(A_1 \text{ and } B) + P(A_2 \text{ and } B)$
- $P(A_1 \text{ and } B) = P(A_1) \times P(B/A_1)$
- $P(A_2 \text{ and } B) = P(A_2) \times P(B/A_2)$
- Let $A_1, A_2, A_3 \dots, A_i, \dots, A_n$ be a set of n mutually exclusive and collectively exhaustive events.

Bayesian Approach

- If B is another event such that P(B) is not zero, then
- $$P(A_1/B) = \frac{P(B/A_1)P(A_1)}{\sum_{i=1}^K P(B/A_i)P(A_i)}$$
- Probability before revision by Bayes' rule- a ***priori*** or simply prior probability
- A probability which has undergone revision in the light of sample information (*via* Bayes' rule)-a ***posterior*** probability

Bayesian Approach

- Posterior probabilities- revised probabilities
- By revising the prior probabilities in the light of the additional information gained
- Are always conditional probabilities- the conditional event being the sample information
- A *priori* probability which is unconditional probability becomes a *posterior* probability

Bayesian Approach-Example

- Assume that a factory has two machines. Past records show that machine 1 produces 30% of the items of output and machine 2 produces 70% of the items. Further 5% of the items produced by machine 1 were defective and only 1% produced by machine 2 were defective. If a defective item is drawn at random, what is the probability that the defective item was produced by machine 1 or machine 2.

Solution

- A_1 = the event of drawing an item produced by machine 1
- A_2 = the event of drawing an item produced by machine 2
- B = the event of drawing a defective item produced either by machine 1 or machine 2
- $P(A_1) = 30\% = 0.30$
- $P(A_2) = 70\% = 0.70$
- $P(B/A_1) = 5\% = 0.05$
- $P(B/A_2) = 1\% = 0.01$
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Solution

(1) Events	(2) Prior probability ($P(A_1)$)	(3) Conditional Probability event B given event A ($P(B/A_i)$)	(4) Joint probability A (A_1 and B) (2) × (3)	(5) Posterior (revised) probability $P(A_1/B)$ (4) ÷ P(B)
A_1	0.30	0.05	0.015	$0.015/0.022$ $=0.682$ $0.007/0.022=$ 0.318
A_2	0.70	0.01	0.007	
Total	100		$P(B)=0.022$	1.000