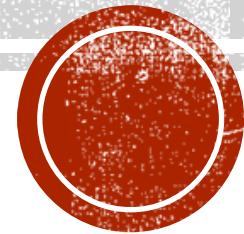


# **BINOMIAL DISTRIBUTION**

Dr Bibhunandini Das



# BINOMIAL DISTRIBUTION

- Binomial distribution- Bernoulli Distribution
- Associated with the name of James Bernoulli
- Binomial distribution- the probability of one set of dichotomous alternatives – success or failure
- The type of process which gives rise to this distribution – Bernoulli trial/ Bernoulli process
  - Wherein an experiment is performed repeatedly, yielding either a success or a failure in each trial and where there is absolutely no pattern in the occurrence of successes or failures



# ASSUMPTIONS OF BERNOULLI PROCESS

- An experiment is performed under the same conditions for a fixed number of trials, say 'n'
- There are only two possible outcomes of the experiment- success and failure. The sample space of possible outcomes on each experimental trial is
  - $S = \{\text{success, failure}\}$
- The probability of a success denoted by  $P$  remains constant from trial to trial. The probability of a failure denoted by  $q$  is equal to  $(1 - P)$ . If the probability of success is not the same in each trial, that will not be binomial distribution



# ASSUMPTIONS OF BERNOULLI PROCESS

- The trials are statistically independent- the outcomes of any trial or sequence of trials do not affect the outcomes of subsequent trials
  - Example- If an experiment is conducted  $n$  times under the stated conditions, what is the probability of obtaining exactly  $x$  successes? Suppose 10 coins are tossed together, what is the probability of obtaining exactly two heads?



# BINOMIAL DISTRIBUTION WITH SOLUTION

- If a coin is tossed once- two outcomes: head or tail
- The probability of obtaining a head or  $p = \frac{1}{2}$
- The probability of obtaining a tail or  $q = \frac{1}{2}$
- Thus  $(q + p) = 1$  and these are terms of the binomial  $(q + p)$



# BINOMIAL DISTRIBUTION WITH SOLUTION

- If two coins are tossed simultaneously- four possible outcomes are there- TT, TH, HT and HH
  - TT-  $qq$  ( $q^2$ )
  - TH-  $qp$
  - HT-  $pq$
  - HH-  $pp$  ( $p^2$ )
- These are terms of the binomial  $(q + P)^2$  because
  - $(q + p)^2 = q^2 + 2qp + p^2$



# BINOMIAL DISTRIBUTION WITH SOLUTION

- In a special case where  $p = q = \frac{1}{2}$  , we have

- $\left(\frac{1}{2} + \frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{2}$

- If three coins are tossed –

- TTT-  $q^3$
- TTH-  $q^2p$
- THT-  $q^2p$
- HTT-  $q^2p$
- THH-  $qp^2$
- HTH -  $qp^2$
- HHT -  $qp^2$
- HHH -  $p^3$

- $(q + p)^3 = q^3 + 3q^2p + 3qp^2 + p^3$



# BINOMIAL DISTRIBUTION

- By expanding the binomial  $(q + p)^n$ , we obtain probability of 0, 1, 2....n heads – the probability distribution is *Binomial Probability Distribution/the Binomial Distribution*
- The Binomial Distribution
  - $P(r) = {}^n C_r q^{n-r} p^r$
- Where  $p$  = Probability of success in a single trial
  - $q = 1 - p$
  - $n$  = Number of trials
  - $r$  = Number of success in  $n$  trials





# OBTAINING BINOMIAL COEFFICIENTS

- The first term is  $q^n$
- The second term is  ${}^n\text{C}_r q^{n-1}p$
- In each succeeding term the power of  $q$  is reduced by 1 and the power of  $p$  is increased by 1
- The coefficient of any term is found by multiplying the coefficient of the preceding term by the power of  $q$  in that preceding term, and dividing the products so obtained by one more than the power of  $p$  in that preceding term



# OBTAINING BINOMIAL COEFFICIENTS

- In expanding  $(q + p)^n$ , we get
- $(q + p)^n = q^n + {}^n\mathbf{C}_1 q^{n-1} p + {}^n\mathbf{C}_2 q^{n-2} p^2 + \dots + {}^n\mathbf{C}_r q^{n-r} p^r + \dots p^n$



# GENERAL RELATIONSHIP

- The number of terms in a binomial expansion is always  $n+1$
- The exponents of  $p$  and  $q$  for any single term, when added together always sum to  $n$
- The exponents of  $q$  are  $n, (n-1), (n-2) \dots 1, 0$ , respectively and the exponents of  $p$  are  $0, 1, 2 \dots (n-1), n$  respectively
- The coefficients for the  $n+1$  terms of the distribution are always symmetrical ascending to the middle of the series and then descending, when  $n$  is odd number,  $n+1$

